Polynomial-Exponential Distribution

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Article Info
Page Number: 2474-2486
Publication Issue:
Vol 71 No. 4 (2022)

Article History
Article Received: *25 April 2022*
Revised: *30 May 2022*
Accepted: *15 July 2022*
Publication: *19 August 2022*

Abstract

The proposed distribution is a continuous probability distribution which has only one parameter and gives better results than Modified Mishra distribution of Sah for statistical modeling of over-dispersed survival life time data of similar nature. The required characteristics of this distribution have been defined and derived. The estimation of parameters has been discussed. We have defined and properly described the Hazard rate function, the mean residual life function and reliability function of this distribution. In order to check reliability and facts of this distribution, we use goodness of fit on some survival life time data which were previously used by other researchers, what we found is that this distribution gives better goodness of fit to the over-dispersed data-sets than Lindley distribution (LD), Mishra distribution (MD), Quadratic-Exponential distribution (QED) and Modified Mishra distribution (MMD).

Keywords: - Probability distribution, Distribution, Parameters, Moments, Polynomial-exponential distribution.
MSC: 60E05, 60E10, 60E15

Introduction

The literature of Statistics is full of concepts of probability theory and probability distributions. Even so, we are always trying to find something new but more factful theory than previously obtained. Meanwhile, we are proposing a new continuous probability distribution which has only one parameter. Since it is made from the product of polynomial function and exponential function that is why it is named Polynomial-exponential distribution (PED). One of the main reasons for the composition and the results it gives is the inclusion of two irrational numbers in the probability density function (pdf) of PED. One of the main reasons for the use of the Normal distribution in every field is the use of two irrational number in their pdf. Using this concept, Sah obtained the 'New Linear-exponential distribution [5]' for the first time, which was found to be suitable for statistical modeling of survival time data in which there is high variation. The main sources of this distribution are MD and MMD. Therefore, origin of this distribution can be considered as MD and MMD, which also depend on only one parameter. The pdf of MD [4] was defined as

$$f_1(y) = \frac{\alpha^3}{(\alpha^2+\alpha+2)}(1+y+y^2)e^{-\alpha y} \qquad (1)$$

Where $\alpha > 0$ and $y > 0$.

Sah studied that in most cases the MD provides a better goodness of fit than the LD for over-dispersed survival time data of similar nature. Lindley defined pdf of LD [2] as

$$f_2(y) = \frac{\alpha^2}{(1+\alpha)}(1+y)e^{-\alpha y} \;;\; \alpha > 0, y > 0 \tag{2}$$

In the study of MMD [7], it has been found that it is a better alternative of LD, MD for similar nature of data sets and pdf was given by

$$f_3(y) = \frac{\alpha^3}{(\pi\alpha^2+\alpha+2)}(\pi+y+y^2)e^{-\alpha y} ;\alpha >, y > 0 \tag{3}$$

The main objective of this paper is to propose a better goodness of fit for statistical modeling of survival time data than LD, MD, QED [5] and MMD. In the first section of this paper, Introduction and literature review of PED are placed. In the second section of this paper, the material and methods are included. In the third section, the results obtained for the proposed distribution are presented in different sub-headings in a systematic manner. The sub-headings are

- Polynomial-exponential distribution (PED) and its related characteristics
- Moments and related measures of PED
- Estimation of the parameters of PED
- The Hazard rate function, the mean residual life function and residual function of PED, and
- Applications of PED.

Concluding remarks of PED is placed in the last and fourth section. The data used for the purpose of applications are secondary in nature. The authors have no ill will towards anyone and no greed of any kind.

The Material and Methods

This section begins with the construction of Probability density function and cumulative distribution function of PED. Two methods have been used to estimate the parameter of this distribution. Goodness of fit has been applied to test validity of this distribution by using some secondary over-dispersed survival time data.

Results

The work of this paper is presented in these different sub-headings.

- Polynomial-exponential distribution (PED) and its related characteristics
- Moments and related measures of PED
- Estimation of the parameters of PED
- The Hazard rate function, the mean residual life function and residual function of PED, and
- Applications of PED.

