Fekete -Szego Results for Some Classes of Analytic Functions using Q-Difference Error Function Related to Shell Shaped Region

K. Saritha[1] and K.Thilagavathi[2*]

School of Advanced Sciences, Vellore Institute of technology, Vellore - 632 014, Tamil Nadu, India.
e-mail: saritha.k@vit.ac.in[1], kthilagavathi@vit.ac.in[2]
** corresponding author.*

Article Info
Page Number: 1565 – 1573
Publication Issue:
Vol. 71 No. 3s (2022)

Article History
Article Received: *22 April 2022*
Revised: *10 May 2022*
Accepted: *15 June 2022*
Publication: *19 July 2022*

Abstract

The main object of this article is to obtain the coefficient estimate using q-difference error function related to shell shaped region.Also we estimate $\left|b_3 - \mu b_2^2\right|$ with the function $h \in \mathcal{E}S^{\lambda}_{q,\ell}(\psi(\zeta))$ and $h \in \mathcal{EC}^{\lambda}_{q,\ell}(\psi(\zeta))$ using subordination and quasi subordination.

Keywords: q-difference operator,error function, Fekete - Szegö functional, shell shaped region, subordination and quasi subordination.

1 Introduction and Definitions

Let $\mathcal{Q}$ denote the holomorphic function in the open unit disc $\mathcal{U}$ is defined by

$$h(\zeta) = \zeta + \sum_{k=2}^{\infty} b_k \zeta^k \qquad \zeta \in \mathcal{U}. \tag{1}$$

For $g \in Q$ given by,

$$g(\zeta) = \zeta + \sum_{k=2}^{\infty} c_k \zeta^k \qquad \zeta \in \mathcal{U}. \tag{2}$$

The Hadamard product of these two functions is given by

$$(h * g)(\zeta) = \zeta + \sum_{k=2}^{\infty} b_k c_k \zeta^k \qquad \zeta \in \mathcal{U}. \tag{3}$$

For two holomorphic functions $h, g \in \mathcal{Q}$, we state that h is subordinate to g which is written as $h \prec g$ (see [5]) if there is a Schwarz function ω which is holomorphic in $\mathcal{U}$ with

$\omega(0) = 0$ and $|\omega(\zeta)| < 1$ for all $\zeta \in \mathcal{U}$, such that $h(\zeta) = g(\omega(\zeta))$, for $\zeta \in \mathcal{U}$. Moreover, if the function g is univalent in U, we have

$$h \prec g \quad \Leftrightarrow \quad h(0) \;=\; g(0) \qquad and \quad h(\mathcal{U}) \subset g(\mathcal{U}).$$

If h and g are holomorphic functions in U, following T.H. MacGregor [4], (also see [6]) we say that h is majorized by g in $\mathcal{U}$ and we write

$$h(\zeta) << g(\zeta), \qquad\qquad (\zeta \in \mathcal{U}) \tag{4}$$

If there is a function ϕ, holomorphic in $\mathcal{U}$, such that

$$|\phi(\zeta)| < 1 \quad and \quad h(\zeta) = \phi(\zeta)\; g(\zeta), \qquad (\zeta \in \mathcal{U}) \tag{5}$$

Here majorization (4) is closely associated with the concept of quasi-Subordination between the holomorphic functions.

Definition 1. If $h \in \mathcal{Q}$ satisfies

$$Re\left\{\frac{\zeta\; h'(\zeta)}{h(\zeta)}\right\} > 0,$$

which is referred to as the starlike function in $\mathcal{U}$

Definition 2. If $h \in \mathcal{Q}$ satisfies

$$Re\left\{1 + \frac{\zeta\; h''(\zeta)}{h'(\zeta)}\right\} > 0,$$

which is referred to as the convex function in $\mathcal{U}$

For $0 < q < 1$ the Jackson's q-derivative of a function $h \in \mathcal{Q}$ is, by definition, given as follows [1],[2]

