Effect of Hall parameters on MHD flow of Nano fluids Due to Non-Coaxial Rotation of a Porous disk and a Fluid at infinity

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Article Info

Page Number: 2339 - 2361

Publication Issue:

Vol 71 No. 4 (2022)

Article History

***Article Received**: 25 March 2022*

***Revised**: 30 April 2022*

***Accepted**: 15 June 2022*

***Publication**: 19 August 2022*

Abstract

The unsteady viscous incompressible nanofluid flow is investigated analytically for copper water nanofluids under the effects of heat transfer and a uniform transverse magnetics field with the thermo physical properties, in which the flow is established between the non-coaxial rotations of a porous disk and a fluid at infinity. In this investigation, we consider that the studied nanofluid is electrically conducting and Hall currents are taken into account. . In a special case, the present numerical solution is validated analytically and numerically with the earlier available results. The effects of major parameters on the dimensionless velocity, temperature and volumetric fraction of nanoparticles are analysed via representative profiles, whereas the skin friction factor and the heat transfer rate are estimated numerically and discussed through tabular illustrations.

Keywords: MHD Flow,Hall Effects, Nanofluids ,Suction,Co-Axial Rotation, Porous Disk.

1. Introduction: The steady MHDflow of viscous incompressible electrically conducting nanofluid due to a non–coaxial rotation of a porous disk and a fluid at infinity in the presence of uniform transverse magnetic field is studied extensively. Hall Effect was first discovered in 1879 by Edwin Hebert Hall while working on his doctoral degree at the Johns Hopkins University in Baltimore, Maryland, USA. The Hall effect is the production of a voltage difference (The Hall voltage) across a current carrying conductor (in presence of Magnetic field), perpendicular to both current and the magnetic field. The effects due to Hall current become significant when the Hall parameter is high. This situation occurs as a result of high magnetic field. The above work is discussed by Kanch and Jana[1], Guria et al.,[2] Ghar at al,[3,4].Das and Jana,[5] .The effects of Hall current with heat transfer are found to arise in many practical applications namely in power generators, Refrigeration coils, electric transformers and heating elements to mention few (Seth el al.,)[6−8].

Nanofluids are homogeneous mixtures of solids and liquids when these solid particles are smaller than 100 nm. An innovative new class of heat transfer fluids that can be engineered by suspending nanoparticles in conventional heat transfer fluids and able to enhance significantly the thermal conductivity and convective heat transfer performance of its base fluids. The main goal of nanofluids is to achieve the highest possible thermal properties at the smallest possible concentrations by uniform dispersion and stable suspension of nanoparticles in host fluids. The first time the term nanofluid was defined in 1995, when Choi coined it while working in a research project at Argonne National Laboratory, USA.

The study of magnetohydrodynamic (MHD) nanofluid flows has wide range of applications in engineering and science such as medicine delivery processing, cancer therapy, and tumour analysis. Due to the interaction between the magnetic field and electrically conducting fluid, the boundary layer control is affected. The semi analytical solution called homology analysis for unsteady magneto hydrodynamic (MHD) fluid flow and heat transfer of a Newtonian fluid, induced by an impulsively stretched plane surface is investigated by Kumari and Nath,[9]Prasad et al.,[10]studied the influence of variable viscosity on viscoelastic magneto hydrodynamic fluid flow and heat transfer over a stretching sheet and noticed that enhancement in the magnetic parameter tend to decrease the skin fraction at the sheet. Chamkha and Aly,[11]presented twodimensional steady MHD free convection boundary-layer flow of an incompressible nanofluid over a semi-infinite vertical permeable plate in the presence of

