Predicting the Behavior of Gold Price Using Markov Chains and Markov Chains of the Fuzzy States

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Article Info
Page Number: 2906-2920
Publication Issue:
Vol. 71 No. 4 (2022)

Article History
Article Received: 25 March 2022
Revised: 30 April 2022
Accepted: 15 June 2022
Publication: 19 August 2022

Abstract
In this work we consider gold prices as a case study. Closing retraction $\mathbf{R_t}$ is studied as a fuzzy concept and several types of fuzzy numbers are applied to $\mathbf{R_t}$: triangular, trapezoidal, parabolic and K-Trapezoidal-Triangular fuzzy numbers. We construct a Markov chain (MC) and a Markov chain with fuzzy states (MCFS) and compare between them. The two models MC and MCFS are used to predict the behavior of gold price. At the end, we estimate the expected return price in specific months. We reach that MCFS has more accuracy than the MC

Keywords- Markov Chain (MC), Markov Chain with Fuzzy States (MCFS), Fuzzy Numbers, Return Rate.

I. INTRODUCTION

Results based on probability theory do not always provide helpful information due to the restriction of being able to handle only quantitative information. Moreover, in real world applications, sometimes there is a lack of data to deal with the statistics of parameters precisely. To conquer these complications, methodologies based on fuzzy set theory have been used in the risk analysis for spreading the basic event uncertainty [1-2].

The idea of fuzzy logic was first advanced by Zadeh (1965) where he defined fuzzy sets in order to describe unclear situations mathematically [30]. Fuzzy matrices were introduced for the first time by *Thomason* (1977), who discussed the convergence of powers of fuzzy matrix [25]. Kruce et al. (1987) introduced the fuzzy Markov chain as a classical Markov chain based on fuzzy probabilities where he used a fuzzy set to denote the transition matrix with the uncertain data in the Markov chains [12]. A fuzzy Markov chain was demonstrated as the concept of fuzzy relation and its compositions [22]. It can be used while the decision maker prefers subjective probabilities to model the uncertainties [27]. Yoshida (1994) constructed a *Markov fuzzy* process, with a *transition possibility measure* [29]. Slowinski (1998) showed that we can use a fuzzy set representation in order to deal with uncertain data and flexible requirements [23].

Fuzzy Markov chains approaches were introduced by Avrachenkov and Sanchez (2000). They analyzed fuzzy Markov chains and its properties in detail [28]. Kuranoa et al. (2006) used fuzzy states to show fuzzy transition probabilities [13]. Pardo and Fuente (2010) used Markovian decision processes with fuzzy states to calculate the best policy to be implemented regarding publicity decisions in a queueing system [21]. Zhou et al. (2013) used fuzzy probability-based Markov chain model to estimate regional long-term electric power demand [31]. Ky and Fuente (2016) used combination of Markov model and fuzzy time series model for forecasting stock market data [14]. Kiral and Uzun (2017) used Markov chain of the fuzzy states to estimate stock market index [10].

In this paper, we consider gold prices as case study. Closing retraction $\mathrm{R_t}$ is studied as a fuzzy concept and several types of fuzzy numbers are applied to $\mathrm{R_t}$: triangular, trapezoidal, parabolic and K-Trapezoidal-Triangular fuzzy numbers. We construct a Markov chain (MC) and a Markov chain with fuzzy states (MCFS) and compare between them. The two models MC and MCFS are used to predict the behavior of gold price. At the end, we estimate the expected return price in specific months.

II. Fuzzy Sets and Fuzzy Numbers

A fuzzy set may be viewed as an extension and generalization of the basic concepts of crisp sets. An important property of a fuzzy set is that it allows partial membership, i.e. between 0 and 1. Zadeh [30] extended the notion of valuation set {0,1} (definitely in / definitely out) to the interval of real values (degree of membership) between 1 and 0, 0.0 represents absolutely false and 1.0 represents absolutely truth [7].