$$\mathcal{D}_q h(\zeta) = \begin{cases} \dfrac{h(\zeta) - h(q\zeta)}{(1-q)\zeta} & \zeta \neq 0 \\ h'(0) & \zeta = 0 \end{cases} \tag{6}$$

and

$$\mathcal{D}_q^2 h(\zeta) = \mathcal{D}_q(\mathcal{D}_q h(\zeta)).$$

From (6), we have

$$\mathcal{D}_q h(\zeta) = 1 + \sum_{k=2}^{\infty} [n]_q b_k\; \zeta^{k-1}$$

where

$$[n]_q = \frac{1-q^n}{1-q},$$

is also known as the fundamental number n. If $q \to 1^-$ then $[n]_q \to n$.

Sokol and Paprocki[7] was introduced the shell shaped domain.Recently, Raina and Sokol [9] was found the coefficient inequalities for $\psi(\zeta) = \zeta + \sqrt{1+\zeta^2}$ of starlike function. if it satisfies the subordination condition given below,

$$\frac{zh'(\zeta)}{h(\zeta)} \prec \psi(\zeta)$$

Sokol and Thomas [11] developed on these findings.

Definition 3. [8] The error function $er\ h$ is defined by

$$er\ h(\zeta) = \frac{2}{\sqrt{\pi}} \int_0^\zeta e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k\ \zeta^{2k+1}}{(2k+1)\ k!}. \tag{7}$$

Definition 4. [8] Let $E_r\ h(\zeta)$ denote the holomorphic function it is derived from (7) and it is given below

$$E_r\ h(\zeta) = \frac{\sqrt{\pi\zeta}}{2}\ er\ h(\zeta) = \zeta + \sum_{k=2}^{\infty} \frac{(-1)^{k-1}}{(2k-1)\ (k-1)!} \zeta^k. \tag{8}$$

The family of an holomorphic function as follows

$$\mathcal{E} = Q * E_r f = \{F : F(\zeta) = (h * E_r f)(\zeta) = z + \sum_{k=2}^{\infty} \frac{(-1)^{k-1} b_k}{(2k-1)(k-1)!} \zeta^k, h \in Q\}. \tag{9}$$

Thus the consequence of (6),(9)for $F \in \mathcal{E}$ we obtain that

$$\mathcal{D}_q\ F(\zeta) = 1 + \sum_{k=2}^{\infty} \frac{(-1)^{k-1} [n]_q b_k}{(2k-1)(k-1)!} \zeta^{k-1}.$$

In the following section, we apply q-operators to the functions associated with the conic sections introduced by Kanas [3].

Definition 5. [8] A function $h \in \mathcal{Q}$ is in the class $\mathcal{E}S^{\lambda}_{q,\ell}(\psi(\zeta))$ if

$$1 + \frac{1}{\ell} \left[(1 + i\ \tan\lambda) \left(\frac{z \mathcal{D}_q h(\zeta)}{h(\zeta)} \right) - i\ \tan\lambda - 1 \right] \prec \psi(\zeta)$$

for $\zeta \in \mathcal{U}$.

Definition 6. [8] A function $h \in \mathcal{Q}$ is in the class $\mathcal{E}\mathcal{C}^{\lambda}_{q,\ell}\psi(\zeta)$ if

$$1 + \frac{1}{\ell} \left[(1 + i\ \tan\lambda) \left(\frac{[\zeta \mathcal{D}_q h(\zeta)]'}{\zeta D_q h(\zeta)} \right) - i\ \tan\lambda - 1 \right] \prec \psi(\zeta)$$

for $\zeta \in \mathcal{U}$.

Let $\phi(\zeta) = 1 + p_1\zeta + p_2\zeta^2 + ...$, $(p_1 > 0)$ be an holomorphic function in the open unit disk $\mathcal{U}$ onto a region starlike with respect to 1 and symmetric with respect to the real axis. Let

$$\varphi(\zeta) = d_0 + d_1\zeta + d_2\zeta^2 + ...$$

and $|d_n| \leq 1$.

