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Dense Elements in B-Almost Distributive Fuzzy Lattices

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Abstract

In this paper we define the concept of a dense element in a B-Almost Distributive Fuzzy Lattice BADFL (B, A) and prove that the set $D_A(B)$ of all dense elements of B is an implicative filter of B. Also, we establish a fuzzy epimorphism of (B, A) into (B^*, A) the set of all closed elements of (B, A) and prove the existence of an fuzzy epimorphism of $D_A(B)$ into $D_A\left(\frac{B^* \times B}{\theta}\right)$. We prove that (B^*, A) and $\left(\left[\frac{B^* \times B}{\theta}\right]^*, A\right)$ are fuzzy isomorphic.

1. Introduction

U. M. Swamy and G. C. Rao [8] proposed the notion of an Almost Distributive Lattice (ADL) as a generalisation of most current ring and lattice theoretic extensions of a Boolean algebra. As an extension of Heyting algebra [1] presented the notion of a Heyting Almost Distributive Lattice (HADL) in [6]. If $(H, \vee, \wedge, \rightarrow, 0, m)$ is a HADL, the set H* containing all closed elements of H is both a bounded pseudo complemented semilattice and a bounded implicative subsemilattice of H, as shown in [4]. Zadeh developed the notion of fuzzy set in [10], which was extended by Goguen in [13] and Sanchez in [11] to define and explore fuzzy relations. We also defined a binary operation $\underline{\vee}$ on (B^*, A) and established that $(B^*, \underline{\vee}, \wedge, *, 0, m)$ is a fuzzy Boolean Algebra. In this study, we expand certain essential features of Dense elements in BADFL using the fuzzy partial order relation given in [12].

In this paper, we explore the concept of dense elements in an BADFL (B, A) and prove that the set $D_A(B)$of all dense elements of B is an implicative filter of B. Also we establish an fuzzy epimorphism of (B, A) into (B^*, A) and prove the existence of an fuzzy epimorphism of $D_A(B)$ into $D_A\left(\frac{B^* \times B}{\theta}\right)$. Finally, we prove that (B^*, A) and $\left(\left[\frac{B^* \times B}{\theta}\right]^*, A\right)$ are fuzzy isomorphic.

Following are some significant results and definitions required for the study of dense element characteristics on B-ADFLs.

2. Preliminaries

Definition 2.1. [7]

Let (L, A) be a BADFL with a maximal element m. Suppose $\Rightarrow$ is a binary operation on L if and only if it satisfies the following conditions:

(1) $A(a \Rightarrow a, m) = 1$

(2) $A\big((a \Rightarrow b) \wedge b, b\big) = 1$

(3) $A(a \wedge (a \Rightarrow b), a \wedge b \wedge m) = 1$

(4) $A(a \Rightarrow (b \wedge c), (a \Rightarrow b) \wedge (a \Rightarrow c)) = 1$

(5) $A((a \wedge b) \Rightarrow c, (a \Rightarrow c) \wedge (b \Rightarrow c)) = 1$ for every $a, b, c \in L$.

Definition 2.2. [5]

Let $(L, \vee, \wedge, \rightarrow, 0, m)$ be a HADL. An implicative filter is a non-empty subset F of L if and only if

(1) $a, b \in F \Rightarrow a \wedge b \in F$,

(2) $a \in F, b \in L \Rightarrow b \rightarrow a \in F$.

Lemma 2.1. [15]

Let $(L, \vee, \wedge, \rightarrow, 0, m)$ be a HADFL. Then for every $a, b, c \in L$, the following hold:

(1) $A(b \wedge m, (a \rightarrow b) \wedge m) > 0$

(2) $A(a \wedge c \wedge m \leq b \wedge m) > 0 \Leftrightarrow A(c \wedge m, (a \rightarrow b) \wedge m) > 0$

(3) $A([a \rightarrow (b \rightarrow c)] \wedge m, [(a \wedge b) \rightarrow c] \wedge m) > 0$

(4) $A([(a \wedge b) \rightarrow c] \wedge m, [(b \wedge a) \rightarrow c] \wedge m) > 0$

(5) $A([a \rightarrow (b \rightarrow c)] \wedge m, [b \rightarrow (a \rightarrow c)] \wedge m) > 0$

Lemma 2.3. [9]

Let (L, A) be a BADFL. Then, for every $x, y \in L$ with $A(x^*, x \Rightarrow 0) = 1$, the following hold.