The fuzzy set $\widetilde{A}$ in the universe of discourse Ω is defined as a set of ordered pairs $(x,\mu_{\tilde{A}(x)})$, i.e. $\tilde{A}=\{(x,\mu_{\tilde{A}(x)}) \mid x\in \Omega\}$where $\mu_{\tilde{A}(x)}$is the degree of membership of x in fuzzy $\tilde{A}$ and it indicates the degree that x belongs to $\tilde{A}$ [4].

Let $\tilde{A}$ be a fuzzy subset of Ω. An α – level of $\tilde{A}$, written $\left[\tilde{A}\,\right]_{\alpha}$, is defined as $\{x\epsilon\, \Omega{:}\, \tilde{A}\ (x) \geq \alpha\}$ for $0 < \alpha \leq 1$. $[\tilde{A}\,]_0$, the support of $\tilde{A}$ is defined as the closure of the union of all the $\left[\tilde{A}\,\right]_{\alpha}$, for $0 < \alpha \leq 1$. The core of $\tilde{A}$ is the set of all elements in Ω with membership degree in $\tilde{A}$ equal to 1.

A fuzzy number N is a fuzzy subset of the real numbers satisfying:

(1) $\exists x{:}\, N(x) = 1$ (2) $[N]_{\alpha}$is a closed and bounded interval for $0 \leq \alpha \leq 1$.

The family of all fuzzy numbers are denoted by R_F.

Triangle Fuzzy Numbers, Trapezoidal Fuzzy Numbers, Parabolic Fuzzy Numbers and K-Trapezoidal-Triangular Fuzzy Numbers are special types of fuzzy numbers ([1], [3], [15-20]).

III. RETURN PRICE R_t AS FUZZY CONCEPT

Return prices for Gold closing prices are transformed into 21 states, from the high loss S_{-10} *to the positive high return* S_{10} *such that each state has the same length* k *; that is, all of them have the same chance of occurrence.*

Suppose that the return price for a month of a certain year $R_t = 0.765$. *Therefore, the position of that* R_t *is tried to be obtained from 21 states by using the relation:*

$i = \frac{R_t}{k}$ (1), *where* i *is the position.*

If we consider k=0.225, then: $i = \frac{0.765}{0.225} = 3.4$.

According to the previous result, the position of R_t *is difficult to be determined certainly and clearly. However, we can conclude that* R_t *is lying between the third and the forth state, which moves away from the third state to be closer to the forth state by 0.4. (i.e. it gets closer to the third state by 0.6).*

So, the membership degree of R_t *in* $\tilde{S}_3 = 4 - 3.4 = 0.6$ *and the membership degree of* R_t *in* $\tilde{S}_4 = 1 - 0.6 = 0.4$.

In general, $i = \left\lfloor \frac{R_t}{k} \right\rfloor$ (2).

$\tilde{S}_i = (i + 1) - \frac{R_t}{k}$ (3) *And* $\tilde{S}_{i+1} = 1 - \tilde{S}_i$ (4).

Let, k = 0.225 as a special case. Then, $\tilde{S}_i$ *will be:*

$$\tilde{S}_i = (i + 1) - \frac{R_t}{0.225}$$

$\tilde{S}_i = \frac{0.225(i+1) - R_t}{0.225}$ (5).

Thus the monthly percentage changes of the market price are transformed into 21 fuzzy states from the high loss S_{-10} *to high return* S_{10}. *Next, we will apply different types of fuzzy numbers to the return price* R_t*:*

<u>Case 1: Triangle Fuzzy Numbers (a_1, a_2, a_3):</u>

By using the equation of a straight line passing through two points

$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$ (6)

Where $the\ points\ are$:

$(x_1, y_1) = \big((i + 1)(0.225), 0\big)$

$(x_2, y_2) =$ $((i)(0.225), 1)$

$\frac{y-0}{x-(i+1)(0.225)} = \frac{1}{-0.225}$

$y = \frac{x-(i+1)(0.225)}{-0.225} = \frac{(i+1)(0.225)-R_t}{0.225}$,

which is similar to eq. (5).

Case 2: Trapezoidal Fuzzy Numbers (a_1, a_2, a_3, a_4):

If $R_t \in (a_2, a_3)$, $then\ \tilde{S}_i = 1$, $i = \left\lfloor \frac{R_t}{0.15} \right\rfloor$.