Brownian diffusion. Turkyilmazoglu [12] analysed the effect of velocity slip on water-based nanofluid flow for four different type of nanoparticles viz. copper (Cu), silver (Ag), copper oxide (CuO), and titanium oxide (TiO2) under the influence of magnetic field. Ibrahim and Shanker[13] analysed the heat transfer aspect of MHD nanofluid flow over a non-isothermal stretching sheet and found that for prescribed heat flux condition, increase in Lewis number results in an increment in the surface temperature. Sheikholeslamiet al.,[14]studied the MHD effect on natural convection heat transfer flow of Al_2O_3water nanofluid and found that Nusselt number is an increasing function of the buoyancy ratio number but it is reversed for Lewis number and Hartmann number. Hayat et al.,[15] analysed the unsteady MHD two-dimensional squeezing flow of a viscous and incompressible nanofluids restricted between two parallel walls under the influence of thermophoresis.

The study of the fluid flow on the surface of rotating disk has got great attentions around the globe from the researcher's due to its many applications in practical problems. Electric power generating system, rotating machinery, corotating turbines, chemical process and computer storage, in the field of aerodynamics engineering, geothermal industry, for lubrication purposes, over the surface of rotating disk the fluid flow is widely applicable. Von Karman's[16] examined the solution of Navier stoke's equations by considering an appropriate transformation. Further, he used the fluid flow over the rotating frame for the first time. The Von Karman's problem and its solution numerically have been discussed by Cochrn[17]. Also, he used two series expansion by solving the limitation in the Von Karman's work. Sheikholeslami etal.[18] used numerical technique for the solution of nanofluid flow over an inclined rotating disk. During the rotation of the disk, Millsaps and Pahlhauen[19]studied the heat transport characteristic. The electric field in radial direction has been considered by, Turkyilmazogu[20]where the heat transfer phenomena in magnetohydro-dynamic (MHD) fluid flow has been investigated. Under the transverse magnetic field influence, Khan et al.[21]considered the non-Newtonian Powell-Eyring fluid over the rotating disk surface. The entropy generation due to porosity of rotating disk in MHD flow has been investigated by Rashidi et al.[22]. Hayat et al.[23]scrutinized the transfer of heat with viscous nanoliquid among two stretchable rotating sheets. The thermal conductivity that depends on temperature in Maxwell fluid over a rotating disk has been studied by Khan et al.[24]. Bachelor[25]was the first researcher, who discussed the fluid flow between the gaps of the rotating frame. The influence of blowing with wall transpiration, suction and mixed convection has investigated by Yan and Soong[26]. Recently Shuaib et al.[27] studied the fractional behaviour of fluid flow through a flexible rotating disk with mass and heat characteristics. The attention of researcher's is increasing towards nanofluid studies day by day due to its many applications in technology that binging facilities in many industrial process of heat transfer. The applications of nanofluid are in drugs delivery, power generation, micromanufacturing process, metallurgical sectors, and thermal therapy, etc. Choi[28] is a researcher who worked for the first time on nanofluid, where he considered it for cooling and coolant purpose in technologies. He found from his work that in a base fluid (water, oil and blood, etc.) by adding the nanoparticles, the heat transfer of thermal conductivity becomes more effective. Using the idea of Choi's idea, many researchers investigated and obtained results using the nanofluids[29,30].A concentric circular pipe with slip flow has been discussed in Turkyilmazoglu[31].By using finite element method (FEM), Hatami et al.[32] finds the solution for the heat transfer in nanofluid with free natural convective in a circular cavity. The Cattaner-Christov heat flux and thermal radiation for an unsteady squeezing MHD flow has been considered by Ganji and Dogonchi[33] they considered the heat of transfer of the nanofluid among two plates. Dilan et al.[34] studied nanofluids effective viscosity based on suspended nanoparticles. A carbon nanotubes based multifunctional hybrid nanoliquid has been considered by Rossella[35]. The influence of SWCNTs on human epithelial tissues is studied by Kaiser et al.[36] Hussanan et al.[37] examined the Oxide nanoparticles for the enhancement of energy in engine nanofluids, kerosene oil and water. Saeed et al.[38] examined nanofluid to improve the heat transfer rate an reduce time for food processing in the industry. Some recent studies related to heat and mass transfer through nanofluids are examined by many researchers[39−43]