Polynomial-exponential Distribution (PED) and its related characteristics

This distribution is proposed for better results than MMD as well as NQED which is a continuous probability distribution with only one parameter (α) and Its probability density function is defined as

$$f(y)=\frac{\alpha^4}{(6+\pi\alpha^3)}(\pi+y^3)e^{-\alpha y};\alpha>0 \text{ and } y>0 \tag{4}$$

Cumulative Distribution Function (cdf): It is obtained as

$$F(Y)=P(Y\le y)=\int_0^y f(y)dy=\frac{\alpha^4}{(6+\pi\alpha^3)}\int_0^y(\pi+y^3)e^{-\alpha y}\,dy$$

$$=\frac{\alpha^4}{(6+\pi\alpha^3)}\left[\frac{(6+\pi\alpha^3)}{\alpha^4}-\frac{(\pi\alpha^3+\alpha^3y^3+3\alpha^2y^2+6\alpha y+6)e^{-\alpha y}}{\alpha^4}\right]$$

$$=1-\frac{\alpha^4}{(6+\pi\alpha^3)}\left[\frac{(6+\pi\alpha^3)e^{-\alpha y}}{\alpha^4}-\frac{(\alpha^3y^3+3\alpha^2y^2+6\alpha y)e^{-\alpha y}}{\alpha^4}\right]$$

$$=1-e^{-\alpha y}-\frac{\alpha y(\alpha^2y^2+3\alpha y+6)}{(6+\pi\alpha^3)}e^{-\alpha y} \tag{5}$$

The expression (5) is the cdf of the PED (4).

Graphical presentations of pdf and cdf for different values of parameter of PED (4) are given below.