Definition 7. [8] A function $h \in \mathcal{Q}$ is in the class $\mathcal{E}S^{\lambda}_{q,\ell}(\phi(\zeta))$, which satisfying the quasi-subordination

$$\frac{1}{\ell} \left[(1 + i\ \tan\lambda) \left(\frac{\zeta \mathcal{D}_q h(\zeta)}{h(\zeta)} \right) - i\ \tan\lambda - 1 \right] \prec_q \phi(\zeta) - 1$$

for $\zeta \in \mathcal{U}$.

Definition 8. [8] A function $h \in \mathcal{Q}$ is in the class $\mathcal{E}C^{\lambda}_{q,\ell}(\phi(\zeta))$ which satisfying the quasi-subordination

$$\frac{1}{\ell}\left[(1+i\ \tan\lambda)\left(\frac{[\zeta\mathcal{D}_q h(\zeta)]'}{\zeta\mathcal{D}_q h(\zeta)}\right)-i\ \tan\lambda-1\right]\prec_q \phi(\zeta)-1$$

for $\zeta \in \mathcal{U}$.

In this paper, we determine the coefficient estimate using q-derivative operator for some classes of function related to shell shaped region and also we establish $\left|b_3-\mu b_2^2\right|$ using subordination and quasi subordination for the function $h \in \mathcal{E}S^{\lambda}_{q,\ell}(\psi(\zeta))$ and $h \in \mathcal{E}\mathcal{C}^{\lambda}_{q,\ell}(\psi(\zeta))$

Here using lemma to prove our main result.

Lemma 9. *[8] If $\delta \in \mathcal{Q}$ then*

$$\left|\delta_2-\eta\delta_1^2\right|\le \max\{1,|\eta|\},$$

Here η is a complex number. The result is sharp for $\delta(\zeta)=\zeta$ or $\delta(\zeta)=\zeta^2$.

2 The Fekete-Szegö Functional Associated with the Shell shaped Region using subordination

First, we prove the result for the function $h \in \mathcal{E}S^{\lambda}_{q,\ell}(\psi(\zeta))$, which satisfies the equation (9).

Theorem 10. *Let $\frac{-\pi}{2}<\lambda<\frac{\pi}{2}$, $0<q<1$ and $\ell \neq 0$. If $h \in \mathcal{E}S^{\lambda}_{q,\ell}(\psi(\zeta))$ satisfies the equation (9) such that*

$$\psi(w(\zeta))=w(\zeta)+\sqrt{1+w^2},$$

where

$$w(\zeta)=\frac{\phi(\zeta)-1}{\phi(\zeta)+1}$$

and

$$\phi(\zeta)=1+p_1\zeta+p_2\zeta^2+...$$

for all $\zeta \in \mathcal{U}$. Then,

$$\left|b_3-\mu b_2^2\right|\le\frac{5|\ell|}{V_3|\vartheta|}\ \max\left\{1,\left|\frac{1}{4}+\left(\frac{9\mu V_3-10V_2}{20\vartheta V_2^2}\right)\ell\right|\right\},\tag{10}$$

where $\vartheta=1+i\ \tan\lambda$, $V_2=[2]_q-1$ and $V_3=[3]_q-1$ and μ is a complex number.

Proof. If $h \in \mathcal{E}S^{\lambda}_{q,\ell}(\psi(\zeta))$, then we have the Schwarz function as a result,

$$1+\frac{1}{\ell}\left[(1+i\ \tan\lambda)\left(\frac{\zeta\mathcal{D}_q h(\zeta)}{h(\zeta)}\right)-1-i\ \tan\lambda\right]=\psi(w(\zeta)).\tag{11}$$

