Some Exponentiated Distributions with Various Applications

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Abstract

The expansion of weibull exponentiated, exponential exponentiated, lognormal exponentiated, gumble exponentiated and exponentiated exponential were discussed. Weibull exponentiated embellishes to match unimodel, monotone and hazardous functions unlike the weibull model. Both the shape and the scale parameter of exponential exponentiated are similar to the gamma and weibull distribution. Specific lognormal (three parameters) and exponentiated gumble (two parameters) are unimodel distributions and can show better equilibrium. The exponential exponentiated distribution provides a highly flexible model for lifetime data. The parameter estimation were performed with the same probability and equal efficiency. The applications were defined by three data sets.

Keywords- Exponentiated weibull, Exponentiated exponential, Exponentiated lognormal, Exponentiated gumble, Extended exponentiated exponential, Maximum likelihood estimators, Goodness of fit.

INTRODUCTION

Evaluation of lifetime data sets are usually done by gamma and weibull. The gamma distribution is not so much perceived as weibull distribution as in the case of gamma distribution the survival function cannot be detected in a closed way unless the shape parameter is a whole number. Data derived from the engineering and technical trainings often find a place in survival data. Mudhokar and Srivastava (1993) and Mudhokar et al (1995) confirmed in their study various sets of data time failures for the proposed weibull distribution. A Marshall and I. Olkin (1997) introduced a new way to add a parameter to a distribution family by utilizing descriptive and weibull families. The enhanced weibull distribution has a unique feature of the standard weibull class described by Gupta et al (1998). Gupta and Kundu (2001) presented a life-time data and noted that the distribution may be substituted. Nadarajah (2005) presented gumble distribution of survival work in clarifying rain data from Orland, Florida. Real-life data sets were also studied by Kakade and Shirke (2006) and found that enhanced lognormal distribution is well matched compared to weibull and descriptive distribution. Raja and Mir (2011) discussed the expansion of other distributions by distinct applications. Bagheri et al (2014) has been very active in the effective pdf estimation and cdf of gumble distribution s. Abu-Youssef et al (2015) introduced a descriptive extension that provides a comparatively flexible model for lifetime data sets in most cases. Suresh and Usha (2016) performed a reduced health trial on a descriptive dose. Raja and Maqbool (2019) also extended application of Poisson and Poisson type distributions. Malik Mansoor and Kumar Devendra (2020) studied exponential weibull model. However the exponentiated gumble distribution is a well fit and can be used in place of weibull. Here exponentiated exponential distribution gives comparatively better fit than three parameter weibull and exponentiated gumble.

Three data sets were utilized by exploiting these distributions together and comparisons were explained.

1. Exponentiated Weibull Distribution

Probability density function

Probability density function (p.d.f) of exponentiated weibull(EW) distribution as considered by Mudhokar et al(1995) with parameter $\alpha, \theta, and \sigma$ is given as

$$f(t;\alpha,\theta,\sigma) = \frac{\alpha\theta}{\sigma}\left[1-\exp(-\left(\frac{t}{\sigma}\right)^{\alpha}\right]\exp\left[-\left(\frac{t}{\sigma}\right)^{\alpha}\right]\left(\frac{t}{\sigma}\right)^{\alpha-1}, \nabla t > 0 \ldots\ldots\ldots\ldots(1)$$

Where α >0, θ>0 are shape parameters and σ >0 is a scale parameter.

Specifies the weibull distribution when θ = 1 and the descriptive distribution when α = 1 and θ = 1.

Survival work associated with random T variance and weibull exponentiated is provided as

$$S(t;\alpha,\theta,\sigma) = P(T \geq t) = 1-\left[1-\exp\left(-(\frac{t_i}{\sigma})^{\alpha}\right)\right]^{\theta} \ldots\ldots\ldots\ldots\ldots\ldots(1.2)$$

Maximum Likelihood Estimators

Maximum likelihood estimators for a three parameter EW are given as.

