Markovian Queueing System with Discouraged Arrivals by Fuzzy Ordering Approach

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Abstract

This article contributes a survey about achievement and cost expedients for Markovian Queueing System with discouraged arrivals by Circumcenter of Incenters fuzzy ordering approach. Ordering approaches are occupying a very remarkable milestone in defuzzification. This approach is converted from fuzzy structure to crisp structure. Our propounded ordering approach can does this conversion very effectively and gives the concrete solutions.

Keywords: Fuzzy numbers, Fuzzy sets, Discouraged arrivals Queueing System, Circumcenter of Incenters fuzzy ordering.

INTRODUCTION

Queueing theory plays a salient segment in our day to day complications. Typically we meet vast congestions in front of Bank counters, Railway booking counters, Hospitals, Pharmacies, Ration shops, Mobile networks, etc…, . The time governance is the top most pursuance of the system manager. In this phase, queueing system occupies massive part.

The preludes [6],[8],[9],[11],[12],[13] and queueing system are most essential in the research field. Now- a- days we are using Fuzzy based applications [14],[25] very casually. The work of job processing in computers are using Queues with discouraged arrivals[2],[3],[4],[18]. The retention of reneged customers queuing system [15] was surveyed by Kumar and Sharma. Natvig [16] analyzed the discouraged arrivals queueing system.

More inventers have used dissimilar ordering techniques to defuzzify the fuzzy numbers. Most of the authors are applied area and distance grounded ordering methods [1],[5],[7],[10],[17],[20],[21]. Some inventers used centroid grounded ordering methods [19],[22],[23]. Westman et.al handled Left and Right Wingspans technique[24]. Our suggested ordering approach is much better to catch the achievement and cost expedients.

PRELUDES

Def 1: A Set $\widetilde{A}$ = {(x, $\phi_{\tilde{A}}$ (x)); x Є U}is named a Fuzzy set if it is indented by a mapping $\phi_{\tilde{A}}$: U →[0,1], here $\phi_{\tilde{A}}$ is membership function and U is the universal set.

Def 2: A Number $\widetilde{A}(a_1, a_2, a_3)$ is named as Triangular Fuzzy Number it should have the membership function $\phi_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1 \quad , & x = a_2 \\ \dfrac{x-a_3}{a_2-a_3}, & a_2 \le x \le a_3 \\ 0, otherwise \end{cases}$

Def 3: A Number $\widetilde{A}(a_1, a_2, a_3, a_4)$ is named as Trapezoidal Fuzzy Number it should have the membership function $\phi_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ 1 \quad , & a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4}, & a_3 \le x \le a_4 \\ 0, otherwise \end{cases}$

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MARKOVIAN QUEUEING SYSTEM WITH DISCOURAGED ARRIVALS

Contemplate the patrons occur in a solitary server line by Poisson fashion with the fuzzy rate $\tilde{\lambda}$ and the exponential service time with fuzzy rate $\tilde{\mu}$ in FCFS manner. When the state of the system expands up, the mean arrival diminishes gradually. The new entry (Fig 1) get discouraging (discouraged arrivals) to join the queue.

Fig 1. Discouraged arrivals

(I). Performance Expedients

Let the system capacity to be finite, say N. Every patron may wait for service particular time duration (T) in the queue. If the service didn't begin after T, he felt renege and can quit from the line without served. Let p be the reneging probability. If the particular patron applied the retention strategy for service he can retain in the same line with q (= 1 - p). The times T follow exponential structure with parameter $\tilde{\xi}$. If n patrons appeared in the system then the probability will be Pn.

By basic queueing theory

$$\tilde{\lambda} P_0 = \tilde{\mu} P_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\left[\left(\frac{\tilde{\lambda}}{n+1}\right)+\tilde{\mu}+(n-1)\tilde{\xi}p\right]P_n = [\tilde{\mu}+n\tilde{\xi}p]P_{n+1}+\left(\frac{\tilde{\lambda}}{n}\right)P_{n-1} \quad \text{.....................................(2)}$$

$$,1 \le \text{n} \le \text{N-1}$$

$$\left(\frac{\tilde{\lambda}}{N}\right)P_{N-1} = [\tilde{\mu}+(N-1)\tilde{\xi}p]P_N \quad \text{...(3)}$$