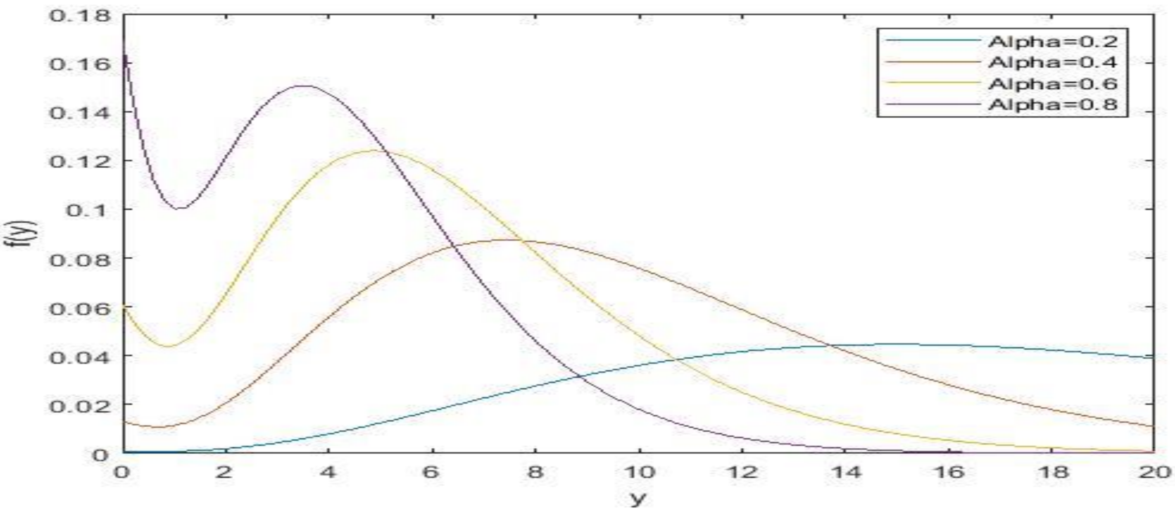

Fig1: Graph of pdf of PED at $\alpha=0.2,0.4,0.6,0.8$

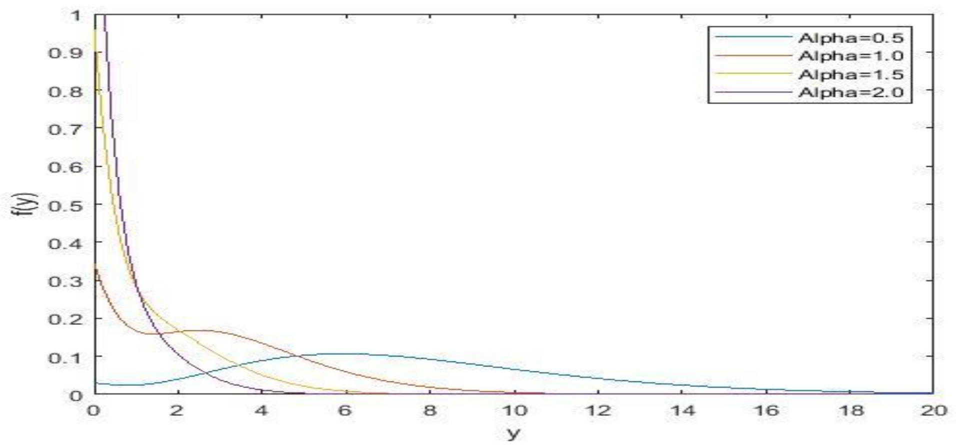

Fig2: Graph of pdf of PED at $\alpha = 0.5, 1.0, 1.5, 2.0$

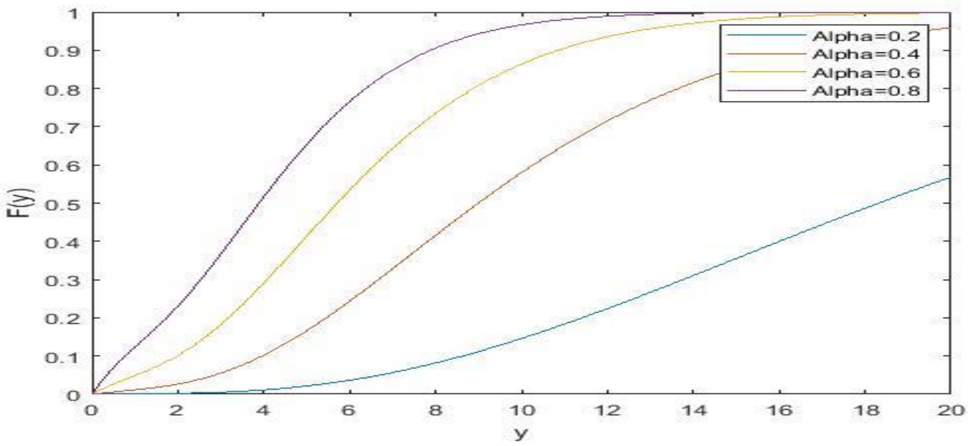

Fig.3: Graph of cdf of PED at $\alpha = 0.2, 0.4, 0.6, 0.8$

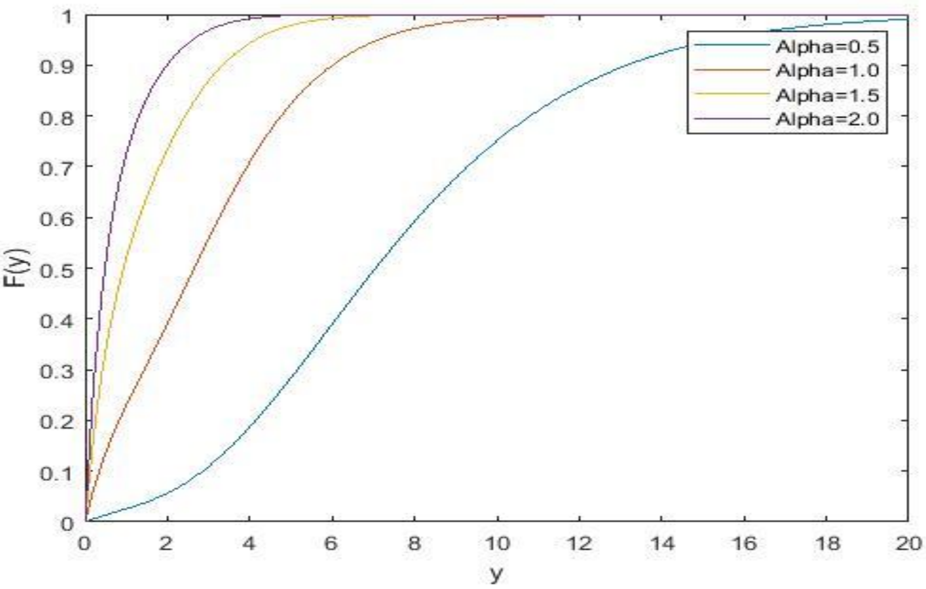

Fig.4: Graph of cdf of PED at $\alpha = 0.5, 1.0, 1.5, 2.0$

Moment Generating Function [$M_Y(t)$]: It has been obtained as

$$M_Y(t)=\int_0^\infty e^{ty} f(y)dy=\frac{\alpha^4}{(\pi\alpha^3+6)}\int_0^\infty e^{ty}(\pi+y^3)e^{-\alpha y}dy$$

$$=\frac{\alpha^4}{(\pi\alpha^3+6)}\int_0^\infty(\pi+y^3)e^{-(\alpha-t)y}dy=\frac{\alpha^4}{(\pi\alpha^3+6)}\left[\pi\int_0^\infty e^{-(\alpha-t)y}dy+\int_0^\infty y^3e^{-(\alpha-t)y}dy\right]$$

$$=\frac{\alpha^4}{(6+\pi\alpha^3)}\left[\frac{\pi}{(\alpha-t)}+\frac{6}{(\alpha-t)^4}\right]=\frac{\alpha^4}{(6+\pi\alpha^3)}\left[\frac{6+\pi(\alpha-t)^3}{(\alpha-t)^4}\right] \quad (6)$$

The expression (6) is the M.G.F. of PED (4).