Let $x_1, x_2, x_3 ....x_n$ be a random sample from EW and the log likelihood can be worked out as

L(α, θ, σ)=n. log (αθ/σ)+

$$(\theta\text{-}1).\sum_{i=1}^{n}\log(g(Ti)) - \sum_{i=1}^{n}(Ti/\sigma)^{\alpha} + (\alpha-1).\sum_{i=1}^{n}\log(Ti/\sigma)\ldots\ldots\ldots(1.3)$$

Where g(Ti)=g(Ti;α,θ)=1-exp(-T/σ)$^{\alpha}$

Differentiate (1.1) with respect to three parameters.

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + (\theta-1).\sum_{i=1}^{n} g_{\alpha}(Ti)/g(Ti) - \sum_{i=1}^{n}(Ti/\sigma)^{\alpha}.\log(Ti/\sigma) + \sum_{i=1}^{n}\log(T/\sigma) = 0\ldots(1.4)$$

$$\frac{\partial L}{\partial \theta} = \frac{n}{\theta} + \sum_{i=1}^{n} \log(g(Ti)) = 0 \ldots\ldots\ldots\ldots \text{(1.5)}$$

$$\frac{\partial L}{\partial \sigma} = -\left(\frac{n\alpha}{\sigma}\right) + (\theta - 1).\sum_{i=1}^{n} g_\sigma(Ti) / g(Ti)) + (\alpha / \sigma).\sum_{i=1}^{n} (Ti/\sigma)^\alpha \ldots\ldots\ldots\ldots \text{(1.6)}$$

where

$$g_\alpha(Ti) = \exp(-(Ti/\sigma)^\alpha.(Ti/\sigma)^\alpha.\log(Ti/\sigma),$$
$$g_\sigma(Ti) = -\left[\alpha.\exp(-(Ti/\sigma)^\alpha).(Ti/\sigma)^\alpha\right]/\sigma$$

From (1.4), (1.5) and (1.6) we obtain the ML Estimates.

2. **Exponentiated Exponential**

Probability density function

The Probability density function of exponentiated exponential is defined by

Gupta and Kunda (2001) with parameters α and λ as

$$f(x, \alpha, \lambda) = \alpha\lambda\left(1 - e^{-\lambda x}\right)^{\alpha - 1}.e^{-\lambda x} \ldots\ldots\ldots\ldots \text{(2.1)}$$

Where $\alpha, \lambda, x > 0$

Here α is the shape parameter and λ is the scale parameter. For α=1 it signify

the exponential family.

The survival function corresponding with exponentiated-exponential density is given as

$$S(x, \alpha, \lambda) = 1 - \left(1 - e^{-\lambda x}\right)^\alpha \ldots\ldots\ldots\ldots \text{(2.2)}$$

The exponentiated exponential represents a parallel system.

Maximum Likelihood Estimators:-

Let $x_1, x_2, x_3 \ldots\ldots x_n$ be a random sample from EE the log likelihood can be as

L(α, λ)=n In α + n In λ+(α-1) $\sum_{i=1}^{n} In(1 - e^{-\lambda xi}) - \lambda \sum_{i=1}^{n} x_i \ldots\ldots$(2.3)

The MLE’s of α and λ can be directly maximized $\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} + \sum_{i=1}^{n} In\left(1 - e^{-\lambda x}\right) = 0 \ldots\ldots$ ……………………………..(2.4)

$$\frac{\partial L}{\partial \lambda}=\frac{n}{\lambda}+(\alpha-1)\sum_{i=1}^{n}\frac{x_i e^{-\lambda x_i}}{1-e^{-\lambda x_i}}-\sum_{i=1}^{n}x_i=0.......\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.5)$$

From (2.4), we obtain the MLE of α as a function of λ, say $\alpha(\lambda^{\wedge})$, as

$$\alpha(\lambda^{\wedge}), =- \frac{n}{\sum_{i=1}^{n} In(1-e^{-\lambda x_i})}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.6)$$

For a known scale parameter, MLE of the standard parameter $\alpha^{\wedge}$, can be obtained directly from (2.6) .If both parameters are unknown, first the scale parameter measurement can be obtained by directly increasing L(α$(\lambda^{\wedge})$, λ) in relation to λ. Once $\lambda^{\wedge}$ is detected $\alpha^{\wedge}$ can be obtained from (2.6) as α^$(\lambda^{\wedge})$, .