(1) $A((x \vee y)^*, x^* \wedge y^*) = 1$

(2) $A(x, y) > 0 \Rightarrow A(y^*, x^*) > 0, A(x^{**}, y^{**}) > 0$

(3) $A(x \wedge x^{**}, x \wedge m) = 1$ and $A(x^{**} \wedge x, x) = 1$

(4) $A((x \wedge y)^*, (x \Rightarrow y^*) \wedge m) = 1$

(5) $A((x \wedge m)^*, x^*) = 1$

(6) $A((x \wedge y)^{**}, x^{**} \wedge y^{**}) = 1$.

3. Dense Elements in B-Almost Distributive Fuzzy Lattices

In this section, we recall from that if (B, A) is a BADFL and $x \in B$ then $B^* = \{{x_\alpha}^* : x_\alpha \in B\}$ of the set of all closed elements of B where $A({x_\alpha}^*, x_\alpha \Rightarrow 0) = 1$.

Definition 3.1

Let (B, A) be a B-ADFL with maximal element m_e. Define $D_{(B,A)} = \{A({x_\alpha}^*, 0) = 1, x_\alpha \in B\}$ Then an element of $D_{(B,A)}$ is called a dense element of (B, A).

It can easily observed that if $d_\alpha \in B$ then $d \in D_{(B,A)} \Leftrightarrow A({d_\alpha}^{**}, m_e.) = 1$. Now we prove that $D_{(B,A)}$ is a implicative filter of (B, A).

Theorem 3.1

Let (B, A) be a BADFL with maximal element m_e.Then $D_{(B,A)}$ is an implicative filter of (B, A).

Proof. Let (B, A) be a BADFL with maximal m_e. Let $x_\alpha, y_\alpha \in D_{(B,A)}$. Then from lemma (2.3) we have

$$A((x_\alpha \wedge y_\alpha)^*, (x_\alpha \Rightarrow y_\alpha{}^*) \wedge m_e) = A((x_\alpha \wedge y_\alpha)^*, (x_\alpha \Rightarrow (y_\alpha \Rightarrow 0) \wedge m_e)$$
$$A((x_\alpha \wedge y_\alpha)^*, (x_\alpha \wedge y_\alpha) \Rightarrow 0 \wedge m_e)$$
$$= A((x_\alpha \wedge y_\alpha)^*, (x_\alpha \Rightarrow 0) \wedge m_e)$$
$$= A((x_\alpha \wedge y_\alpha)^*, x_\alpha{}^* \wedge m_e)$$
$$= A((x_\alpha \wedge y_\alpha)^*, 0)$$
$$= 1 \qquad \ldots\ldots\ldots\ldots.. \qquad (1)$$

Let $d_\alpha \in D_{(B,A)}$, $x \in B$, then we get $A(d_\alpha \wedge m_e, (x_\alpha \Rightarrow d_\alpha) \wedge m_e) > 0$. This implies $A((d_\alpha \wedge m_e.)^{**}, [(x \Rightarrow d_\alpha) \wedge m_e.]^{**}) > 0$ and hence

$$A(d_\alpha{}^{**} \wedge m_e{}^{**}, [(x_\alpha \Rightarrow d_\alpha)^{**} \wedge m_e{}^{**}]) > 0$$
$$A(m_e \wedge m_e{}^{**}, (x_\alpha \Rightarrow d_\alpha)^{**} \wedge m_e{}^{**}) > 0$$
$$A(m_e, (x_\alpha \Rightarrow d_\alpha)^{**}) > 0$$

Therefore $A((x \Rightarrow d_\alpha)^*, 0) = 1 \qquad \ldots\ldots\ldots \qquad (2)$

From (1) and (2) $D_{(B,A)}$ is implicative filter of (B, A).