Moments of PED:

It is the very important section to study about variation, shape and size of the proposed distribution. Statistical moments are useful for studying descriptive measures. In this section we have discussed about the sub-topics mentioned below

-The r^{th} moment about origin

-The first four moment about the mean and

-Dispersion, Skewness and Kurtosis of the PED.

The r^{th} moment about origin: It can be obtained as

$$\mu'_r=E[Y^r]=\int_0^\infty y^r f(y)dy=\frac{\alpha^4}{(\pi\alpha^3+6)}\left[\int_0^\infty y^r(\pi+y^3)e^{-\alpha y}dy\right] \quad (7)$$

$$=\frac{\alpha^4}{(\pi\alpha^3+6)}\left[\pi\int_0^\infty y^r\, e^{-\alpha y}dy+\int_0^\infty y^{r+3}\, e^{-\alpha y}dy\right]=\frac{\alpha^4}{(\pi\alpha^3+6)}\left[\frac{\pi\,\Gamma(r+1)}{\alpha^{(r+1)}}+\frac{\pi\,\Gamma(r+4)}{\alpha^{(r+4)}}\right]$$

$$=\frac{\alpha^4}{(\pi\alpha^3+6)}\left[\frac{\pi\, r!}{\alpha^{(r+1)}}+\frac{\pi\,(r+3)!}{\alpha^{(r+4)}}\right]$$

$$\therefore\ \mu'_r=\frac{r!}{\alpha^r}\left[\frac{\left\{\pi\alpha^3+(r+3)(r+2)(r+1\right\}}{(6+\pi\alpha^3)}\right] \quad (8)$$

The expression (8) is the r^{th} moment about origin of PED (4). Putting the value of $r=1,2,3,4$ in the expression (8), the first four moments about origin are obtained as

$$\mu'_1==\frac{1}{\alpha}\left[\frac{(\pi\alpha^3+24)}{(\pi\alpha^3+6)}\right] \quad (9)$$

$$\mu'_2==\frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^3+60)}{(\pi\alpha^3+6)}\right] \quad (10)$$

$$\mu'_3==\frac{3!}{\alpha^3}\left[\frac{(\pi\alpha^3+120)}{(\pi\alpha^3+6)}\right] \quad (11)$$

$$\mu'_4==\frac{4!}{\alpha^4}\left[\frac{(\pi\alpha^3+210)}{(\pi\alpha^3+6)}\right] \quad (12)$$

ISSN: 2094-0343
2326-9865

Central Moments of PED:
The first four central moments of PED (4) have been obtained as

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - (\mu_1')^2 = \frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^3+60)}{(\pi\alpha^3+6)}\right] - \left[\frac{1}{\alpha}\frac{(\pi\alpha^3+24)}{(\pi\alpha^3+6)}\right]^2 = \left[\frac{\left\{(\pi\alpha^3)^2+60\pi\alpha^3+144)\right\}}{\left\{\alpha(\pi\alpha^3+6)\right\}^2}\right] \quad (13)$$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3$$

$$= \frac{3!}{\alpha^3}\left[\frac{(\pi\alpha^3+120)}{(\pi\alpha^3+6)}\right] - 3\left[\frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^3+60)}{(\pi\alpha^3+6)}\right]\right]\left[\frac{1}{\alpha}\frac{(\pi\alpha^3+24)}{(\pi\alpha^3+6)}\right] + 2\left[\frac{1}{\alpha}\frac{(\pi\alpha^3+24)}{(\pi\alpha^3+6)}\right]^3$$

$$= \left[\frac{\left\{6(\pi\alpha^3+20)(\pi\alpha^3+6)^2 - 6(\pi\alpha^3+60)(\pi\alpha^3+24)(\pi\alpha^3+6) + 2(\pi\alpha^3+24)^3\right\}}{\left\{\alpha(\pi\alpha^3+6)\right\}^3}\right]$$

$$= \frac{2(\pi\alpha^3)^3 + 396(\pi\alpha^3)^2 + 684(\pi\alpha^3) + 1728}{\left[\alpha(\pi\alpha^3+6)\right]^3} \quad (14)$$

$$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'(\mu_1')^2 - 3(\mu_1')^4$$

$$= \frac{4!}{\alpha^4}\left[\frac{(\pi\alpha^3+210)}{(\pi\alpha^3+6)}\right] - 4\left[\left\{\frac{3!}{\alpha^3}\frac{(\pi\alpha^3+120)}{(\pi\alpha^3+6)}\right\}\left\{\frac{1}{\alpha}\frac{(\pi\alpha^3+24)}{(\pi\alpha^3+6)}\right\}\right]$$