3. Exponentiated Lognormal Distribution

Probability density function

Probability density function (P.d.f) of exponential Lognormal distribution is

defined by three parameters (α,μ,σ) as

f(x; (α,μ,σ))= $\alpha\left(\varphi(In(x);\mu,\sigma)\right)^{\alpha-1}.\phi\left(In(x);\mu,\sigma\right).x^{-1},\ldots\ldots\ldots(3.1)$

x, α >0, $--\infty<\mu<\infty$

where $\varphi(In(x);\mu,\sigma)$ and $\phi(In(x);\Diamond$ are the c.d.f and p.d.f of the normal distribution with mean and standard deviation as μ and σ.

The survival function corresponding with exponentiated lognormal distribution density is given as

S(x, μ σ. α) =1- ($\varphi(In(x);\mu,\sigma)$)$^{\alpha}$ where x>0

Maximum Likelihood Estimators

Let $x_1,x_2,x_3.......x_n$ be a random sample from EL distribution the log likelihood function can be as

$$L(\alpha,\mu,\sigma/x)=nIn\alpha-\sum_{i=1}^{n}In(xi)+(\alpha-1)\sum_{i=1}^{n}In\varphi(In(xi);\mu,\sigma)+\sum_{i=1}^{n}In\phi(In(xi);\mu,\sigma)\ ..(3.2)$$

To find the values of the parameters α,μ,σ that maximize L(α,μ,σ/x) we can solve the equations which are as follows:-

$$\frac{\partial L}{\partial \alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}In\,\varphi(In(xi);\mu,\sigma)=0.......\ \ldots\ldots\ldots(3.3)$$

$$\frac{\partial L}{\partial \mu} = (\alpha - 1)\sum_{i=1}^{n} \frac{\varphi_{\mu}{}^{'}(In(xi);\mu,\sigma)}{\varphi(In(xi);\mu,\sigma)} + \sum_{i=1}^{n} \frac{\varphi_{\mu}{}^{'}(In(xi);\mu,\sigma)}{\phi(In(xi);\mu,\sigma)} = 0.........(3.4)$$

$$\frac{\partial L}{\partial \sigma} = (\alpha - 1)\sum_{i=1}^{n} \frac{\varphi_{\sigma}{}^{'}(In(xi);\mu,\sigma)}{\varphi(In(xi);\mu,\sigma)} + \sum_{i=1}^{n} \frac{\varphi_{\sigma}{}^{'}(In(xi);\mu,\sigma)}{\phi(In(xi);\mu,\sigma)} = 0.........(3.5)$$

From (3.3), (3.4) and (3.5) MLE of α, μand σ is obtained.

4. Exponential Gumble Distribution

Probability density function

The Probability density function (Pd.f) of exponential gumble distribution was led by Nadarajah(2005) with parameters α and σ as

$$f(x;\alpha,\sigma) = \frac{\alpha}{\sigma}\left[\exp\left(-e^{-\frac{x}{\sigma}}\right)\right]^{\alpha}.e^{-\frac{x}{\sigma}}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

Where α and σ>0 and $-\infty < x < \infty$

Where α is a shape parameter and σ is scale parameter.

Here when α=1 it reduces to standard gumble distribution.

The survival function corresponding with exponentiated-gumble density is given as

$$S(x,\alpha,\sigma)=1-\frac{\alpha}{\sigma}\left(\exp(-e^{\frac{x}{\sigma}})\right)^{\alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.2)$$

The survival function of the exponentiated gumble distribution is the α th power of the survival function of the gumble distribution.