Now (1) The Predicted aggregate of patrons in the System (N_S) = $\sum_{n=1}^{N} nP_n$

$$\Rightarrow \qquad (N_S) = \sum_{n=1}^{N} n\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{\tilde{\lambda}}{\tilde{\mu}+(k-1)\tilde{\xi}p}\right]P_0$$

(2) The Predicted aggregate of patrons Served (E(C S)) = $\sum_{n=1}^{N} \tilde{\mu}P_n$

$$\Rightarrow \text{(E(C S))} = \tilde{\mu}\sum_{n=1}^{N}\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{\tilde{\lambda}}{\tilde{\mu}+(k-1)\tilde{\xi}p}\right]P_0$$

(3) Average Reneging Rate (Rr) = $\sum_{n=1}^{N}(n-1)\tilde{\xi}pP_n$

$$\Rightarrow \text{(Rr)} = \sum_{n=1}^{N}(n-1)\tilde{\xi}p\frac{1}{n!}\left[\prod_{k=1}^{n}\frac{\tilde{\lambda}}{\tilde{\mu}+(k-1)\tilde{\xi}p}\right]P_0$$

(4) Average Retention Rate (RR) = $\sum_{n=1}^{N}(n-1)\tilde{\xi}qP_n$

$$\Rightarrow \text{(RR)} = \sum_{n=1}^{N}(n-1)\tilde{\xi}q\frac{1}{n!}\left[\prod_{k=1}^{n}\frac{\tilde{\lambda}}{\tilde{\mu}+(k-1)\tilde{\xi}p}\right]P_0$$

Where $P_0 = \dfrac{1}{1+\sum_{n=1}^{N}\left[\frac{1}{n!}\prod_{k=1}^{n}\frac{\tilde{\lambda}}{\tilde{\mu}+(k-1)\tilde{\xi}p}\right]}$

(II). Cost Expedients

symbols

$\lambda_{lost} = \lambda P_N$ = Average rate for a patron is lost.

P_N = Probability for if N patrons in the system.

N_s = Predicted aggregate of patrons in the system.

R_r = Mean reneged rate.

R_R = Mean retention rate.

C_s = The service cost / unit time.

C_h = The holding cost / unit time.

C_l = The cost for every lost patron /unit time.

C_r = The cost for every reneged patron / unit time.

C_R = The cost for every retained patron / unit time.

R = The cost of revenue by the service for every patron / unit time.

TEC = Total expected cost of the system.

TEP = Total expected profit of the system.

TER = Total expected revenue of the system.

Now,

(i) TEC = $\widetilde{Cs}\tilde{\mu} + \widetilde{C_h}\widetilde{N_s} + \widetilde{C_l}\tilde{\lambda}\widetilde{P_N} + \widetilde{C_r}\widetilde{R_r} + C_R\widetilde{R_R}$

(ii) TER =$\tilde{R}\widetilde{N_s} - \tilde{R}\tilde{\lambda}\widetilde{P_N} - \tilde{R}\widetilde{R_r}$

(iii) TEP = TER − T EC

CIRCUMCENTER OF INCENTERS FUZZY RANKING METHOD

Ordering techniques are very sensational for defuzzification. As yet, innumerable inventers handled the centroid grounded techniques for defuzzification. Here we launching a new Incenter grounded ordering technique for defuzzification. We detect a gravity point (balancing point) circumcenter of Incenters of trapezium for ordering the generalized trapezoidal fuzzy numeral $\tilde{A} = (p, q, r, s; w)$.

The trapezium *PUTS* is partitioned to ΔPEV, ΔVSP & ΔSTV (Fig 2) and get the incenters

I_1, I_2& I_3. Then We make a $\Delta(I_1, I_2, I_3)$ by these non-collinear incenters and get the circumcenter '**S**' of this triangle and this is equidistant to all incenters.