$$+6\left[\left\{\frac{2!}{\alpha^2}\left[\frac{(\pi\alpha^3+60)}{(\pi\alpha^3+6)}\right]\right\}\left\{\frac{1}{\alpha}\frac{(\pi\alpha^3+24)}{(\pi\alpha^3+6)}\right\}^2\right] - 3\left[\frac{1}{\alpha}\frac{(\pi\alpha^3+24)}{(\pi\alpha^3+6)}\right]^4$$

$$= \left[\frac{\left\{\begin{array}{l}24(\pi\alpha^3+210)(\pi\alpha^3+6)^3 - 24(\pi\alpha^3+120)(\pi\alpha^3+24)(\pi\alpha^3+6)^2 \\ +12(\pi\alpha^3+60)(\pi\alpha^3+24)^2(\pi\alpha^3+6) - 3(\pi\alpha^3+24)^4\end{array}\right\}}{\left\{\alpha(\pi\alpha^3+6)\right\}^4}\right]$$

$$= \frac{\left[9(\pi\alpha^3)^4 + 2808(\pi\alpha^3)^3 + 20736(\pi\alpha^3)^2 + 93312(\pi\alpha^3) + 93312\right]}{\left[\alpha(\pi\alpha^3+6\right]^4} \quad (15)$$

Nature of PED (4) according to dispersion: To know about the nature of variability, we have obtained Index of dispersion (I) as

$$I = \frac{Variance}{Mean} = \frac{(\pi^2\alpha^6+60\pi\alpha^3+144)}{[\alpha(\pi\alpha^3+6)(\pi\alpha^3+24)]} \quad (16)$$

If $I = 1$ then

$$(\pi^2\alpha^6+60\pi\alpha^3+144) = [\alpha(\pi\alpha^3+6)(\pi\alpha^3+24)]$$

After a little simplification, we get a 7th degree polynomial equation in terms of α as follows

$$f(\alpha) = (\pi^2\alpha^6+144)(\alpha-1) + 30(\pi\alpha^3)(\alpha-2) = 0 \quad (17)$$

The expression (17) may be solved by using Newton-Rapson or Regula-Falsi method. It is found that at $\alpha = 1.555528765$, $f(\alpha) = 0$ .Hence, it has been observed that PED will be equi-dispersed if $\alpha = 1.555528765$, over-dispersed if $\alpha < 1.555528765$ and under dispersed if $\alpha > 1.555528765$.

$$C.V. = \left(\frac{\sigma_Y}{\mu_1'}\right)100 = \left[\frac{\sqrt{(\pi^2\alpha^6 + 60\pi\alpha^3 + 144)}}{[\alpha(\pi\alpha^3 + 6)(\pi\alpha^3 + 24)]}\right]100 \tag{18}$$

The expression (18) is the co-efficient of variation of PED (4).

Nature of PED according to skewness: The co-efficient of skewness based on moments is obtained as

$$\beta_1 = \frac{\mu_3^{\ 2}}{\mu_2^3} = \frac{[2(\pi\alpha^3)^3 + 396(\pi\alpha^3)^2 + 648(\pi\alpha^3) + 1728]^2}{(\pi^2\alpha^6 + 60\pi\alpha^3 + 144)^3} \tag{19}$$

$$\gamma_1 = \frac{\mu_3}{\mu_2^{3/2}} = \frac{[2(\pi\alpha^3)^3 + 396(\pi\alpha^3)^2 + 648(\pi\alpha^3) + 1728]}{(\pi^2\alpha^6 + 60\pi\alpha^3 + 144)^{3/2}} \tag{20}$$

It has been observed that $1 < \gamma_1 < \infty$. Hence, PED is positively skewed by shape.

Nature of PED according to kurtosis: The co-efficient of kurtosis based on moments is obtained as

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\left[9(\pi\alpha^3)^4 + 2808(\pi\alpha^3)^3 + 20736(\pi\alpha^3)^2 + 93312(\pi\alpha^3) + 93312\right]}{(\pi^2\alpha^6 + 60\pi\alpha^3 + 144)^2} \tag{21}$$

It has been observed that $(3/\sqrt{2}) < \beta_2 < \infty$ and hence PED is leptokurtic by size.

Estimation of Parameters of PED:

Here, we have used only two methods to estimate the parameter of PED.