Maximum Likelihood Estimators

Let $x_1, x_2, x_3.......x_n$ be a random sample from EG distribution the log likelihood function can be as

$$L(\alpha,\sigma)=n\ln\alpha - n\ln\sigma - \frac{1}{\sigma}\sum_{i=1}^{n} x_i - \alpha\sum e^{\frac{-xi}{\sigma}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.3)$$

Therefore to obtain the MLE's of α and σ we can directly maximize (4.3) with respect to α and σ or we can solve the non-linear normal equations which are as follows:-

$$\frac{\partial L}{\partial \alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n} e^{-\frac{x_i}{\sigma}} = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.4)$$

$$\frac{\partial L}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^2}\sum_{i=1}^{n} x_i - \frac{\alpha}{\sigma^2}\sum_{i=1}^{n} x_i.e^{\frac{-x_i}{\sigma}} = 0\ldots\ldots\ldots\ldots\ldots\ldots\ldots(45)$$

From (4.4) and (4.5) MLE of α, and σ is obtained.

5. Extended Exponentiated Exponential Distribution:-

Assume that X is a random variable with a given survival function $\bar{F}(x)$, the Marshall-Olkin extension of the real family is defined as a distribution family with survival function as:

$\bar{G}(x) = \lambda\bar{F}(x) / (1 - \bar{\lambda}\bar{F}(x))$, $-\infty < x < \infty$, $\lambda > 0$, $\bar{\lambda} = 1-\lambda$. (5.1)

Abu-Youssef et al (2015)

Presented a new alternative of the Marshall-Olkin extended family of distributions by selecting in (5.1) the exponentiated exponential distribution with survival function 1 which shows

$\bar{G}(x) = \lambda - \lambda(1 - e^{-\beta x})^{\alpha}\ \lambda + \bar{\lambda}(1 - e^{-\beta x})^{\alpha}$, $x > 0$, $\alpha > 0, \lambda > 0$, $\beta > 0$…(5.2)

Maximum Likelihood Estimators:-

Suppose x1, x2, ..., xn is a random sample size n from Marshall Olkin extended descriptive distribution and the function of likelihood will be

$$Ln(\alpha, \beta, \lambda) = \prod_{i=1}^{n} g(x_i, \alpha, \beta, \lambda) = \prod \frac{\alpha\beta\lambda(1 - e^{-\beta x_i})^{\alpha-1} e^{-\beta x_i}}{(\bar{\lambda}(1 - e^{-\beta x_i})^{\alpha} + \lambda)^2} \text{ ……….(5.3)}$$

and the log- likelihood function is

$$Ln = \sum_{i=1}^{n}(\alpha - 1)(\log(1 - e^{-\beta x_i})) - \beta x_i - 2\log((\bar{\lambda}(1 - e^{-\beta x_i})^{\alpha} + \lambda))$$

$$+ n\log(\alpha\beta\lambda) \text{……………………………………………………..(5.4)}$$

The Maximum Likelihood Estimation (MLE) of α, β and λ are attained as the result of $\partial L/\partial\alpha = 0$,

$$\partial Ln(\alpha, \beta, \lambda)/\partial\alpha = \partial/\partial\alpha \prod_{i=1}^{n} g(x_i, \alpha, \beta, \lambda) = \prod \frac{\alpha\beta\lambda(1 - e^{-\beta x_i})^{\alpha-1} e^{-\beta x_i}}{(\bar{\lambda}(1 - e^{-\beta x_i})^{\alpha} + \lambda)^2} \text{...(5.5)}$$

$\partial L/\partial\beta = 0$,

$$\partial Ln(\alpha, \beta, \lambda)/\partial\beta = \partial/\partial\beta \prod_{i=1}^{n} g(x_i, \alpha, \beta, \lambda) = \prod \frac{\alpha\beta\lambda(1 - e^{-\beta x_i})^{\alpha-1} e^{-\beta x_i}}{(\bar{\lambda}(1 - e^{-\beta x_i})^{\alpha} + \lambda)^2} \text{...(5.6)}$$

and

$\partial L/\partial\lambda = 0$.

$$\partial Ln(\alpha, \beta, \lambda)/\partial\lambda = \partial/\partial\lambda \prod_{i=1}^{n} g(x_i, \alpha, \beta, \lambda) = \prod \frac{\alpha\beta\lambda(1 - e^{-\beta x_i})^{\alpha-1} e^{-\beta x_i}}{(\bar{\lambda}(1 - e^{-\beta x_i})^{\alpha} + \lambda)^2} \text{...(5.7)}$$

From (5.1),(5.6) and (5.7) we can have MLE.