We note that

$$\frac{\zeta\mathcal{D}_q h(\zeta)}{h(\zeta)}=1+\frac{1-[2]_q}{3}b_2\zeta+\left(\frac{[3]_q-1}{10}b_3+\frac{1-[2]_q}{9}b_2^2\right)\zeta^2+...\tag{12}$$

and

$$\psi(w(\zeta)) = 1 + \frac{P_1}{2}\zeta + \left(\frac{P_2}{2} - \frac{P_1^2}{8}\right)\zeta^2 + \left(\frac{P_3}{2} - \frac{P_1P_2}{4}\right)\zeta^3 + ... \tag{13}$$

Applying (11), (12) and (13), we obtain that

$$b_2 = \frac{3\ell P_1}{2(1-[2]_q)\ (1+i\ \tan\lambda)} \tag{14}$$

and

$$b_3 = \frac{5\ell}{([3]_q-1)\ (1+i\ \tan\lambda)}\left[P_2 - \left(\frac{1}{4} + \frac{\ell}{2(1-[2]_q)\ (1+i\ \tan\lambda)}\right)P_1^2\right]. \tag{15}$$

Hence by (14) and (15), we get the following

$$\begin{aligned} b_3 - \mu b_2^2 &= \frac{5\ell}{([3]_q-1)\ (1+i\ \tan\lambda)}\left[P_2 - \left(\frac{1}{4} + \frac{\ell}{2(1-[2]_q)\ (1+i\ \tan\lambda)}\right)P_1^2\right] \\ &\quad - \mu\left[\frac{3\ell P_1}{2(1-[2]_q)\ (1+i\ \tan\lambda)}\right]^2 \\ &= \frac{5\ell}{([3]_q-1)\ (1+i\ \tan\lambda)}\times \\ &\quad \left\{P_2 - \left[\frac{1}{4} + \left(\frac{1}{2(1-[2]_q)\ (1+i\ \tan\lambda)} + \frac{9\mu([3]_q-1)}{20(1-[2]_q)^2\ (1+i\ \tan\lambda)}\right)\ell P_1^2\right]\right\} \\ &= \frac{5\ell}{([3]_q-1)\ (1+i\ \tan\lambda)}\left\{P_2 - \left[\frac{1}{4} + \left(\frac{10(1-[2]_q) + 9\mu([3]_q-1)}{20(1-[2]_q)^2\ (1+i\ \tan\lambda)}\right)\ell P_1^2\right]\right\}. \end{aligned}$$

Thus the above can be reduces as

$$b_3 - \mu b_2^2 = \frac{5\ell}{([3]_q-1)\ (1+i\ \tan\lambda)}(P_2 - tP_1^2),$$

where

$$t = \frac{1}{4} + \left(\frac{10(1-[2]_q) + 9\mu([3]_q-1)}{20(1+[2]_q)^2\ (1+i\ \tan\lambda)}\right)\ell = \left[\frac{1}{4} + \left(\frac{-10V_2 + 9\mu V_3}{20\vartheta V_2^2}\right)\ell\right].$$

Hence proved. □

Next, we establish the result for the function $h \in \mathcal{EC}_{q,\ell}^{\lambda}(\psi(\zeta))$, which satisfies the equation (9). The following theorem demonstrated in the same manner.

Theorem 11. *Let* $\frac{-\pi}{2} < \beta < \frac{\pi}{2}$, $0 < q < 1$ *and* $\ell \neq 0$. *If* $h \in \mathcal{EC}_{q,\ell}^{\lambda}(\psi(\zeta))$ *satisfying* (9) *such that*

$$\psi(w(\zeta)) = w(\zeta) + \sqrt{1+w^2},$$

where

$$w(\zeta) = \frac{\phi(\zeta)-1}{\phi(\zeta)+1}$$

and

$$\phi(\zeta) = 1 + p_1\zeta + p_2\zeta^2 + ...$$

for all $\zeta \in \mathcal{U}$. *Then,*

$$\left|b_3 - \mu b_2^2\right| \leq \frac{5|\ell|}{2[3]_q|\vartheta|}\ \max\left\{1, \left|\frac{1}{4} + \left(\frac{9\mu[3]_q - 5[2]_q^2}{10[2]_q^2\vartheta}\right)\ell\right|\right\}, \tag{16}$$