(a) *The Method of Moments*:

We have used the first moment about origin to get the estimated value of the parameter α and we get the following expression

$$\mu'(\pi\alpha^4 + 6\alpha) - (\pi\alpha^3 + 24) = 0 \tag{22}$$

The expression (22) is the 4th degree polynomial equation in α .To solve the expression (22), replace the population mean by the sample mean and it may be solved by using the Newton-Rapson or Regula-Falsi method

(b) *The Method of Maximum Likelihood*:

Let Y follows PED having population size N and a sample ($(y_1, y_2, ..., y_n)$ of size n has been drawn from this population. The likelihood function of PED (4) can be derived as

$$L = \prod_{i=1}^{n} f(y;\alpha) = \left(\frac{\alpha^4}{(\pi\alpha^3+6)}\right)^n \left[\prod_{i=1}^{n}(\pi + y^3)\right] e^{-n\alpha\bar{y}} \tag{23}$$

The log likelihood equation can be obtained as

Or, $\ln L = n\ln(\alpha)^4 - n\ln(\pi\alpha^3+6) + \sum_{i=1}^{n} \ln(\pi + y_i^3) - n\alpha\bar{y}$

Differentiation $\ln L$ with respect to α , we get

Or, $\frac{\partial \ln L}{\partial \alpha} = \frac{4n}{\alpha} - \frac{n(3\pi\alpha^2)}{(\pi\alpha^3+6)} - n\bar{y} = 0$

Or, $\bar{y} == \left[\frac{4}{\alpha} - \left[\frac{3\pi\alpha^2}{(\pi\alpha^3+6)}\right]\right] = \frac{4\pi\alpha^3 + 24 - 3\pi\alpha^3}{\alpha(\pi\alpha^3+6)} = \frac{(\pi\alpha^3+24)}{\alpha(\pi\alpha^3+6)}$ (24)

Or, $\bar{y}\alpha(2+\pi\alpha^2) - (\pi\alpha^2+6) = 0$ (25)

The expression (24) is the mean of PED. Replacing the population mean by the sample mean and solving the expression (25) by using Regula-Falsi method for α ,we get an estimate of α .

***The Reliability Function, Hazard Rate Function and Mean Residual Life Function of PED*:**

The Reliability Function:

It helps to determine the guarantee or warranty period of the product in the manufacturing sector as well as production engineering sector because it can measure the survival time without failure of the manufactured products. Let Y be a continuous random variable which follows the PED. Let the survival life time of any product component also follows PED. The complement of the cumulative distribution function of PED is called the reliability function of PED. The reliability function of PED is denoted by R(t) and it can be formulated as

$$R(t) = P(Y > t) = \int_t^\infty f(y)dy = 1 - F(t)$$

$$= \frac{\left[(\pi\alpha^3+6) + \alpha t(\alpha^2 t^2 + 3\alpha t + 6)\right]}{(\pi\alpha^3+6)} e^{-\alpha t} \tag{26}$$

At $t = 0$, $R(t) = 1$ and $t = \infty, \lim_{t\to\infty}\left[R(t)\right] = 0$. So, we may say that the value of reliability function is inversely proportional to the working time of a system.

Graphical representations of reliability function of PED (4) with varying values of α are given below.