Goodness of Fit

Nine models namely gamma, weibull, lognormal, gumble, exponentiated weibull, exponentiated exponential, exponentiated lognormal, exponentiated gumble and extended exponentiated exponential have been fitted to three real data sets. The distributions along with Probability density function are given as under:-

<u>Distribution</u> <u>P.d. f</u>

Gamma $f(x, \alpha, \lambda) = \frac{\lambda^{\alpha}}{\Gamma\alpha} x^{\alpha-1}.e^{-\lambda.x}$; $\alpha, \lambda, x > 0$

Weibull $f(x, \alpha, \lambda) = \alpha\lambda\,(x\lambda)^{\alpha-1}\ .e^{(-\lambda x)^{\alpha}}$; $\alpha, \lambda, x > 0$

Lognormal $f(x, \mu, \sigma) = \frac{\exp\left(-\frac{1}{2}\left(\frac{In(x)-\mu}{\sigma}\right)^2\right)}{x.\sigma.\sqrt{2\bar{\lambda}}}$; $-\infty < \mu < \infty, \sigma > 0$

Gumble $f(x, \sigma) = \frac{1}{\sigma}\exp^{\frac{-x}{\sigma}}.\exp\left(-e^{\frac{-x}{\sigma}}\right)$; $\sigma > 0$

Exponentiated weibull $f(t;\alpha,\theta,\sigma) = \frac{\alpha\theta}{\sigma}\left[1-\exp(-\left(\frac{t}{\sigma}\right)^{\alpha}\right]\exp\left[-\left(\frac{t}{\sigma}\right)^{\alpha}\right]\left(\frac{t}{\sigma}\right)^{\alpha-1}$, t >o

Exponentiated exponential $f(x,\alpha,\lambda) = \alpha\lambda\left(1-e^{-\lambda x}\right)^{\alpha-1}.e^{-\lambda x}$, $\alpha, \lambda, x > 0$

Exponentiated lognormal $f(x, (\alpha, \mu, \sigma)) = \alpha\left(\varphi(In(x);\mu,\sigma)\right)^{\alpha-1}.\phi\left(In(x);\mu,\sigma\right).x^{-1}$,

x, $\alpha > 0$, $--\infty < \mu < \infty$

Exponentiated gumble $f(x;\alpha,\sigma) = \frac{\alpha}{\sigma}\left[\exp\left(-e^{-\frac{x}{\sigma}}\right)\right]^{\alpha}.e^{-\frac{x}{\sigma}}$, $\alpha, \sigma > 0$

Extended Exponentiated exponential f(x) = αβλ(1 − e −βx) α−1 e −βx (λ¯(1 − e −βx) α + λ) 2 , x > 0.

Data Set1:-The data pertaining to failure times of the conditioning system of an aero plane.

23,261,87,7,120,14,62,47,225,71,246,21,42,20,5,

12,120,11,3, 14, 71,11,14,11,16, 90, 1,16,52,95.

Table1:- Distribution with MLE'S, Log-likelihood and Anderson's value.

Distribution	MLE'S	Log likelihood	Anderson's Value
Gamma	$\hat{\alpha}$=0.8119, $\hat{\lambda}$=0.0136	-152.167	0.078
Weibull	$\hat{\alpha}$=0.8536, $\hat{\lambda}$=0.0183	-151.9.70	0.069
Lognormal	$\hat{\mu}$=3.358, $\hat{\lambda}$=1.3190	-151.621	0.065
Gumble	$\hat{\alpha}$=27.792, $\hat{\lambda}$=55.106	-151.256	0.062
Exponentiated weibull	$\hat{\alpha}$= 3.824, $\hat{\theta}$= 0.1732 $\hat{\sigma}$=82.235	-149.567	0.057
Exponentiated exponential	$\hat{\alpha}$=0.8093, $\hat{\lambda}$=0.0145	-152.206	0.079
Exponentiated lognormal	$\hat{\alpha}$= 0.1543 $\hat{\mu}$= 3.1353 $\hat{\sigma}$=0.3648	-148.659	0.055
Exponentiated gumble	$\hat{\alpha}$=1.9881, $\hat{\lambda}$=49.0638	-148.537	0.054
Extended exponentiated exponential	$\hat{\alpha}$=0.8119, β= 1.46 $\hat{\lambda}$=0.0136	1381.572	0.051