We deal with single value of (a_2, a_3), we take the *midpoint* $\frac{a_2+a_3}{2}$ [9]:

$$\frac{\left(i-\frac{1}{3}\right)(0.225) + (i+\frac{1}{3})(0.225)}{2}$$

$= (0.225)i$

$= a_2$ in the triangle fuzzy number case.

Case 3: Parabola "Triangle Shape" Fuzzy Numbers (a_1, a_2, a_3):

By referring to the parabolic membership function, we note that the relationship is the square of the triangle membership function

$$\tilde{S}_i = [\frac{(i+1)(0.225) - R_t}{0.225}]^2$$

As concluded above, the principle of the fuzziness in the estimation of the states for the return in prices is identical to the relationship of the triangle; this means that the use of this type of fuzzy numbers gives less accurate results. For example:

Take $R_t = 0.5175$, $i = \frac{0.5175}{0.225} = 2.3, this\ i = \lfloor 2.3 \rfloor = 2.$

In general, $by\ \ Eq\ (6)$:

$$\tilde{S}_i = (i+1) - \frac{R_t}{0.225}$$

$R_t = 0.5175,\ \tilde{S}_2 = 3 - \frac{0.5175}{0.225} = 0.7.$

In parabola case: $S_2 = (0.7)^2 = 0.49$

$error = 0.7 - 0.49 = 0.21.$

Case 4: K-Triangle-Trapezoidal Fuzzy Numbers

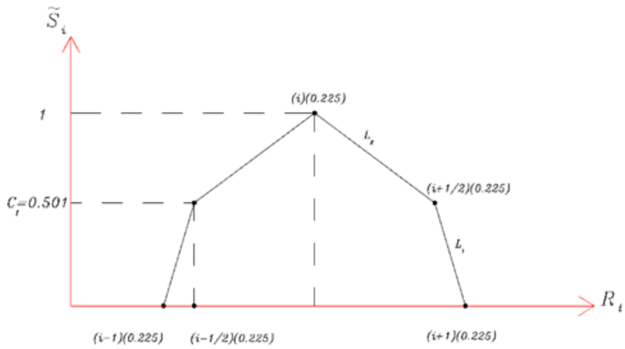

Figure 1: Coordinates K-triangle-trapezoidal fuzzy state

For line L_1:

$$\tilde{S}_i = (0.501)\left(\frac{(i+1)(0.225)-R_t}{0.1125}\right) \quad (8)$$

For line L_2:

$$\tilde{S}_i = 1-(0.499).\left(\frac{R_t-i(0.225)}{0.1125}\right) \quad (9)$$

the closer the C_1 value to 0.5, the more accurate the result (Figure 3).

The best result when $C_1 = 0.5$ **(**triangle fuzzy number case).

IV. Important Remark

In [9-10] and [26], it was mentioned that:

$$\tilde{S}_i = \frac{(i+1)(0.225-R_t)}{0.225}$$

Actually this is not true as we reached in eq. 5. However, in [11] they mentioned the correct equation:

$$\tilde{S}_i = \frac{\boldsymbol{(i+1)(0.225)-R_t}}{\boldsymbol{0.225}}$$

V. The Sample

The study includes monthly data between January 2010 and July 2020. The monthly weighted average of the gold price received from Istanbul.

The return R_t were calculated as monthly percentage of the gold price $R_t = ((P_t - P_{t-1})/P_{t-1}) * 100\%$, ([5], [26]), where t denotes the sessions $(t = 2, 3, \ldots, 127)$.

The average return μ_R is approximately 0.46%, where the standard deviation is 3.69%, for the given period which is 8 times higher than expected return.