where $\vartheta = 1 + i\ \tan\beta$ *and* μ *is a complex number.*

Proof. If $h \in \mathcal{E}S^{\lambda}_{q,\ell}(\psi(\zeta))$ then we have the Schwarz function as a result,

$$1+\frac{1}{\ell}\left[(1+i\ \tan\lambda)\left(\frac{\zeta\mathcal{D}_q h(\zeta)}{h(\zeta)}\right)-1-i\ \tan\lambda\right]=\psi(w(\zeta)). \tag{17}$$

We note that

$$\frac{\zeta\mathcal{D}_q h(\zeta)}{h(\zeta)}=1+\frac{1-[2]_q}{3}b_2\zeta+\left(\frac{[3]_q-1}{10}b_3+\frac{1-[2]_q}{9}b_2^2\right)\zeta^2+... \tag{18}$$

and

$$\psi(w(\zeta))=1+\frac{P_1}{2}\zeta+\left(\frac{P_2}{2}-\frac{P_1^2}{8}\right)\zeta^2+\left(\frac{P_3}{2}-\frac{P_1P_2}{4}\right)\zeta^3+.... \tag{19}$$

Similarly, we will obtain $\left(\frac{[\zeta\mathcal{D}_q h(\zeta)]'}{\zeta\mathcal{D}_q h(\zeta)}\right)$ and applying the equations (17) and (19), we can easily find the value of b_2 and b_3. Then, by using the same approach as applied in the above Theorem 10, one can have

$$b_3-\mu b_2^2=\frac{5\ell}{2[3]_q(1+i\ \tan\lambda)}(P_2-tP_1^2),$$

where

$$t=\frac{1}{4}+\left(\frac{9\mu[3]_q-5[2]_q^2}{10[2]_q^2\vartheta}\right)\ell.$$

Hence proved. □

3 The Fekete-Szegö Functional Associated with the Shell shaped Region using quasi subordination

The quasi-subordination established by Robertson in [10] is an extension of the concept of subordination.

In this section, we are going to establish the result for the function using quasi-subordination .

Theorem 12. *Let* $\frac{-\pi}{2}<\lambda<\frac{\pi}{2}$, $0<q<1$ *and* $\ell\neq 0$. *If* $h\in\widetilde{\mathcal{E}S}^{\lambda}_{q,\ell}(\psi(\zeta))$ *satisfies the equation* (9) *and consider*

$$\psi(w(\zeta))=w(\zeta)+\sqrt{1+w^2},$$

where

$$w(\zeta)=\frac{\phi(\zeta)-1}{\phi(\zeta)+1}$$

for all $\zeta\in\mathcal{U}$. *Then,*

$$\left|b_3-\mu b_2^2\right|\leq\frac{5|\ell|}{V_3|\vartheta|}\ \max\left\{1,\left|d_0+\left(\frac{1}{4}-\frac{9\mu\ell d_0V_3}{10V_2^2\ \vartheta}\right)d_0\right|\right\}, \tag{20}$$

and

$$|b_2| \leq \frac{3|\ell| d_0}{2V_2\ |\vartheta|}$$
$$|b_3| \leq \frac{5|\ell|}{V_3\ |\vartheta|}\left[|d_1| + \max\left\{d_0, \left|d_0 + \frac{d_0^2}{4}\right|\right\}\right]$$

where $\vartheta = 1 + i\ \tan\lambda$, $V_2 = [2]_q - 1$ *and* $V_3 = [3]_q - 1$ *and* μ *is a complex number.*

Proof. If $h \in \mathcal{ES}^{\lambda}_{q,\ell}(\psi(\zeta))$, then we have the schwarz function as a result

$$\frac{1}{\ell}\left[(1+i\ \tan\lambda)\left(\frac{\zeta D_q h(\zeta)}{h(\zeta)}\right) - i\ \tan\lambda\right] \prec_q \psi(\zeta) - 1.$$