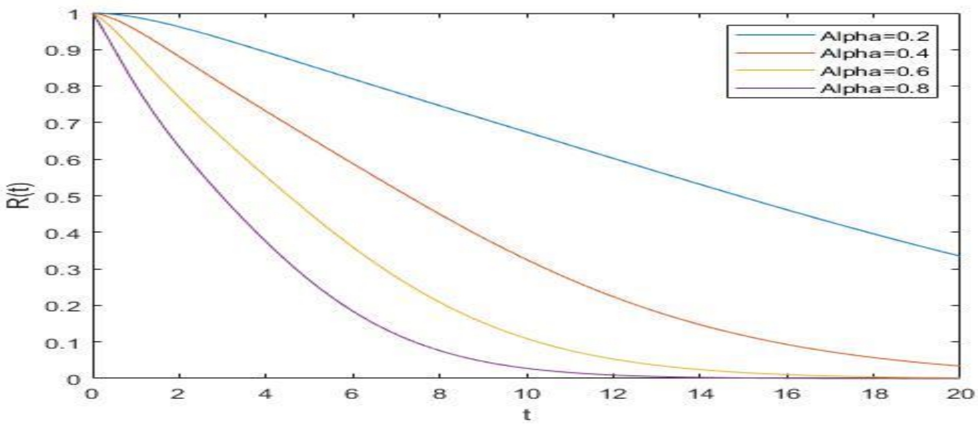

Fig.5. Graph of R(t) of PED at $\alpha = 0.2, 0.4, 0.6, 0.8$

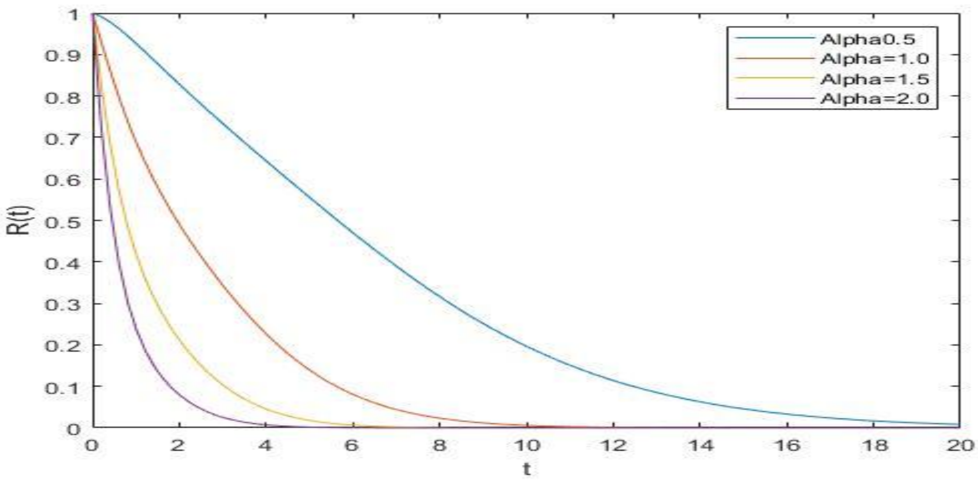

Fig.6. Graph of R(t) of PED at $\alpha = 0.5, 1.0, 1.5, 2.0$

The Hazard Rate Function:

The ratio of probability density function to the reliability function is known as the hazard rate function and it is formulated as

$$h(y) = \frac{f(y)}{R(y)} = \frac{\alpha^4(\pi + y^3)}{[(\pi\alpha^3 + 6) + \alpha y(\alpha^2 y^2 + 3\alpha y + 6)]} \quad (27)$$

The hazard rate function is also known as failure rate function and hence $h(t)$ of PED (4) until time 't' is thus obtained as

$$h(t) = \frac{\alpha^4(\pi + t^3)}{[(\pi\alpha^3 + 6) + \alpha t(\alpha^2 t^2 + 3\alpha t + 6)]} \quad (28)$$

At $t = 0$, $\quad h(y = t = 0) = \frac{\pi\alpha^4}{(\pi\alpha^4 + 6)} > 0 \quad (29)$

We can observe that h(y) is an increasing function of y and α .

Mean Residual Life Function

It may be defined as the mean additional life time of a particular component given that the component has survived until time 't'. Let Y be the life of a component which follows PED. The mean residual life function is given by

$$m(y)=E\left[Y-y/Y>y\right]=\frac{\int_{y}^{\infty}\left[1-F(t)\right]dt}{1-F(y)} \tag{30}$$

To obtain *m(y)* of the PED (4), we have to calculate the following measures

$$F(Y)=P(Y\le y)=\int_{0}^{y}f(y)dy=\frac{\alpha^4}{(\pi\alpha^3+6)}\int_{0}^{y}(\pi+y^3)e^{-\alpha y}\,dy\,;\ \text{x}>0,\ \alpha>0$$

$$=1-e^{-\alpha y}-\frac{\alpha y(\alpha^2y^2+3\alpha y+6)}{(6+\pi\alpha^3)}e^{-\alpha y} \tag{31}$$

and

$$1-F(y)=e^{-\alpha y}+\frac{\alpha y(\alpha^2y^2+3\alpha y+6)}{(6+\pi\alpha^3)}e^{-\alpha y}$$

$$1-F(t)=e^{-\alpha t}+\frac{\left[\alpha t(\alpha^2t^2+3\alpha t+6)\right]}{(\pi\alpha^3+6)}e^{-\alpha t} \tag{32}$$

$$\int_{y}^{\infty}\{1-F(t)\}dt=\int_{y}^{\infty}\left[e^{-\alpha t}+\frac{\left[\alpha t(\alpha^2t^2+3\alpha t+6)\right]}{(\pi\alpha^3+6)}e^{-\alpha t}\right]dt=\left[\frac{(\pi\alpha^3+\alpha^3y^3+6\alpha^2y^2+18\alpha y+24)}{\alpha(6+\pi\alpha^3)}\right]e^{-\alpha y} \tag{33}$$

Putting the value of $\int_{y}^{\infty}\left[1-F(t)\right]dt$ and $1-F(y)$ in equation (30), the mean residual life function of PED can be obtained as

$$m(y)=\frac{(\pi\alpha^3+24)+\alpha y(18+6\alpha y+\alpha^2y^2)}{\alpha\left[(\pi\alpha^3+6)+\alpha y(6+3\alpha y+\alpha^2y^2)\right]} \tag{34}$$

$$\text{At }\ y=0\,,\ m(y=0)=\frac{(\pi\alpha^3+24)}{\alpha(\pi\alpha^3+6)}=\mu_1' \tag{35}$$

It can also be seen that at $y=0$,the mean residual life function is the mean of PED (4). It can also be observed that *m*(*y*) is a decreasing function of y and α .