VI. THE MODELS

A) The Markov Chain Model

Closing returns of the gold price are transformed into 21discrete categorical states from high loss S_{-10} to the positive high return S_{10} according to functions below. For this aim, we define k integer numbers which are based on R_t as $k-1<\frac{R_t+0.12\%}{0.24\%}\leq k$ where: $-2.28\%<R_t\leq 2.28\%$

The k-th state for $k\in\{-9,\dots,9\}$ is as follows [10]:

$$S_{10}=\begin{cases}1, & R_t>2.28\%\\ 0, & Otherwise\end{cases}$$

$$S_k=\begin{cases}1, & (2k-1)0.12\%<R_t\leq(2k+1)0.12\%\\ 0, & Otherwise\end{cases}$$

$$S_{-10}=\begin{cases}1, & R_t\leq 2.28\%\\ 0, & Otherwise\end{cases}$$

As shown in Table (1) and figure (2), we transform126 closing returns of Gold price to the defined 21 discrete states. Then, we calculate all transitions numbers of states from the present session to the next session for the considered period. We also use conditional probabilities of the Markov chain to obtain one-step.

The transition probability matrix P is shown in Table (2) in the appendix, which is the one step transition probability matrix estimated by Maximum Likelihood Method (M.L.E) [6]:

$$\widehat{P_{ij}}=n_{ij}\,/\sum_j n_{ij}\geq 0 \qquad (10)$$

B) The Markov Chain with Fuzzy States Model

Closing returns of the Gold price are transformed into 21 fuzzy states from the high loss $\widetilde{S}_{-10}$to high return $\widetilde{S}_{10}$ shown in Figure 3.We use triangular fuzzy numbers to obtain the membership degree of R_t to the fuzzy states.

We define fuzzy state components of the returns $\tilde{S}_i$ and $\tilde{S}_{i+1}$ as follows:

If $-2.25\%<R_t<2.25\%$ then $i=\left[\!\left[\frac{R_t}{0.225}\right]\!\right]$ and $\tilde{S}_i=\frac{(i+1)(0.225)-R_t}{0.225}$,

$$\tilde{S}_{i+1}=1-\tilde{S}_i$$

If $R_t\leq -2.25\%$ or $R_t\geq 2.25\%$ then $\tilde{S}_{-10}=1$ or $\tilde{S}_{10}=1$ respectively [11].

Therefore, R_t numbers are transformed to the triangular fuzzy numbers for the considered time.

Within this framework, MCFS that depends on the fuzzy set theory gives more precision and realistic description to the problems than the MC.

Then we obtain the probabilistic transition matrix of the fuzzy states by using $\widetilde{P}=S\overline{P}=SPQ$

[21].

Firstly, monthly percentage changes of the gold price are transformed into 21 fuzzy states from the high loss S_{-10} to high return S_{10}(see table 3).

We classify the states as triangular fuzzy numbers.

If $-2.25 < R_t < 2.25\%$ then $i = \left\lfloor \frac{R_t}{0.225} \right\rfloor$

$$\tilde{S}_i = \frac{(i+1)(0.225) - R_t}{0.225}, \qquad \tilde{S}_{i+1} = 1 - \tilde{S}_i$$

If $R_t \leq -2.25$ or $R_t \geq 2.25$ then: $\tilde{S}_{-10} = 1\,, \tilde{S}_{10} = 1$

To obtain the Markov chain of the fuzzy states:

$$\tilde{P} = SPQ$$

P = the transition probability matrix

$$\boldsymbol{Q} = \begin{bmatrix} \mu_{\tilde{S}_{-10}}(-10) & \mu_{\tilde{S}_{-9}}(-10) & \cdots & \mu_{\tilde{S}_{10}}(-10) \\ \mu_{\tilde{S}_{-10}}(-9) & \mu_{\tilde{S}_{-9}}(-9) & \cdots & \mu_{\tilde{S}_{10}}(-9) \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{\tilde{S}_{-10}}(10) & \mu_{\tilde{S}_{-9}}(10) & \cdots & \mu_{\tilde{S}_{10}}(10) \end{bmatrix}$$

Where $\mu_{\tilde{S}_{-10}}, \mu_{\tilde{S}_{-9}}, \dots, \mu_{\tilde{S}_{10}}$ denote the membership functions of fuzzy states of $\{\tilde{S}_{-10}, \tilde{S}_{-9}, \dots \tilde{S}_{10}\}$ so, $\mu_{\tilde{S}_{-10}}(-10)$ is the degree of possibility that the ith position of closing return price R_t with $i = -10$ belongs to the fuzzy state $\tilde{S}_{-10}$. If there are values of R_t having the same position i, then we have a group of finite numbers (n) for the same $\mu_{\tilde{S}_i}(i)$, where $\mu_{\tilde{S}_i}(i)$ is a fuzzy number defined as:

$ht\ (\bar{A}) = \bar{A}_i$

$ht\ (\bar{A}_i) = Max\left\{\mu_{\tilde{S}_{i1}}(i), \mu_{\tilde{S}_{i2}}(i), \dots, \mu_{\tilde{S}_{in}}(i)\right\} = \mu_{\tilde{S}_i}(i)$ [3].

Since, $\tilde{S}_{i+1} = 1 - \tilde{S}_i$ so, $\mu_{\tilde{S}_{i+1}}(i) = 1 - \mu_{\tilde{S}_i}(i)$

$$\boldsymbol{S}=\begin{bmatrix} \frac{P_{-10}\mu_{\tilde{S}_{-10}}(-10)}{P(\tilde{S}_{-10})} & \frac{P_{-9}\mu_{\tilde{S}_{-10}}(-9)}{P(\tilde{S}_{-10})} & \cdots & \frac{P_{10}\mu_{\tilde{S}_{-10}}(10)}{P(\tilde{S}_{-10})} \\ \frac{P_{-10}\mu_{\tilde{S}_{-9}}(-10)}{P(\tilde{S}_{-9})} & \frac{P_{-9}\mu_{\tilde{S}_{-9}}(-9)}{P(\tilde{S}_{-9})} & \cdots & \frac{P_{10}\mu_{\tilde{S}_{-9}}(10)}{P(\tilde{S}_{-9})} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{P_{-10}\mu_{\tilde{S}_{10}}(-10)}{P(\tilde{S}_{10})} & \frac{P_{-9}\mu_{\tilde{S}_{10}}(-9)}{P(\tilde{S}_{10})} & \cdots & \frac{P_{10}\mu_{\tilde{S}_{10}}(10)}{P(\tilde{S}_{10})} \end{bmatrix}$$

We assume that the initial probabilities P_i are equal, so P_i = 1/21. With P_i we calculate the fuzzy initial state probabilities using:

$$P(\tilde{S}_i)=\sum_{S=-10}^{10} P_s\mu_{\tilde{S}_i}(s).$$

$P_i =$ The probability of beginning in state (i).

The transition probability matrix corresponding to the fuzzy states $\{\tilde{S}_{-10}, \tilde{S}_{-9}, \ldots \tilde{S}_{10}\}$, $\tilde{P} = [P(\tilde{A}_j/\tilde{A}_i)]$, obtained with: $\tilde{P}$ =SPQ [21] which is Table (4), where the closing returns of gold price are considered as a stochastic process with 21 fuzzy state $\{\tilde{S}_{-10}, \tilde{S}_{-9}, \ldots \tilde{S}_{10}\}$ space with Markov chain structure.

VII. Estimating Closing Return $\hat{R}_t$

A) Using Markov Chain

$\hat{R}_t = \sum x_i\, P(x_i)$, where x_i denotes the middle point of the discrete categorical states for i=–9,.., 9 and the boundaries of the states for i= −10,10.

We get $\hat{R}_t = -1.92\%$.(see table 5).

B) Using Markov Chain with Fuzzy States

$\hat{R}_t = \sum x_i\, P(x_i)$ where x_i denotes the middle point of the fuzzy states for $i = -9, \ldots, 9$ and the boundaries of the states for $= -10, \ldots, 10$.

We get $\hat{R}_t = -0.9802\%$ and the actual value of closing return R_t=-1,259% on October, 2012.

In table (6) we have shown the predicted probability of closing return $\tilde{P}(x_i)$ on October, 2012 with MCFS model and the expected return $(\hat{R}_t)$. In Table 7, we have shown some estimation results for other months chosen randomly. We used the mean absolute error MAE to measure how our prediction is close to the eventual outcomes.