Theorem 3.2

Let (B, A) be a BADFL with maximal element m_e and $\alpha_f: (B, A) \to (B^*, A)$ be defined by $A(\alpha_f(x_\alpha), x_\alpha{}^{**}) = 1$ for all $x_\alpha \in B$ and suppose $x_\alpha, y_\alpha \in B$. Then

(1) α_f is fuzzy isotone

(2) $A(\alpha_f(x_\alpha \wedge y_\alpha), \alpha_f(x_\alpha) \wedge \alpha_f(y_\alpha)) = 1$

(3) $A(\alpha_f(x_\alpha \vee y_\alpha), \alpha_f(x_\alpha) \underline{\vee} \alpha_f(y_\alpha)) = 1$

(4) $A(ker\ ker\ (\alpha_f), D_{(B,A)}) = 1$

Proof. Let $x_\alpha, y_\alpha \in B$

(i) Assume $A(x_\alpha, y_\alpha) > 0$ then by [lemma 2.3]

$$A(x_\alpha{}^{**}, y_\alpha{}^{**}) = A(\alpha_f(x_\alpha) \wedge \alpha_f(y_\alpha)) > 0$$

Therefore α_f is fuzzy isotone

(ii) $A(\alpha_f(x_\alpha \wedge y_\alpha), (x_\alpha \wedge y_\alpha)^{**}) = A(\alpha_f(x_\alpha \wedge y_\alpha), x_\alpha{}^{**} \wedge y_\alpha{}^{**})$

$$= A\left(\alpha_f(x_\alpha \wedge y_\alpha), \left(\alpha_f(x_\alpha) \wedge \alpha_f(y_\alpha)\right)\right)$$
$$= 1$$

(iii) $A(\alpha_f(x_\alpha \vee y_\alpha), (x_\alpha \vee y_\alpha)^{**}) = A(\alpha_f(x_\alpha \vee y_\alpha), (x_\alpha{}^* \wedge y_\alpha{}^*)^*)$

$$= A(\alpha_f(x_\alpha \vee y_\alpha), x_\alpha{}^{**} \underline{\vee} y_\alpha{}^{**})$$
$$= A\left(\alpha_f(x_\alpha \vee y_\alpha), \left(\alpha_f(x_\alpha) \underline{\vee} \alpha_f(y_\alpha)\right)\right)$$
$$= 1$$

(iv) Let $x_\alpha \in D_{(B,A)}$. Then

$$A(x_\alpha{}^*,0) = A(x_\alpha{}^{**}, m_e{}^*)$$
$$= A\big(\alpha_f(x_\alpha), m_e{}^*\big)$$
$$= 1$$

Thus $x_\alpha \in ker(\alpha_f) = \alpha_f^{-1}(m_e)$

Conversely suppose that $x_\alpha \in ker(\alpha_f) = \alpha_f^{-1}(m_e)$. Then

$$A\left(m_e, \alpha_f(x_\alpha)\right) = A(m_e, x_\alpha{}^{**})$$
$$= A(m_e{}^*, x_\alpha{}^{***})$$
$$= A(0, x_\alpha{}^*)$$
$$= 1$$

and hence $x_\alpha \in D_{(B,A)}$.

Therefore $ker(\alpha_f) = D_{(B,A)}$.