Comprehensive review on nanofluid flows have been made by Keblinski et al.,[44] Wang et al.,[45] Eastman et al.,[46] Choi et al.,[47] Buongiorno [48] and Kakac and combined with Pramuanjaroenkij [49]. Magnetohydrodynamic flow of nonofluids due to a rotating disk has numerous applications in many areas, such as rotating machinery, cooling and heating process of computer devices and crystal growth processes. In view of its wide applications in industrial and other technological fields, the problem of flow due to a rotating disk has been extended to nanofluids. Bachok et al.,[50] have presented the flow and heat transfer over a rotating porous disk in a nanofluid. Rashidi et al.,[51] have analyzed the entropy generation in a steady MHD flow due to a rotating porous disk in a nanofluid. Hussain et al.,[52] have studied the radiative magneto-nanofluid flow over an accelerated moving ramped temperature plate with Hall effects. They have used the Laplace transform technique to solve the mathematical model and reported that the augmentation of Hall current leads to speed up the nanofluid velocity components whereas the solid volume fraction of nanoparticles has a reverse effect on it. The double diffusive MHD natural convection flow of Brinkman type nanofluid with diffusion-thermo and chemical reaction effects has been presented by Kumar et al.,[53] and they solved it analytically by Laplace Transform technique.

It is the aim of the present work to study the steady magnetohydrodynamic flow of a viscous incompressible electrically conducting nanofluid due to non-coaxial rotations of non-conducting porous disk and a fluid at infinity in the presence of a uniform transverse magnetic field. The disk and the fluid at infinity rotate with same angular velocity. The disk and the fluid at infinity are maintained at two different constant temperatures and the viscous dissipation and Joule heating are considered in the energy equation.

Exact solutions are obtained for the governing equations. The numerical computations are performed using MATLAB. Effects of the pertinent parameters on the fluid velocity components, temperature, shear stresses as well as rate of heat transfer at the disk are presented graphically and tabulated.