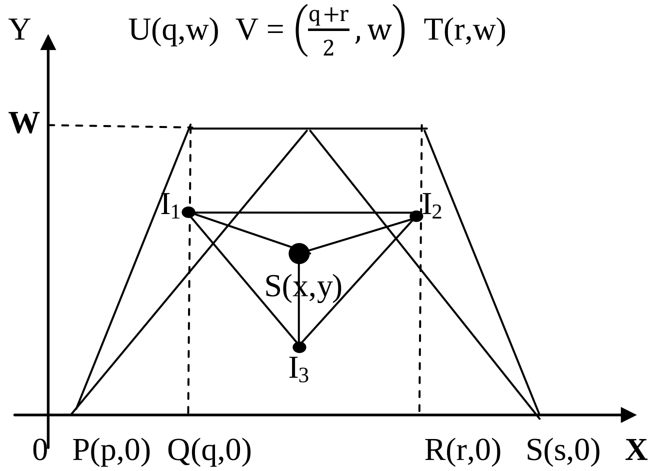

Fig 2. Circumcenter of Incenters

The incenters of ΔPEV, ΔVSP & ΔSTV are

$I_1(x_{i1},y_{i1})=\left(\frac{pl_1+qm_1+\frac{q+r}{2}n_1}{l_1+m_1+n_1},\frac{w(m_1+n_1)}{l_1+m_1+n_1}\right)$; $I_2(x_{i2},y_{i2})=\left(\frac{pl_2+\frac{q+r}{2}m_2+sn_2}{l_2+m_2+n_2},\frac{wm_2}{l_2+m_2+n_2}\right)$ and

$I_3(x_{i3},y_{i3})=\left(\frac{\frac{q+r}{2}l_3+rm_3+sn_3}{l_3+m_3+n_3},\frac{w(l_3+m_3)}{l_3+m_3+n_3}\right)$ mutually. Where

$l_1=\sqrt{(\frac{q+r}{2}-q)^2}$, $m_1=\sqrt{(\frac{q+r}{2}-p)^2+W^2}$, $n_1=\sqrt{(q-p)^2+W^2}$,

$l_2=\sqrt{(\frac{q+r}{2}-s)^2+W^2}$, $m_2=\sqrt{(p-s)^2}$, $n_2=\sqrt{(\frac{q+r}{2}-p)^2+W^2}$,

$l_3=\sqrt{(r-s)^2+W^2}$, $m_3=\sqrt{(\frac{q+r}{2}-s)^2+W^2}$, $n_3=\sqrt{(\frac{q+r}{2}-r)^2}$

Then we detect the Circumcenter S(x,y) by solving

$$([\mathbf{x}-\mathbf{x_{i1}}]^2+[\mathbf{y}-\mathbf{y_{i1}}]^2) = ([\mathbf{x}-\mathbf{x_{i2}}]^2+[\mathbf{y}-\mathbf{y_{i2}}]^2) = ([\mathbf{x}-\mathbf{x_{i3}}]^2+[\mathbf{y}-\mathbf{y_{i3}}]^2)$$

The ordering function of $\tilde{A}$ is $R(\tilde{A}) = \tilde{\mathbf{x}}\ \mathbf{x}\ \tilde{\mathbf{y}}$.

ILLUSTRATION

Since long years ago, in India, particularly in Tamilnadu, the most prestigious pongal festival is celebrating by all Tamil people. Usually The Government of Tamilnadu is dispatching the pongal gift hampers through the ration shops. During those periods a very long queue is formed by the people in front of the ration shops. Think about a particular ration shop dispatched the pongal gifts along a single server queue. If occurring people observe a huge crowd in the queue they may not add in that line. So, when the state of the system grows up, the rate of average occurrence decreases gradually. It means, this discouraged the new entries (Discouraged arrivals) for that line. In this situation we consider only a finite number of people (N) and single server modeled queue.

Let N = 10 and q = 0, 0.1, 0.2, …….., 1.

Consider the arrival rate $\tilde{\lambda}$ = [1,2,3,4;1], the service rate $\tilde{\mu}$ = [3,4,5,6;1] and time distribution parameter $\tilde{\xi}$ = [0.1,0.2,0.3,0.4;1] / hr individually .

Here $\tilde{\lambda}$ = [1,2,3,4;1] has the membership function of the form

$$\phi_{\tilde{\lambda}}(x) = \begin{cases} \dfrac{(x-1)}{(2-1)}, & 1 \le x \le 2 \\ 1 \quad , & 2 \le x \le 3 \\ \dfrac{(x-4)}{(3-4)}, & 3 \le x \le 4 \\ 0, otherwise \end{cases}$$

This same procedure continues for all other trapezoidal fuzzy numerals.