.**Theorem 3.3**

Let (B, A) be a BADFL. Then for any element x_α of B there exists $d \in D_{(B,A)}$ such that $A(x_\alpha \wedge m_e, x_\alpha{}^{**} \wedge d) = 1$

Proof. Suppose that

$$A(x_\alpha \wedge m_e, x^{**} \wedge x_\alpha \wedge m_e) = A\big(x_\alpha \wedge m_e, x_\alpha{}^{**} \wedge (x_\alpha{}^{**} \Rightarrow x_\alpha)\big) = 1$$

It is enough to prove that $x_\alpha{}^{**} \Rightarrow x_\alpha \in D_{(B,A)}$.

$$A(x_\alpha{}^{**}, (x_\alpha \wedge m_e)^{**}) = A(x^{**}, [x_\alpha{}^{**} \wedge (x_\alpha{}^{**} \Rightarrow x_\alpha)]^{**})$$
$$= A(x_\alpha{}^{**}, x_\alpha{}^{**} \wedge (x_\alpha{}^{**} \Rightarrow x_\alpha)^{**})$$
$$= 1$$

so that $A(x_\alpha{}^{**}, (x_\alpha{}^{**} \Rightarrow x_\alpha)^{**}) > 0$ and hence $A((x_\alpha{}^{**} \Rightarrow x_\alpha)^*, x_\alpha{}^*) > 0$ ………… (1)

on the other hand, $A(0, x_\alpha{}^* \wedge x_\alpha{}^{**}) = A(0, x_\alpha \wedge m_e) > 0$

$\Rightarrow A\big(x_\alpha{}^*, x_\alpha{}^{**} \Rightarrow (x_\alpha \wedge m_e)\big) = A(x^*, (x_\alpha{}^{**} \Rightarrow x_\alpha) \wedge m_e)$

so that $A((x_\alpha{}^{**} \Rightarrow x_\alpha)^*, [(x_\alpha{}^{**} \Rightarrow x_\alpha) \wedge m_e]^*) = A((x_\alpha{}^{**} \Rightarrow x_\alpha)^*, x_\alpha{}^{**}) > 0$ …………... (2)

From equation (1) and (2) we get

$$A((x_\alpha{}^{**} \Rightarrow x_\alpha)^*, 0) = 1$$

Thus $x_\alpha{}^{**} \Rightarrow x_\alpha \in D_{(B,A)}$.

Corollary 3.1

Let (B, A) be a BADFL and $x_\alpha, y_\alpha \in B$ such that $A(x_\alpha{}^{**}, y_\alpha{}^{**}) = 1$. Then there exists $d_\alpha \in D_{(B,A)}$ such that $A(x_\alpha \wedge d_\alpha \wedge m_e . y_\alpha \wedge d_\alpha \wedge m_e) = 1$.

Proof. Let $x_\alpha, y_\alpha \in B$ by above theorem, there exist $d_{\alpha_1}, d_{\alpha_2} \in D_{(B,A)}$ such that $A\big(x_\alpha \wedge m, x_\alpha{}^{**} \wedge d_{\alpha_1}\big) = 1$ and $A\big(y_\alpha \wedge m, y_\alpha{}^{**} \wedge d_{\alpha_2}\big) = 1$.

Let $A\big(d_\alpha, d_{\alpha_1} \wedge d_{\alpha_2}\big) = 1$.

Then d is dense element of B and

$$A(x_\alpha \wedge d_\alpha \wedge m_e, x_\alpha \wedge m_e \wedge d_\alpha) = A\big(x_\alpha \wedge d_\alpha \wedge m_e, d_{\alpha_1} \wedge d_{\alpha_1} \wedge d_{\alpha_2}\big)$$

$$= A\left(x_\alpha \wedge d_\alpha \wedge m_e, {y_\alpha}^{**} \wedge \ d_{\alpha_1} \wedge d_{\alpha_2}\right)$$
$$= A\left(x_\alpha \wedge d_\alpha \wedge m_e, {y_\alpha}^{**} \wedge d_{\alpha_2} \wedge d_{\alpha_1}\right)$$
$$= A\left(x_\alpha \wedge d_\alpha \wedge m_e, {y_\alpha}^{**} \wedge d_{\alpha_2} \wedge d_{\alpha_2} \wedge d_{\alpha_1}\right)$$
$$= A\left(x_\alpha \wedge d_\alpha \wedge m_e, y_\alpha \wedge m_e \wedge d_{\alpha_1} \wedge d_{\alpha_2}\right)$$
$$= A(x_\alpha \wedge d_\alpha \wedge m_e, y_\alpha \wedge m_e \wedge d_\alpha)$$
$$= A(x_\alpha \wedge d_\alpha \wedge m_e, y_\alpha \wedge d_\alpha \wedge m_e)$$
$$= 1$$

Theorem 3.4

Let (B, A) be a BADFL with maximal element m Define $\alpha_f: (B, A) \to (B^*, A)$ by $\left(\alpha_f(x_\alpha), {x_\alpha}^{**}\right) = 1$, $x_\alpha \in B$. Then α_f is an fuzzy epimorphism.