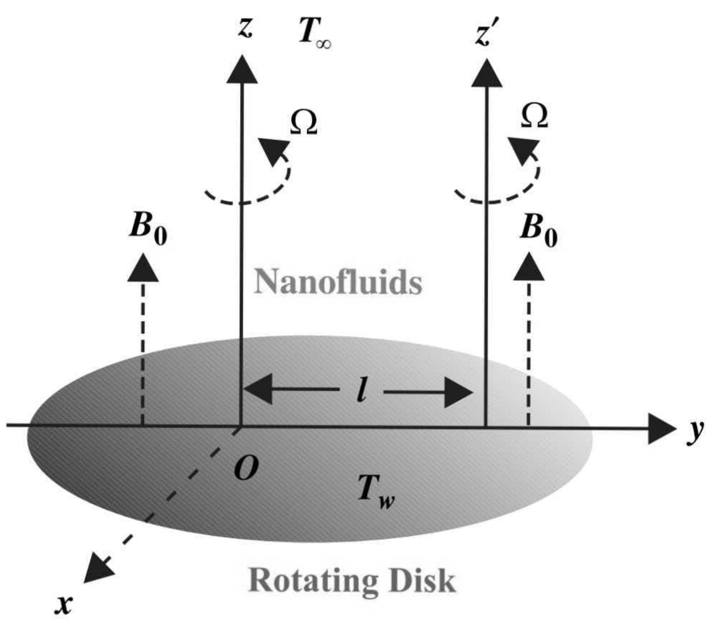


Fig 1.Geometry of the problem

2. Formulation of the problem and its solution:

Consider the steady boundary layer flow of a nanofluid occupying the space Z > 0 and is bounded by an infinite porous disk at Z= 0. Due to the symmetry about the plane Z = 0, it sufficient to consider the problem in the upper half space only. The axes of rotation of the both the disk and that of the fluid at infinity to be in the plane x = 0. The disk and the fluid at infinity are rotating with the same uniform angular velocity **Ω** about the z and z^1 axes respectively and the distance between the axes of rotation is denoted by l as depicted in Fig. 1. The disk is an electrically non-conducting and maintained at temperature$T_w$while the temperature of the ambient fluid is T_∞ at a large distance from the disk.A uniform transverse magnetic field $\mathbf{B_0}$is applied perpendicular to the disk being subject to the disk being subject to uniform suction W_0. The fluid is a water based nanofluid containing four types of nano particles namely copper (Cu), aluminium oxide (Al_2O_3), titanium dioxide (T_iO_2) and silver (Ag). The nanoparticles are assumed to have a uniform shape and size. Moreover it is assumed that both the base fluid and nanoparticles are in thermal equilibrium state. The thermo physical properties of the nanofluid are given in Table.

The geometry of the problem suggest that the velocity field in the flow is of the form

$$\left.\begin{aligned} &u = -\Omega y, v = \Omega x, w = -W_0 \text{ at } z = 0 \\ &u = -\Omega(y - l), v = \Omega x \text{ at } z \to \infty \end{aligned}\right\} \quad (2.1)$$

where u, v and w are respectively the velocity components along x, y and z-direction. The boundary conditions 2.1 shows that the motion is a summation of a helical and translatory motion with the velocity field being

$$u = -\Omega y + f(z), \quad v = \Omega x + g(z), \quad w = h(z) \quad (2.2)$$

Thermo physical properties of water and nanoparticles.

Properties	Water/base Fluid	Cu (Copper)	Ag (Silver)	Al_2O_3 (Alumina)	TiO_2(Titanium) Oxide)
P (kg/m^3)	9997.1	8933	10500	3970	4250
c_p(J/kgK)	4179	385	235	765	686.2
K(W/mK)	0.613	401	429	40	8.9538
Φ	0.0	0.05	0.1	0.15	0.2
σ(S/m)	5.5×10^{-6}	59.6×10^6	-	35×10^6	2.6×10^6

where f,g,h , z are unknown functions and they represent the flow dueto non-coaxial relations of a porous disk and a fluid at infinity.

The equation of continuity gives $\frac{\partial h}{\partial z}= 0$ which on integration yields h = c =-ω_0 everywhere in the flow where w_0 >0 for the suction and w_0 <0 for the blowing at the disk.

On the use of 2.2, the Navier-stoke's eqns. of motion along the x,y and z-direction are

$$g\Omega=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+ \Omega^2 x+ W_0\frac{df}{dz} +v_{nf}\frac{d^2f}{dz^2}+ \frac{B_0}{\rho_{nf}}J_y \quad (2.3)$$

$$-f\Omega=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+ \Omega^2 y+W_0\frac{df}{dz}+v_{nf}\frac{d^2g}{dz^2}-\frac{B_0}{\rho_{nf}}J_x \quad (2.4)$$

$$-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial z}= 0 \quad (2.5)$$

where p is the pressure of nanofluid, v_{nf} the kinematic viscosity of the nanofluid, μ_{nf} the dynamic viscosity of the nanofluid

$$\mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}}, \ \rho_{nf}= (1-\varphi)\rho_f +\varphi\rho_s,$$

$\sigma_{nf} = \sigma_f\left(1+ \frac{3(\sigma-1)\varphi}{(\sigma+2)-(\sigma-1)\varphi}\right), \sigma = \frac{\sigma_s}{\sigma_f}$ (2.6)where φ is the solid volume fraction of nano particles (φ = 0 correspond to a regular fluid),$\rho_f$the density of the base fluid, ρ_s the density of the nanoparticle,σ_f the electrical conductivity of the base fluid, σ_s the electrical conductivity of the nanoparticle.