By the circumcenter of Incenters Fuzzy Ordering approach we got

$R(\tilde{\lambda})$ = R(1,2,3,4;1) = 2.47 x 0.83 = 2.05, Similarly $R(\tilde{\mu})$ = 3.74 , $R(\tilde{\xi})$ = 0.2

(I). Performance Expedients

By using the queueing models we caught the performance expedients concerning q.

S.No	q	N_s	E(CS)	R_r	R_R
1	0	0.5180	1.6425	0.0207	0
2	0.1	0.5211	1.6450	0.0186	0.0020
3	0.2	0.5240	1.6474	0.0166	0.0041
4	0.3	0.5269	1.6495	0.0144	0.0062
5	0.4	0.5299	1.6513	0.0122	0.0081
6	0.5	0.5330	1.6532	0.0101	0.0101
7	0.6	0.5361	1.6552	0.0080	0.0120
8	0.7	0.5392	1.6574	0.0060	0.0140
9	0.8	0.5421	1.6598	0.0039	0.0156
10	0.9	0.5452	1.6624	0.0019	0.0171
11	1.0	0.5483	1.6645	0	0.0185

Table 1 : Performance Expedients Vs q

Table 1 describes that the increment in the probability q makes a steady increment in $\mathbf{N_s}$, **E(CS)** and $\mathbf{R_R}$. Here, the $\mathbf{R_r}$ decrease subsequently on increasing q. If q = 0, then R_R = 0 means that no patron has retained. If q = 1 then Rr = 0, it means every reneging patrons have retained.

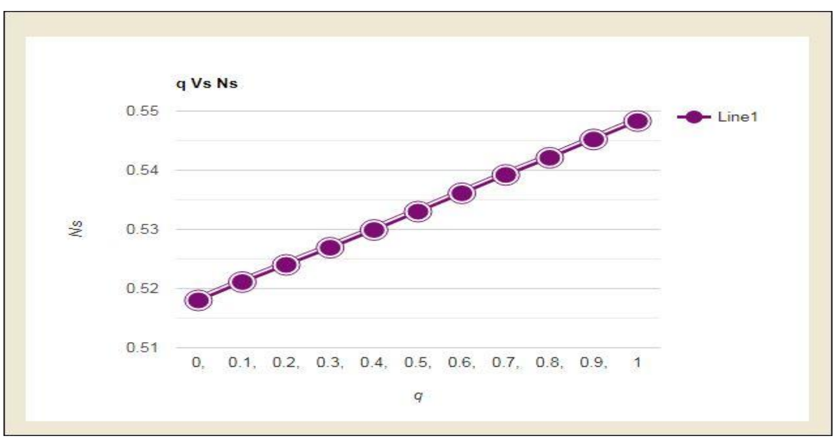

Fig 3. q Vs Ns

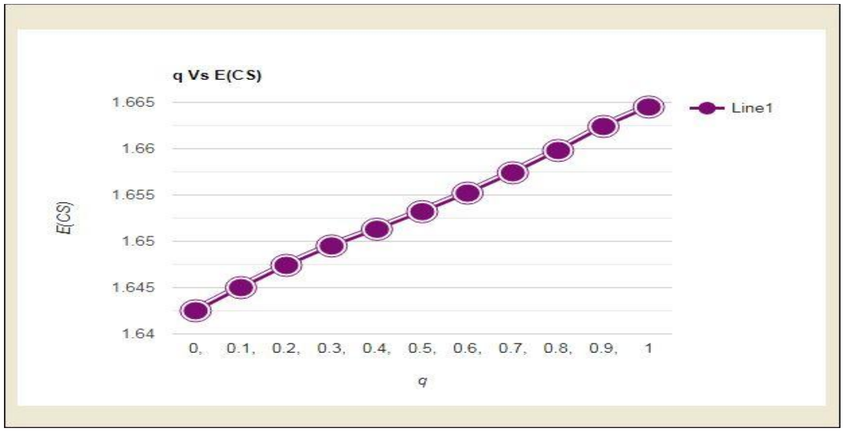

Fig 4. q Vs E(CS)