Proof. For any$x_\alpha, y_\alpha \in B$, we have

$$A((x_\alpha \Rightarrow y_\alpha)^{**}, [(x_\alpha \Rightarrow y_\alpha) \wedge m_e]^{**}) = A((x_\alpha \Rightarrow y_\alpha)^{**}, [(x_\alpha \Rightarrow y_\alpha) \wedge (x_\alpha \Rightarrow m_e)]^{**})$$
$$= A((x_\alpha \Rightarrow y_\alpha)^{**}, [x_\alpha \Rightarrow (y_\alpha \wedge m_e)]^{**})$$
$$= A((x_\alpha \Rightarrow y_\alpha)^{**}, [x_\alpha \Rightarrow ({y_\alpha}^{**} \wedge d_\alpha)]^{**})$$

for some dense elements d_α in B.

$$= A((x_\alpha \Rightarrow y_\alpha)^{**}, [(x_\alpha \Rightarrow {y_\alpha}^{**}) \wedge (x_\alpha \Rightarrow d_\alpha)]^{**})$$
$$= A((x_\alpha \Rightarrow y_\alpha)^{**}, (x_\alpha \Rightarrow {y_\alpha}^{**})^{**} \wedge (x_\alpha \Rightarrow d_\alpha)^{**})$$
$$= A((x_\alpha \Rightarrow y_\alpha)^{**}, (x_\alpha \Rightarrow {y_\alpha}^{**})^{**} \wedge m_e)$$
$$= A((x_\alpha \Rightarrow y_\alpha)^{**}, ({x_\alpha}^{**} \Rightarrow {y_\alpha}^{**}) \wedge m_e)$$
$$= A((x_\alpha \Rightarrow y_\alpha)^{**}, {x_\alpha}^{**} \Rightarrow {y_\alpha}^{**})$$
$$\Rightarrow A\left(\alpha_f(x_\alpha \Rightarrow y_\alpha), \alpha_f(x_\alpha) \Rightarrow \alpha_f(y_\alpha)\right) = 1$$

Thus α_f is an epimorphism from theorem 3.2

Theorem 3.5

Let (B, A) be a BADFL and x_α be an element of B. Then x_α is dense if and only if there is an element y_α of B such that $A(x_\alpha \wedge m_e, {y_\alpha}^{**} \Rightarrow y_\alpha) = 1$.

Proof. Assume x_α is a dense element of B. Then

$$A({x_\alpha}^{**} \Rightarrow x_\alpha, m_e \Rightarrow x_\alpha) = A\left({x_\alpha}^{**} \Rightarrow x_\alpha, m_e \wedge (m_e \Rightarrow x_\alpha)\right)$$
$$= A({x_\alpha}^{**} \Rightarrow x_\alpha, m_e \wedge m_e \wedge x_\alpha)$$
$$= A({x_\alpha}^{**} \Rightarrow x_\alpha, x_\alpha \wedge m_e)$$
$$= 1.$$

Conversely assume that $A(x_\alpha \wedge m_e, {y_\alpha}^{**} \Rightarrow y_\alpha)$ for some $y_\alpha \in B$. Then from the proof of theorem 3.3. we have $A(x_\alpha \wedge m_e, {y_\alpha}^{**} \wedge y_\alpha) = 1$, (i.e.) ${y_\alpha}^{**} \Rightarrow y_\alpha$ is a dense element of B so that $x_\alpha \wedge m_e$ is a dense element of B and hence x_α is a dense element of B. Since $A({x_\alpha}^*, (x_\alpha \wedge m_e)^*) = 1$.