We note that

$$\frac{\zeta D_q h(\zeta)}{h(\zeta)} = 1 + \frac{1-[2]_q}{3} b_2\zeta + \left(\frac{[3]_q - 1}{10} b_3 + \frac{1-[2]_q}{9} b_2^2\right)\zeta^2 + ... \tag{21}$$

and

$$\psi(w(\zeta)) = 1 + \left(\frac{P_1}{2}\right)\zeta + \left(\frac{P_2}{2} - \frac{P_1^2}{8}\right)\zeta^2 + \left(\frac{P_3}{2} - \frac{P_1 P_2}{4}\right)\zeta^3 + ...$$

Then we obtain that

$$\frac{1}{\ell}\left[(1+i\ \tan\lambda)\left(\frac{\zeta D_q h(\zeta)}{h(\zeta)}\right) - i\ \tan\lambda\right] = \varphi(\zeta)[\psi(w(\zeta)) - 1] \tag{22}$$

with

$$\begin{aligned}\varphi(\zeta)[\psi(w(\zeta)) - 1] = \frac{P_1\ d_0}{2}\zeta + \left[\frac{P_1\ d_1}{2} + d_0\left(\frac{P_2}{2} - \frac{P_1^2}{8}\right)\right]\zeta^2 \\ + \left[d_0\left(\frac{P_3}{2} + \frac{P_1\ P_2}{4}\right) + d_1\left(\frac{P_2}{2} - \frac{P_1^2}{8}\right)\right]\zeta^3 + ...\end{aligned} \tag{23}$$

with $|d_n| < 1$ and $|P_1| < 1$. Thus by applying (21), (22) and (23), we obtain that

$$b_2 = \frac{3\ell d_0 P_1}{2(1-[2]_q)\ (1+i\ \tan\lambda)} \tag{24}$$

and

$$b_3 = \frac{5\ell}{([3]_q - 1)\ (1+i\ \tan\lambda)}\left[d_1\ P_1 + d_0\left(P_2 + \frac{P_1^2}{4}\right)\right]. \tag{25}$$

Hence by (24) and (25), we get the following

$$b_3 - \mu b_2^2 = \frac{5\ell}{([3]_q - 1)\ (1+i\ \tan\lambda)}\left[d_1\ P_1 + d_0\left(P_2 + \frac{P_1^2}{4}\right)\right] - \mu\left[\frac{3\ell d_0 P_1}{2(1-[2]_q)\ (1+i\ \tan\lambda)}\right]^2$$

using $\vartheta = 1 + i\ \tan\lambda$, $V_2 = [2]_q - 1$ and $V_3 = [3]_q - 1$, then the above can be reduced to

$$\begin{aligned}
\left|b_3 - \mu b_2^2\right| &= \left|\frac{5\ell}{V_3\ \vartheta}\left[d_1 P_1 + d_0\left(P_2 + \frac{P_1^2}{4}\right)\right] - \frac{9\mu\ell^2 d_0^2 P_1^2}{2V_2^2\ \vartheta^2}\right| \\
&= \left|\frac{5\ell}{V_3\ \vartheta}\left[d_1 P_1 + d_0 P_2 + \frac{d_0\ P_1^2}{4} - \frac{9\mu\ell d_0^2 P_1^2 V_3}{10V_2^2\ \vartheta}\right]\right| \\
&= \left|\frac{5\ell}{V_3\ \vartheta}\left\{d_1 P_1 + \left[P_2 + \left(\frac{1}{4} - \frac{9\mu\ell d_0 V_3}{10V_2^2\ \vartheta}\right)P_1^2\right]d_0\right\}\right| \\
&\leq \frac{5|\ell|}{V_3\ |\vartheta|}\left\{1 + \left|P_2 + \left(\frac{1}{4} - \frac{9\mu\ell d_0 V_3}{10V_2^2\ \vartheta}\right)P_1^2\right|d_0\right\} \\
&\leq \frac{5|\ell|}{V_3\ |\vartheta|}\left\{1 + \left|P_2 + tP_1^2\right|d_0\right\}
\end{aligned}$$