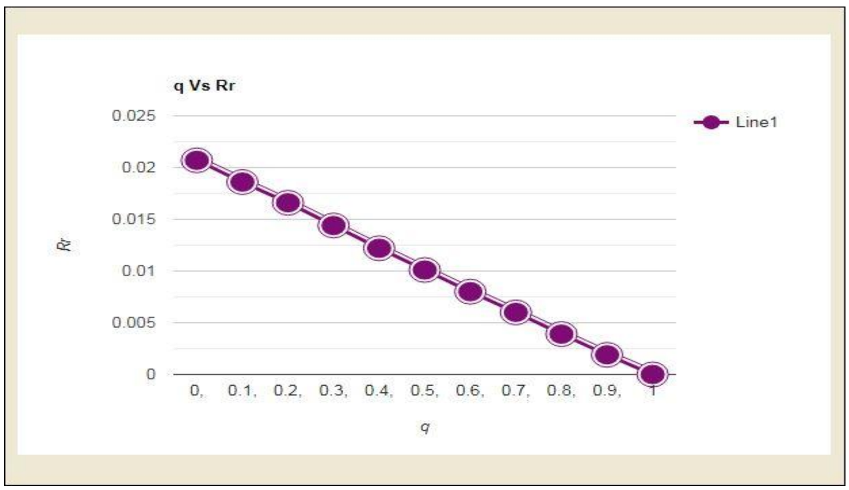

Fig 5. q Vs Rr

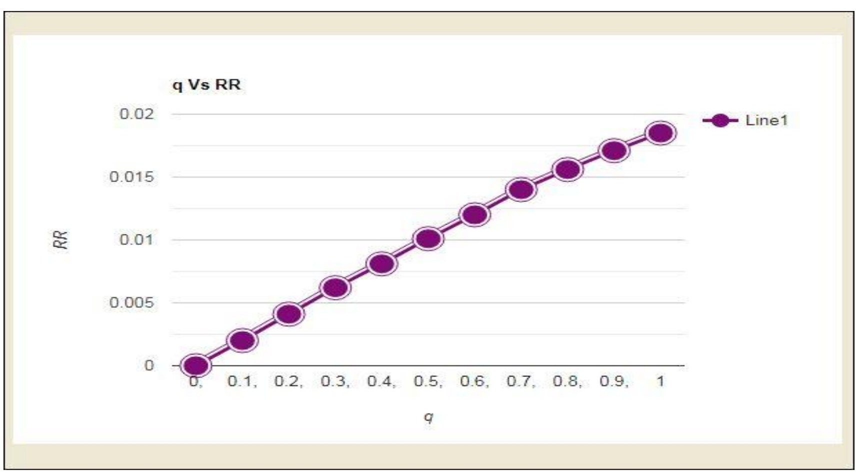

Fig 6. q Vs R_R

Figures 3, 4, 6 show that the increment in the probability q makes a steady increment in $\mathbf{N_s}$, **E(CS)** and $\mathbf{R_R}$.. Figure 5 shows that the $\mathbf{R_r}$ decrease subsequently on increasing q.

(II). Cost Expedients

Consider the arrival rate $\tilde{\lambda}$ = [1,2,3,4], the service rate $\tilde{\mu}$ = [3,4,5,6] and time distribution parameter $\tilde{\xi}$ = [0.1,0.2,0.3,0.4] per hour respectively . Also N = 10, $\widetilde{C_l}$ = [100,200,300,400], $\widetilde{Cs}$ = [10,20,30,40], $\widetilde{C_h}$ = [8,9,10,11] $\widetilde{C_r}$ = [30,40,50,60] and $\tilde{R}$ = [300,400,500,600].

By the circumcenter of Incenters Fuzzy Ordering approach we got
R($\tilde{\lambda}$) = R(1,2,3,4) = 2.47 x 0.83 = 2.05, Similarly R($\tilde{\mu}$) = 3.74 , R($\tilde{\xi}$) = 0.2, R($\widetilde{C_l}$) = 205.23, R($\widetilde{Cs}$) = 20.52, R($\widetilde{C_h}$) = 9.12, R($\widetilde{C_r}$) = 37.46, R($\tilde{R}$) = 374.62

Here we calculate the variation in Cost Expedients with respect to q.