$MAE = \frac{1}{n}\sum_{i=1}^{n}|x_i - x|$where e_i denotes the error. From Table 4-7, one can see that MCFS model is better than MC model for forecasting the gold return price.

VIII. THE LONG RUN BEHAVIOR OF MC & MCFS

The MC and MCFS models can be used to predict for long run time using: $\pi P = \pi,$ when the MC is Ergodic (satisfied since after a number of steps there are no zero entries in the matrix) where π is the limiting stationary distribution [8] and [24].We find that:P^{12} (hence the limit of P^n as n goes to infinity) consists approximately of identical rows each of which is: (0.2382, 0.0030, 0.0282, 0.0391, 0.0591, 0.0309, 0.0118, 0.0080, 0.0155, 0.0394, 0.0164, 0.0213, 0.0245, 0.0335, 0.0119, 0.0291, 0.0195, 0.0249, 0.0083, 0.0350, 0.2979).

We see that the highest ratio is for the highest return $\tilde{S}_{10} \approx 30\%$ and then for the highest loss $\tilde{S}_{-10} \approx 24\%$.

In the long run, no matter the state of gold price in a month, the number of gain months will be approximately equal to the number of loss. Categories of the states and the conditional transition probability matrices are calculated by MATLAB program. If we apply the law of expectation using equation:

$$\hat{R}_{t\ t\to\infty} = \sum_{i=1}^{21} \pi_i x_i$$

We get, $\hat{R}_t = 0.1104\%$ and its present return state as in table 8.

CONCLUSION

According to our results, one can minimize the steps of finding the degree of belonging to the return price and can only rely on the concept of Fuzzy to find the $\mathrm{R_t}$ site of the relationship $\mathrm{i} = \frac{\mathrm{R_t}}{\mathrm{k}}$.

After applying several types of fuzzy number, we found that Triangle Fuzzy Number are the best fuzzy numbers to get the most accurate results.

We have predicted the behavior of the Gold return price for next month. Using the MC and MCFS models. The results gave sensitive and significant information to the investors about investment opportunities of the Gold return price for the monthly buying and selling strategies when the present return is known. In risky months, when a monthly return substantially increased or decreased, the next month's return also substantially increased or decreased for both models. The transition probabilities of monthly returns in non-risky months would be significantly lower than those in risky months for both models.

The MCFS model can be used to forecast the returns for smaller time (less than one month) intervals, which may give more investment opportunities. Investors can earn higher than the average return in risky months in a short run. Besides the MCFS model can be used to predict the returns for smaller time (one day) and also different classifications and fuzzy sets, which may give more investment opportunities. The probability distribution of gold showed that, the investors can gain higher return in the long run.

Our results, by predicting the behavior of gold prices in the long term, indicate that the stability will be reached after 12 months. From the last reading, on July, 2020, prices were constantly increasing. According to our results, the market will witness a decline in gold prices until reaching the stage of stability, and this is what is mostly happening the last few months.

Acknowledgment: The authors would like to thank Palestine Technical University Kadoorie for the support the authors obtained from their respective university while executing the research.