The boundary condition for f and g are

f=0,y=0 at z=0

f=Ωl,g=0 as z→∞ for t > 0 (2.7)

Making reference to cowling when the strength of the magnetic field is very large, the generalized Ohm's law is modified to include Hall current so that

$$\vec{J}+\frac{\omega_e \tau e}{B_0}(\vec{J}X\vec{B}) =\sigma_{nf}\left(\vec{E} + \vec{q} \ x\vec{B}\right)$$

where $\vec{q}$,$\vec{B}$, $\vec{E}$, $\vec{J}$,ω_e and τ_e are respectively the velocity vector, the magnetic field vector, the electric field vector, cyclotron frequency and electron collision time.

∇.B= 0 (The solenoidal relation)Integrating w.r.t.y

B_y = Constant = $\mathbf{B_0}$

where $B = (0, B_y, 0)$ (flow only in y direction).

The Conservation of electric charge is given by $\nabla.\sigma = 0$.

$\Rightarrow J_z$ = Constants, where $\vec{J} = (J_x, J_y, J_z)$ $J_z = 0$

The first term of the left hand side of (2.8) comes from the electron drag of the ions.

The second term represents the effects due to Hall currents and has to develop with the idea that electrons and ions can decouple and move separately. It is assumed that the magnetic Reynolds number ($Rem \leq 1$) for the flow is very small, so that induced magnetic field can be neglected in comparison with the imposed field $\mathbf{B_0}$. This assumption is justified since the magnetic Reynolds number is generally very small for metallic liquid or partially ionized fluid. Liquid metals can be used in a range of applications because they are non-flammable, nontoxic environmentally safe. That is why, liquid metals have number for technical applications in source exchanges, electronic pumps, amount heat exchangers and also used as a heat engine fluid. Moreover in nuclear power plants sodium, alloys, lead-bismuth and bismuth are extensively used in heat transfer process. Besides that mercury play its role as a fluid in high-temperature Rankine cycles and also used in reactors in order to reduce the temperature of the system. For power plants which are exerted at extensively high temperature, sodium is treated as a heat-engine fluid. The remaining is given in Eqn. (2.1)

$$\Rightarrow J_x + mJ_y = \sigma_{nf}[E_x + v\mathbf{B_0}] \qquad (2.9)$$

$$J_y - mJ_a = \sigma_{nf}[E_y - u\mathbf{B_0}] \qquad (2.10)$$

Wherem =,$\omega_e\, \tau_e$stands for Hall parameter which can take positive or negative values. In general, for an electrically conducting fluid, Hall currents affect the flow in the presence of a strong magnetic field. Further it is assumed that $\omega_e\, \tau_e \sim o(1)$ and $\omega_i\, \tau_i \leq 1$, where ω_e ,ω_i are the cyclotron frequencies of electrons and ions and τ_e , τ_i are the collisiontimes of electrons and ions. In writing the magneticinduction equation, the ion slip effects arising outof imperfect coupling between ions and neutrals as wellas the electron pressure gradient are neglected. The effectof Hall currents gives rise to a force in the z-direction,which induces a cross-flow in that direction and hence theflow becomes three-dimensional.To simplify the problem, we assume that there is no variation of flow quantities in z direction.

In the free-stress, the magnetic field is uniform, so that there is no current and hence, we have

$J_x \to 0, J_y \to 0$ as $z \to \infty$ (2.11)

In view of (2.11), we obtain from (2.9) and (2.10)

$\sigma_{nf}[E_x + v\mathbf{B_0}] = 0 \; \sigma_{nf} Ex + \sigma_{nf} v \mathbf{B_0} = 0$