S.No	q	C_R	**TEC**	**TER**	**TEP**
1	0	0	**82.60**	**185.66**	**103.60**
2	0.1	5	**82.48**	**187.59**	**105.11**
3	0.2	10	**82.38**	**189.47**	**107.09**
4	0.3	15	**82.31**	**191.32**	**109.01**
5	0.4	20	**82.21**	**193.11**	**110.90**
6	0.5	25	**82.19**	**195.93**	**113.74**
7	0.6	30	**82.08**	**197.71**	**115.63**
8	0.7	35	**81.99**	**199.64**	**117.65**
9	0.8	40	**81.88**	**201.62**	**119.74**
10	0.9	45	**81.77**	**203.53**	**121.76**
11	1.0	50	**81.65**	**205.40**	**123.75**

Table 2 : Cost Expedients Vs q

Table 2 describes that the increment in the probability q makes a steady decrease in TEC and steady increase in TER and TEP.

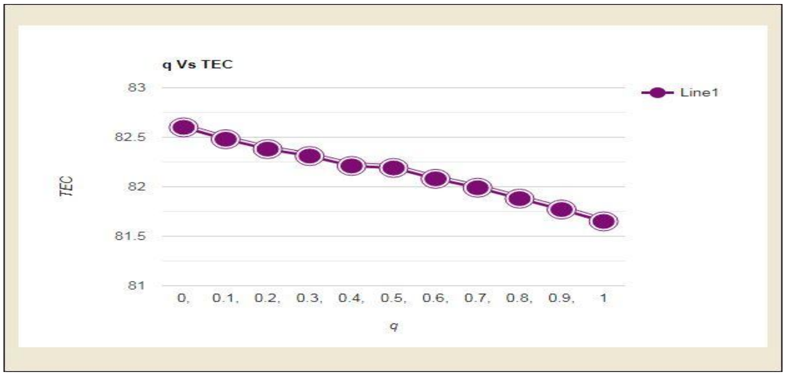

Fig 7. q Vs TEC

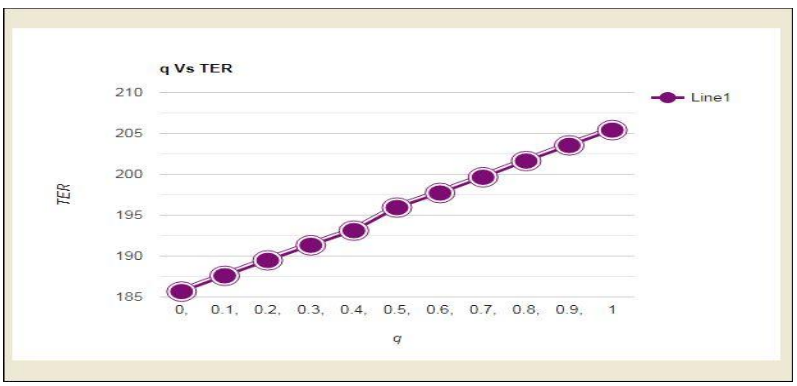

Fig 8. q Vs TER

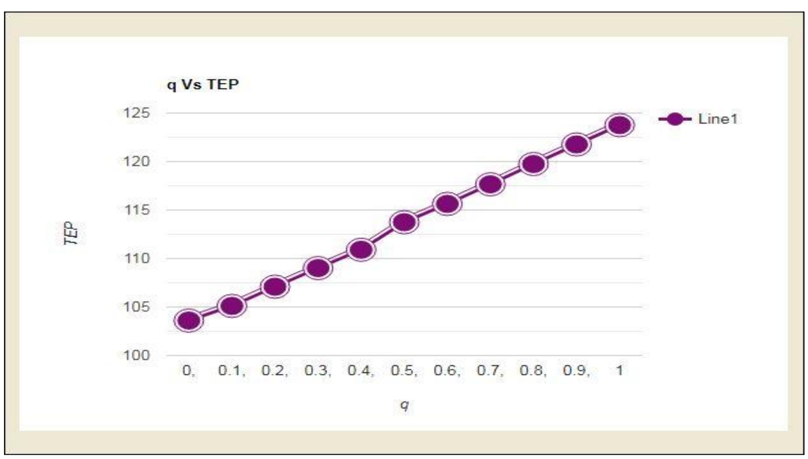

Fig 9. q Vs TEP

Figure 7 shows that the increment in the probability q makes a steady decrement in TEC and Figures 8,9 describe that the increment in the probability q makes a steady increment in TER and TEP.

CONCLUSION

This article analyzed the discouraged arrivals Markovian Queueing system with a new incenter grounded fuzzy ordering approach. This approach is more loyal and stranger than the others. The results which we got by this approach are very powerful and concrete. In the profit wise, this will very beneficial to take verdicts.

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