Data Set 2:-The data pertains to runs scored by a cricketer in 27 innings at national level:-

28,20,6,4,23,127,25,45,41,67,68,3,17,2,

105,98,55,68,15,3,42,45,7,20, 34,9 ,6,

Table2:- Distribution with MLE'S, Log-likelihood and Anderson's value.

Distribution	MLE'S	Log likelihood	Anderson's Value
Gamma	$\hat{\alpha}$=0.7235 , $\hat{\lambda}$=0.0127	-125.654	0.956
Weibull	$\hat{\alpha}$=1.040, $\hat{\lambda}$=36.985	-124.021	0.716
Lognormal	$\hat{\mu}$=3.053, $\hat{\lambda}$=1.174	-125.022	0.916
Gumble	$\hat{\alpha}$= 21.432 , $\hat{\lambda}$=25.944	-124.059	0.702
Exponentiated weibull	$\hat{\alpha}$= 3.521, $\hat{\theta}$= 0.1452 $\hat{\sigma}$=67.235	-125.078	0.928
Exponentiated exponential	$\hat{\alpha}$= 0.8126, $\hat{\lambda}$=0.0153	-125.945	0.959
Exponentiated lognormal	$\hat{\alpha}$= 0.578, $\hat{\mu}$=3.836 $\hat{\sigma}$=0.7834	-125.965	0.961
Exponentiated gumble	$\hat{\alpha}$= 1.873 , $\hat{\lambda}$=45.264	-124.843	0.843
Extended exponentiated exponential	$\hat{\alpha}$=0.8119, β= 1.46 $\hat{\lambda}$=0.0136	-121.784	0.827

Data Set 3:- Nichols and Padgett data set consisting of 100 observations on breaking stress of carbon fibers (in gba):-

3.7 2.74 2.73 2.5 3.6 3.11 3.27 2.87 1.47 3.11

4.42 2.41 3.19 3.22 1.69 3.28 3.09 1.87 3.15 4.9

3.75 2.43 2.95 2.97 3.39 2.96 2.53 2.67 2.93 3.22

3.15 2.35 2.55 2.59 2.38 2.81 4.2 3.33 2.55 3.39

3.31 3.31 2.85 2.56 3.56 2.81 2.77 2.17 2.83 1.92

1.41 3.68 2.97 1.36 0.98 2.76 4.91 3.68 1.84 1.59

2.17 1.17 5.08 2.48 1.18 3.19 1.57 0.81 5.56 1.73

1.59 2.0 1.22 1.12 1.71 1.84 3.65 2.05 0.39 3.68

2.48 0.85 1.61 2.79 4.7 3.51 2.17 1.69 1.25 4.38

2.03 1.8 1.57 1.08 2.03 1.61 2.12 1.89 2.88 2.82

Table3:- Distribution with MLE'S, Log-likelihood and Anderson's value.

Distribution	MLE'S	Log likelihood	Anderson's Value
Gamma	$\hat{\alpha}$=0.6754 , $\hat{\lambda}$=0.1324	-134.976	0.743
Weibull	$\hat{\alpha}$=0.9678, $\hat{\lambda}$=5.678	-115.763	0.843
Lognormal	$\hat{\mu}$=2.609, $\hat{\lambda}$=2.419	-117.046	0.744
Gumble	$\hat{\alpha}$= 17.435 , $\hat{\lambda}$=19.865	-116.052	0.789
Exponentiated weibull	$\hat{\alpha}$= 4.564, $\hat{\theta}$= 0.1432 $\hat{\sigma}$=67.235	-117.078	0.710
Exponentiated exponential	$\hat{\alpha}$= 0.984, $\hat{\lambda}$=0.0453	-117.956	0.783
Exponentiated lognormal	$\hat{\alpha}$= 0.436, $\hat{\mu}$=2.342 $\hat{\sigma}$=0.6753	-117.893	0.776
Exponentiated gumble	$\hat{\alpha}$= 1.345, $\hat{\lambda}$=34.876	-115.67	0.734
Extended exponentiated exponential	$\hat{\alpha}$=0.763, β= 1.45 $\hat{\lambda}$=0.0125	-114.31	0.713