$\Rightarrow Ex = -\Omega \mathbf{B_0} x, E_y = -\Omega B_0 (y - l)$ (2.12)

everywhere in the flow. It is known that

$J_X + mJ_Y = \sigma_{nf} [-\Omega \mathbf{B_0} x + v \mathbf{B_0}]$

$J_X + mJ_Y = \sigma_{nf} \mathbf{B_0} [-\Omega x + v]$

$J_X = \sigma_{nf} \mathbf{B_0} [-\Omega x + v] - mJ_Y$ from (2.9)

$J_y = \sigma_{nf} E_y - \sigma_{nf} u \mathbf{B_0} + mJ_X$ from (2.10)

$J_X = \sigma_{nf} \mathbf{B_0}[-\Omega x + v] - m\sigma_{nf}\left(E_y - uB_0\right) + m J_X] J_X = \sigma_{nf} \mathbf{B_0}[-\Omega x + v] - m\sigma_{nf} E_y + m\sigma_{nf} u \mathbf{B_0} - m^2 J_X]$

$J_X + m^2 J_X = \sigma_{nf} \mathbf{B_0}[m_u - \Omega x + v] - m\sigma_{nf} E_y$

$J_X[1+m^2] = \sigma_{nf} \mathbf{B_0}(m_u - \Omega x + v) - m \sigma_{nf}(-\Omega \mathbf{B_0}(y-l))$

$[1+m^2] = \sigma_{nf} \mathbf{B_0}(m_u - \Omega x + v + m\sigma_y - m\Omega l)$

$f = u + \Omega y, \; g = v - \Omega x$

$J_X[1+ m^2] = \sigma_{nf} \mathbf{B_0}[mf + g - m\Omega l)]$

$J_X[1+ m^2] = \sigma_{nf} \mathbf{B_0}[g - m(\Omega l - f)]$

$J_X = \frac{\sigma_{nf}}{1+m^2}[g - m(\Omega l - f)]$ (2.13)

$J_y = \sigma_{nf}\left(E_y - u\mathbf{B_0}\right) + m J_X)$

$J_y = \sigma_{nf}[-\Omega \mathbf{B_0}(y - l) - B_0 u] + \frac{m\sigma_{nf} B_0}{1+m^2}[g - m(\Omega l - f)]$

$J_y = -\Omega \mathbf{B_0} y \sigma_{nf} + \Omega \mathbf{B_0} l \, \sigma_{nf} + \frac{m\sigma_{nf}\mathbf{B_0}}{1+m^2} g \frac{m\sigma_{nf}\mathbf{B_0}}{1+m^2} (\Omega l f)$

$J_y = -\mathbf{B_0}\sigma_{nf}[u + \Omega y] + \Omega \mathbf{B_0} l \, \sigma_{nf} + \frac{m}{1+m^2} \sigma_{nf} \mathbf{B_0}[g - m(\Omega l - f)]$

$= -\mathbf{B_0}\sigma_{nf} + \mathbf{B_0}\sigma_{nf} \Omega l + \frac{m}{1+m^2} \sigma_{nf} \mathbf{B_0}[g - m(\Omega l - f)]$

$= \frac{[(1+m^2)(\Omega l - f)+mg - m^2(\Omega l - f)]\mathbf{B_0}\sigma_{nf}}{(1+m^2)}$

$J_y = \frac{\mathbf{B0}\sigma_{nf}}{(1+m^2)}[mg + (\Omega l - f)]$ (2.14)

Substituting equation (2.13) & (2.14) in equation (2.3) to (2.5)

$-g\Omega = \Omega^2 x + \frac{1}{\rho_{nf}}\frac{\partial f}{\partial x} + \omega_0 \frac{\partial f}{\partial z} v_{nf} \frac{\partial^2 f}{\partial z^2} + \frac{\sigma_{nf} B_0^2}{\rho_{nf}(1+m^2)}[(\Omega l - f) + mg]$ (2.15)

$-f\Omega = -\Omega^2 y + \frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} + \omega_0 \frac{\partial f}{\partial z} v_{nf} \frac{\partial^2 g}{\partial z^2} - \frac{-\sigma_{nf} B_0^2}{\rho_{nf}(1+m^2)}[g - m(\Omega l - f)]$ (2.16)