Definition 3.2

Let (B, A) be a BADFL and ϕ: $(B^*, A) \times (B, A) \to (B, A)$ be a map such that

(i) $f_a: (B, A) \to (B, A)$ defined by $A\left(f_a(d_\alpha), f(a_\alpha, d_\alpha)\right) = 1$ is an fuzzy endomorphism.

(ii) $A\left(f(a_\alpha \wedge b_\alpha, d_\alpha), f\left(a_\alpha, f(b_\alpha, d_\alpha)\right)\right) = 1$

(iii) If $A(a_\alpha, b_\alpha) > 0$ then $A\big(f(b_\alpha, d_\alpha), f(a_\alpha, d_\alpha)\big) > 0$

(iv) $A(f(0, d_\alpha), m_e) = 1, A(f(m_e, d_\alpha), d_\alpha) = 1$

for any $a_\alpha, b_\alpha \in B^*$ and $d_\alpha \in B$. Then f is said to be an admissible map.

Definition 3.3

Let (B, A) be a BADFL and $f: (B^*, A) \times (B^*, A) \to (B, A)$ be an admissible map. Define the relation θ_f on $(B^*, A) \times (B, A)$ by $(a_\alpha, d_\alpha)\theta_f(b_\alpha, e_\alpha) \Leftrightarrow A(a_\alpha, b_\alpha) = 1$ and $A\big(f(a_\alpha, d_\alpha \wedge m_e), f(a_\alpha, e_\alpha \wedge m_e)\big) = 1$. Then θ_f is an equivalence relation on $(B^*, A) \times (B, A)$. We denote the equivalence class $\frac{(a,d)}{\theta_f}$ by $[a_\alpha, d_\alpha]_f$.

Lemma 3.1

Let (B, A) be an BADFL. Then for any $a_\alpha, b_\alpha \in B^*$and $d_\alpha, e_\alpha \in B$.

$A(b_\alpha, a_\alpha) > 0$ and $A\big(f(a_\alpha, d_\alpha \wedge m_e), f(a_\alpha, e_\alpha \wedge m_e)\big) = 1$.

Then $A\Big(f\big(b_\alpha, d_\alpha \wedge m_e, f(b_\alpha, e_\alpha \wedge m_e)\big)\Big) = 1$.

(i) $A([(a_\alpha, d_\alpha), (b_\alpha, e_\alpha)]) > 0 \Leftrightarrow A(a_\alpha, b_\alpha) > 0$ and $\big(f(a_\alpha, d_\alpha \wedge m_e), f(a_\alpha, e_\alpha \wedge m_e)\big) > 0$

Proof. Let $A(b_\alpha, a_\alpha) > 0$ and $A\big(f(a_\alpha, d_\alpha \wedge m_e), f(a_\alpha, e_\alpha \wedge m_e)\big) = 1$

Now

$$\begin{aligned}
A\big(f(b_\alpha, d_\alpha \wedge m_e), f(a_\alpha \wedge b_\alpha, d_\alpha \wedge m_e)\big) &= A\Big(f(b_\alpha, d_\alpha \wedge m_e), f\big(b_\alpha, f(a_\alpha, d_\alpha \wedge m_e)\big)\Big) \\
&= A\Big(f(b_\alpha, d_\alpha \wedge m_e), f\big(b_\alpha, f(a_\alpha, e_\alpha \wedge m_e)\big)\Big) \\
&= A\big(f(b_\alpha, d_\alpha \wedge m_e), f(b_\alpha \wedge a_\alpha, e_\alpha \wedge m_e)\big) \\
&= A\big(f(b_\alpha, d_\alpha \wedge m_e), f(a_\alpha \wedge b_\alpha, e_\alpha \wedge m_e)\big)
\end{aligned}$$

$$A\big(f(b_\alpha, d_\alpha \wedge m_e), f(b_\alpha, e_\alpha \wedge m_e)\big) = 1$$

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