where $t = \frac{1}{4} - \frac{9\mu\ell d_0 V_3}{10V_2^2\ \vartheta}$. Then the above inequality can be written as

$$\left|b_3 - \mu b_2^2\right| \leq \frac{5|\ell|}{([3]_q - 1)\ |1 + i\ \tan\lambda|}\left\{1 + \left|P_2 + tP_1^2\right|d_0\right\}$$

where

$$t = \frac{1}{4} - \frac{9\mu\ell d_0([3]_q - 1)}{10(1 - [2]_q)^2\ (1 + i\ \tan\lambda)}.$$

Hence proved. □

The following result can be established by the same technique as applied in the Theorem 12.

Theorem 13. *Let* $\frac{-\pi}{2} < \lambda < \frac{\pi}{2}$, $0 < q < 1$ *and* $\ell \neq 0$. *If* $h \in \widetilde{\mathcal{EC}}_{q,\ell}^{\lambda}(\psi(\zeta))$ *satisfies the equation* (9) *and let* $\psi(w(\zeta)) = w(\zeta) + \sqrt{1 + w^2}$, *where*

$$w(\zeta) = \frac{\phi(\zeta) - 1}{\phi(\zeta) + 1}$$

for all $\zeta \in \mathcal{U}$. *Then,*

$$\left|b_3 - \mu b_2^2\right| \leq \frac{5|\ell|}{2[3]_q|\vartheta|}\ \max\left\{1, \left|d_0 + \left(\frac{1}{4} - \frac{9\mu\ell d_0[3]_q}{5[2]_q^2\ \vartheta}\right)d_0\right|\right\}, \tag{26}$$

where $\vartheta = 1 + i\ \tan\lambda$, $V_2 = [2]_q - 1$ *and* $V_3 = [3]_q - 1$ *and* μ *is a complex number.*

References

[1] JACKSON: *On q definite integrals*, Quart .J.Pure Apply.Math. **41** (1910), 193-203.

[2] JACKSON: *On q functions and a certain difference operator*, Transactions of the Royal Society of Edinburgh. **46** (1908), 253-281.

[3] S. Kannas, D. Raducanu: *Some subclass of analytic functions related to conic domains,* Math. Slovaca **64** (2014), 1183–1196.

[4] T.H. Maсgregor: *Majorization by univalent functions*, Duke Math. J. **34** (1967) 95–102.

[5] S.S. Miller, P.T. Mocanu: *Differential Subordinations*, Monographs and Textbooks in Pure and Applied Mathematics **225**, Dekker, New York, 2000.

[6] G. Murugusundaramoorthy, K. Thilagavathi: *Majorization of Starlike and Convex functions of Complex order involving linear operators*, Math. Slovaca, **66**(1) (2016), 127–134.

[7] E. Paprocki, J. Sokol: *The extremal problems in some subclass of strongly starlike functions*, Zeszyty Nauk. Politech. Rzeszowskiej Mat. **20** (1996), 89–94.

[8] C. Ramachandran, L. Vanitha, S. Kannas: *Certain ressults on q-starlike and q-convex error functions,* Math. Slovaca, **68**(2) (2018), 361–368.

[9] R.K. Raina, J. Sokol: *On Coefficient estimates for a certain class of starlike functions*, Hacet. J. Math. Stat. **44** (2015), 1427–1433.

[10] M.S. Robertson: *Quasi-subordination and coefficient conjectures*, Bull. Amer. Math. Soc. **76** (1970), 1–9.

[11] J. Sokol, D.K. Thomas: *Further results on a class of starlike functions related to the Bernoulli lemniscate* Houston J. Math. **44** (2018), 83–95.