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Appendix

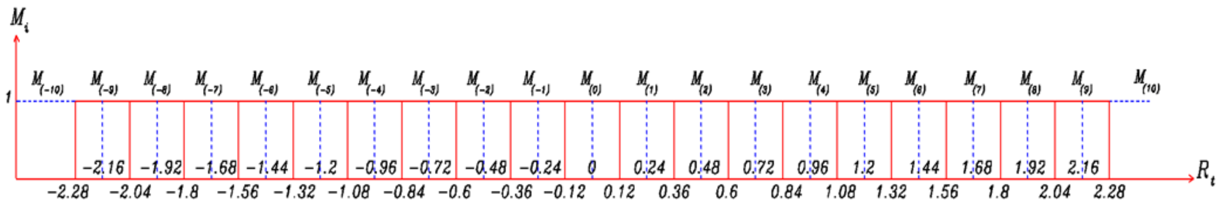

Figure 2: Discrete categorical state

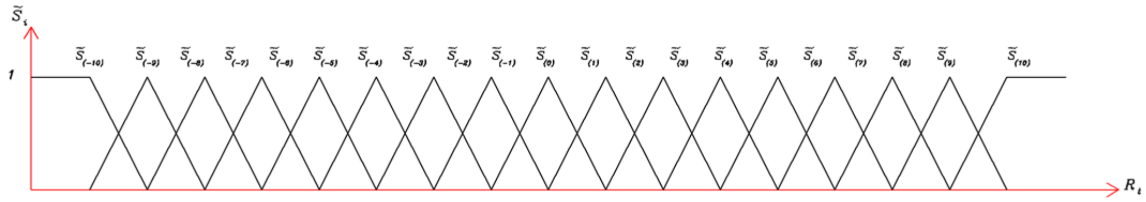

Figure 3: Fuzzy State for Gold Return Price

Table 1: Transformed States of the monthly Closing Returns for Some months

Date	R_t	S_{-10}	S_{-9}	S_{-8}	S_{-7}	S_{-6}	S_{-5}	S_{-4}	S_{-3}	S_{-2}	S_{-1}	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
01-2010	-2.12%	.44	.56	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
02-2010	1.80%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.99	.01	0
03-2010	2.95%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
11-2010	1.58%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.96	.04	0	0
12-2010	-2.37%	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
01-2011	0.80%	0	0	0	0	0	0	0	0	0	0	0	0	0	.46	.54	0	0	0	0	0	0

Table 2: The One Step Probability Transition Matrix P

	S_{-10}	S_{-9}	S_{-8}	S_{-7}	S_{-6}	S_{-5}	S_{-4}	S_{-3}	S_{-2}	S_{-1}	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_	0.2581	0.0644	0.0644	0.0323	0.0644	0.0323	0.0323	0.0000	0.0000	0.0323	0.0644	0.0000	0.0323	0.0323	0.0000	0.0000	0.0968	0.0323	0.0323	0.0323	0.0968
S_	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

S_{-}	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-}	0.6000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000
S_{-}	0.5714	0.0000	0.0000	0.0000	0.0000	0.1429	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2857	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-}	0.3333	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.3334	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000
S_{-}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
S_{-}	0.0000	0.0000	0.0000	0.0000	0.4000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.2000	0.2000
S_0	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3334
S_1	0.5000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8000
S_3	0.3333	0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3334
S_4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
S_5	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3333
S_7	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000
S_8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
S_9	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.2000	0.0000	0.0000	0.4000
S_1(	0.2000	0.0000	0.0286	0.0571	0.0286	0.0000	0.0000	0.0000	0.0286	0.0286	0.0286	0.0571	0.0000	0.0286	0.0286	0.0286	0.0000	0.0000	0.0000	0.0857	0.3713

Table 2: Transformed fuzzy states of the closing returns for some months

Date	R_t	S_{-10}	S_{-9}	S_{-8}	S_{-7}	S_{-6}	S_{-5}	S_{-4}	S_{-3}	S_{-2}	S_{-1}	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
01-2010	-2.12%	***.44***	***.56***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***
02-2010	1.80%	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***.99***	***.01***	***0***
03-2010	2.95%	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***1***
11-2010	1.58%	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***.96***	***.04***	***0***	***0***
12-2010	-2.37%	***1***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***
01-2011	0.80%	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***0***	***.46***	***.54***	***0***	***0***	***0***	***0***	***0***	***0***