Applications of PED:

To test validity of the theoretical work, goodness of fit of PED (4) has been applied to the following to data.

Example (1): Survival times (in days) of guinea pigs infected with virulent tubercle bacilli, reported by Bjerkedal [1].

Survival Time (In days)	0 -80	80-160	160-240	240-320	320-400	400-480	480-560
Observed frequency	8	30	18	8	4	3	1

Example (2): Mortality grouped data for blackbirds species reported by Paranjpe and Rajarshi [3]

Survival Time (In days)	0 -1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	>8
Observed frequency	192	60	50	20	12	7	6	3	2

In order to compare with the expected frequencies of PED, we have also included the theoretical frequencies of LD, MD, QED and MMD in the same table as

Table I: Expected V Observed of Example (1)

Survival Time (In days)	Observed frequency	Expected frequency				
		Lindley	Mishra	QED	MMD	PED
0 – 80	8	16.1	10.8	10.8	10.8	7.5
80 – 160	30	21.9	24.8	24.8	24.8	26.4
160 – 240	18	15.4	19.0	19.0	19.0	21.9
240 – 320	8	9.0	10.1	10.1	10.1	10.6
320 – 400	4	5.5	4.5	4.5	4.5	3.9
400 – 480	3	1.8	1.8	1.8	1.8	1.2
480 – 560	1	2.3	1.0	1.0	1.0	0.5
Total	72	72.0	72.0	72.0	72.0	72.0
$\hat{\alpha}$		0.011	0.016431	0.016519	0.016514	0.022085796
χ^2 (df)		7.7712(3)	1.98(3)	1.98(3)	1.98(3)	0.73(3)
μ_1' =181.11111		μ_2' =43911.11111				

ISSN: 2094-0343

2326-9865

Table II: Expected V Observed Frequencies of Example (2)

Survival Time (In Days)	Observed frequency	Expected frequency				
		LD	**MD**	**QED**	**MMD**	**PED**
0-1	192	173.5	142.8	145.9	165.0	166.4
1-2	60	98.6	104.5	101.2	83.3	75.4
2-3	50	46.5	58.7	57.6	51.9	51.8
3-4	20	20.1	27.5	27.5	27.4	31.2
4-5	12	8.1	11.5	12.0	13.4	15.8
5-6	7	3.2	4.5	4.8	6.2	7.0
6-7	6	1.4	1.6	1.8	2.8	2.8
7-8	3	0.4	0.5	0.7	1.2	1.1
>8	2	0.3	0.4	0.5	0.8	0.5
Total	352	352.0	352.0	352.0	352.0	352.0
$\hat{\alpha}$		.984	1.311017	1.269721	1.10616888	1.411461832
χ^2 (df)		49.85(4)	46.37(4)	39.64(4)	17.60(5)	14.98(5)
$\mu_1' = 1.56818$		$\mu_2' = 5.005682$				

From the tables given above we can observe that the calculated value of Chi-square is lower than all other distributions mentioned in the table.

Concluding Remarks:

In this paper, several structural properties of PED have been obtained. The moments about origin and about the mean have been discussed and derived. Parameter of this distribution has been estimated by the method of moments and the maximum likelihood. The reliability function, hazard rate function and mean residual life function of this distribution have been obtained and discussed. The highlighted remarks about the PED (4) are

* It has been observed that it will be Equi-dispersed when $\alpha = 1.555528765$,Over-dispersed when $\alpha < 1.555528765$ and Under-dispersed when $\alpha > 1.555528765$.
* It has been observed that $1 < \gamma_1 < \infty$. Hence, it is positively skewed in shape.
* It has been obtained that $(3/\sqrt{2}) < \beta_2 < \infty$ and hence it is leptokurtic by size.
* From tables I and II, it has been observed that PED (4) gives a better fit to the similar data-sets than LD [2], MD [4], QED [5] and MMD [7] under statistical homogeneity.

Conflict of Interest:

The authors of this paper declare that there is no any kind of conflict of interest.

Acknowledgement:

The authors would like to sincerely thank all the team members including the referees of this paper for their constructive comments and suggestions on this paper.

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