The following table gives a comparison between the MLE's Loglikelihood, and Anderson's Value and can be interpreted as :-

Conclusion

The probability density function of exponentiated weibull, exponentiated exponential, exponentiated lognormal, exponentiated gumble and extended exponentiated exponential distributions and its applications for three data sets were discussed. It appears that the weibull defined by the two parameters is more suitable to fit the unimodel, monotone and risk functions. The exponential has a framework and scale similar to the gamma and weibull distribution and can be considered as an alternative and equally superior to the weibull and gamma in most cases. Three parameter extended exponentiated exponential distribution offers more flexible model for real time data sets. The initial set of data relates to the failure times of the flight status suspension system. The exponential exponentiated and gamma provides the best balance followed by Weibull. Thus the gamma weibull and the extended exponentiated exponential can be exchanged with each other in a particular case. In second data set regarding runs scored by a cricketer extended exponentiated exponential, exponentiated lognormal and exponentiated exponential gives better fit followed by gamma. Hence exponentiated lognormal, extended exponentiated exponential and gamma can be replaced as an alternative with each other. In third data set extended exponentiated exponential followed by exponentiated weibull gives better fit. One can fit other distributions and can look for flexibility and advancement.

References

1. Mudholkar, G.S., Srivastava, D.K. (1993): Exponentiated weibull family for analyzing bathtub failure-rate data. IEEE Transmission Reliability 42(2): 299-302
2. Mudholkar, G.S., Srivastava, D.K., Freimer, M. (1995): The exponentiated weibull Family: a reanalysis of the bus-motor-failure data. Tech. 37(4): 436-445.
3. Gupta, R.C; Gupta,P.L and Gupta, R.D (1998); Modelling failure time data by lehman alternative. Communications Statistics, Theory and Methods 27,887-904.
4. Gupta, R.D and D, Kundu (2001);Exponentaited exponential family: An alternative to Gamma and Weibull distributions, Biometrical Journal,43(1),117-130
5. Nadarajah,S.(2005);The exponentiated gumble distribution with climate applicaton,Environmetrics,17,13-23.
6. Kakde, C.S and Shirke, D.T(2006);On exponentiated lognormal distribution. Int. J. Agricult. Stat. Sci,2(2)319-326.
7. Kakde,C.S and Shirke,D.T(2007);Some Inferences on Exponentiated Gumble Distribution. Int. J. Agricult. Stat. Sci,3(1)212-218.
8. Raja. T A and Mir A H(2011). On extension of some exponentiated distributions with application. Int J Contemp Math Sci, 6:393–400.
9. Abu-Youssef, S.E Mohammed, B I and Sief M G(2015); An extended exponentiated exponential distribution and its properties . Int. J of Computer Applications 121(5) ,0975–8887
10. Bagheri S F, Alizadeh M and Nadaraj S (2016). Efficient estimation of the pdf and the cdf of the exponentiated gumbel distribution.
11. Suresh K K and Usha K(2016) A truncated life test in acceptance sampling plan based on exponentiated exponential distribution. Int.Conf. on Emerging Trends in Eng,Tech. and Sc.
12. Raja. T A and Maqbool S. Some poisson type distributions with application . Int J Agricult. stat Sci,15(2):559-761.
13. Malik R Mansoor and Kumar Devendra (2020) Relations for single and product moments of exponential weibull distribution based on progressively censored data. International Journal of Agricultural and Statistical Sciences, 16(1):465-477