Table 3: Probabilistic transition matrix of the fuzzy states

	S_{-10}	S_{-9}	S_{-8}	S_{-7}	S_{-6}	S_{-5}	S_{-4}	S_{-3}	S_{-2}	S_{-1}	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
S_{-10}	0.2581	0.0644	0.0644	0.0323	0.0644	0.0323	0.0323	0.0000	0.0000	0.0323	0.0644	0.0000	0.0323	0.0323	0.0000	0.0000	0.0968	0.0323	0.0323	0.0323	0.0968
S_{-9}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
S_{-8}	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-7}	0.6000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000
S_{-6}	0.5714	0.0000	0.0000	0.0000	0.0000	0.1429	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2857	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-5}	0.3333	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.3334	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-4}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000
S_{-3}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_{-2}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
S_{-1}	0.0000	0.0000	0.0000	0.0000	0.4000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.2000	0.2000
S_0	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3334
S_1	0.5000	0.0000	0.0000	0.0000	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_2	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.8000
S_3	0.3333	0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3334
S_4	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
S_5	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3333	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
S_6	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.6667	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3333
S_7	0.5000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.5000
S_8	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
S_9	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2000	0.0000	0.0000	0.2000	0.0000	0.0000	0.4000
S_{10}	0.2000	0.0000	0.0286	0.0571	0.0286	0.0000	0.0000	0.0000	0.0286	0.0286	0.0286	0.0571	0.0000	0.0286	0.0286	0.0286	0.0000	0.0000	0.0000	0.0857	0.3713

Table 4:Estimated closing return ($\hat{R}_t$) with MC model for October, 2012

	S_{-10}	S_{-9}	S_{-8}	S_{-7}	S_{-6}	S_{-5}	S_{-4}	S_{-3}	S_{-2}	S_{-1}	S_0	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
$P(x_i)$	0.5	0	0	0	0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x_i	-2.40	-2.16	-1.92	-1.68	-1.44	-1.20	-0.96	-0.72	-0.48	-0.24	0.00	0.24	0.48	0.72	0.96	1.20	1.44	1.68	1.92	2.16	2.40

Table 5: Estimated closing return with MCFS model for October 2012

	$\tilde{S}_{-10}$	$\tilde{S}_{-9}$	$\tilde{S}_{-8}$	$\tilde{S}_{-7}$	$\tilde{S}_{-6}$	$\tilde{S}_{-5}$	$\tilde{S}_{-4}$	$\tilde{S}_{-3}$	$\tilde{S}_{-2}$	$\tilde{S}_{-1}$	$\tilde{S}_0$	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{S}_3$	$\tilde{S}_4$	$\tilde{S}_5$	$\tilde{S}_6$	$\tilde{S}_7$	$\tilde{S}_8$	$\tilde{S}_9$	$\tilde{S}_{10}$
$P(x_i)$	0.587	0.013	0.096	0.108	0.370	0.078	0.026	0.064	0.021	0.053	0.045	0.021	0.020	0.036	0.009	0	0.049	0.078	0.033	0.030	0.263
x_i	-2.25	-2.03	-1.80	-1.58	-1.35	-1.13	-0.90	-0.86	-0.45	-0.23	0.00	0.23	0.45	0.68	0.90	1.13	1.35	1.58	1.80	2.03	2.25

Table 6: Estimated R_t for some months

Date	$\hat{R}_t$ with MC model	$\hat{R}_t$ with MCFS model	Actual (R_t)	$\lvert e_i \rvert$ with MC model	$\lvert e_i \rvert$ with MCFS model
October ,2012	-1.92%	-0.98%	-1.26%	0.66%	0.28%
October , 2019	0.72%	-0.52%	-1.53%	2.25%	1.01%
November,2010	1.20%	0.88%	1.58%	0.38%	0.70%
May ,2020	1.20%	0.89%	1.06%	0.16%	0.17%
February,2018	0%	-0.38%	-0.62%	0.62%	0.24%
MAE	0.81%	0.48%			

Table 8: Present state for long run R ̂(t) with MCFS

R_t	$\tilde{S}_{-10}$	$\tilde{S}_{-9}$	$\tilde{S}_{-8}$	$\tilde{S}_{-7}$	$\tilde{S}_{-6}$	$\tilde{S}_{-5}$	$\tilde{S}_{-4}$	$\tilde{S}_{-3}$	$\tilde{S}_{-2}$	$\tilde{S}_{-1}$	$\tilde{S}_0$	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{S}_3$	$\tilde{S}_4$	$\tilde{S}_5$	$\tilde{S}_6$	$\tilde{S}_7$	$\tilde{S}_8$	$\tilde{S}_9$	$\tilde{S}_{10}$
0.1104	0	0	0	0	0	0	0	0	0	0	0.51	0.49	0	0	0	0	0	0	0	0	0