$-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial z} = 0$ (2.17)

on the use of infinity Conditions, equation(2.15)&(2.16) yield

$g = 0, f = \Omega l$ as $z \rightarrow \infty$

now the equations (2.15)&(2.16) becomes

$$0 = -\Omega^2 x + \frac{1}{\rho_{nf}}\frac{\partial p}{\partial z} \quad (2.18)$$

$-\Omega^2 l = -\Omega^2 y + \frac{1}{\rho_{nf}}\frac{\partial p}{\partial z}$(2.19)

$-g\,\Omega = \omega_0 \frac{\partial f}{\partial z} + v_{nf} \frac{\partial^2 f}{\partial z^2} + \frac{\sigma_{nf} B_0^2}{\rho_{nf}(1+m^2)}[(\Omega l - f) + mg]$

$\Rightarrow v_{nf} \frac{\partial^2 f}{\partial z^2} + \omega_0 \frac{\partial f}{\partial z} + \Omega g + \frac{\sigma_{nf} B_0^2}{\rho_{nf}(1+m^2)}[(\Omega l - f) + mg]$ (2.20)

Similarly,

$v_{nf} \frac{\partial^2 f}{\partial z^2} + \omega_0 \frac{\partial f}{\partial z} - \Omega^2 l + f\Omega - \frac{\sigma_{nf} B_0^2}{\rho_{nf}(1+m^2)}[g - m(\Omega l - f)] = 0$(2.21)

Now combining equation (2.20)&(2.21), (2.20) + (2.21)

$v_{nf}\left(\frac{\partial^2 f}{\partial z^2} + \frac{\partial^2 g}{\partial z^2}\right) + \omega_0\left(\frac{\partial f}{\partial z} + \frac{\partial f}{\partial z}\right) + \Omega[g + \Omega l - k] + \frac{\sigma_{nf} B_0^2}{\rho_{nf}(1+m^2)}[(\Omega l - f) + mg] - [g + m(\Omega l - f)] = 0$(2.22)

Introducing upon non- dimensional variables $\eta = \sqrt{\frac{\Omega}{\upsilon_f}}z$, $F = 1 - \left[\frac{f}{\Omega l} + i\frac{f}{\Omega l}\right]$

any $\eta \rightarrow 0$ (ie) $v_{nf} = v_f$

$i=\sqrt{-1}\sqrt{2} = \sqrt{2^{\frac{1}{8}}}$(2.23)

f + ig = (1 - F) Ωl

$= \frac{1}{2\sqrt{2}} = -\frac{1}{2}2^{\frac{-3}{2}}$

and combining (2.20) and (2.21)

$\left(\frac{v_f}{\Omega l}\frac{\partial^2 f}{\partial z^2}\right)\left(\frac{\omega_0}{\Omega l}.\frac{\partial F}{\partial z}\right) + \Omega[\Omega l - (f - g)]$

$x_2\frac{\partial^2 f}{\partial \boldsymbol{\eta}^2} + x_1\frac{\partial F}{\partial \boldsymbol{\eta}} - \left(\frac{x_3 M^2}{1+m^2} + \left(ix_1 + \frac{x_3 m M^2}{1+m^2}\right)\right)F=0$(2.24)

where

$$\left.\begin{aligned} x_1 &= \left[(1-\varphi)+\varphi\left(\frac{\rho_s}{\rho_f}\right)\right], \quad x_2 = \frac{1}{1-\varphi^{\,2.5}}, \\ x_3 &= \left[1+\frac{3(\sigma-1)\varphi}{(\sigma+2)-(\sigma-1)\varphi}\right]\sigma = \frac{\sigma_s}{\sigma_f} \end{aligned}\right\} \qquad (2.25)$$

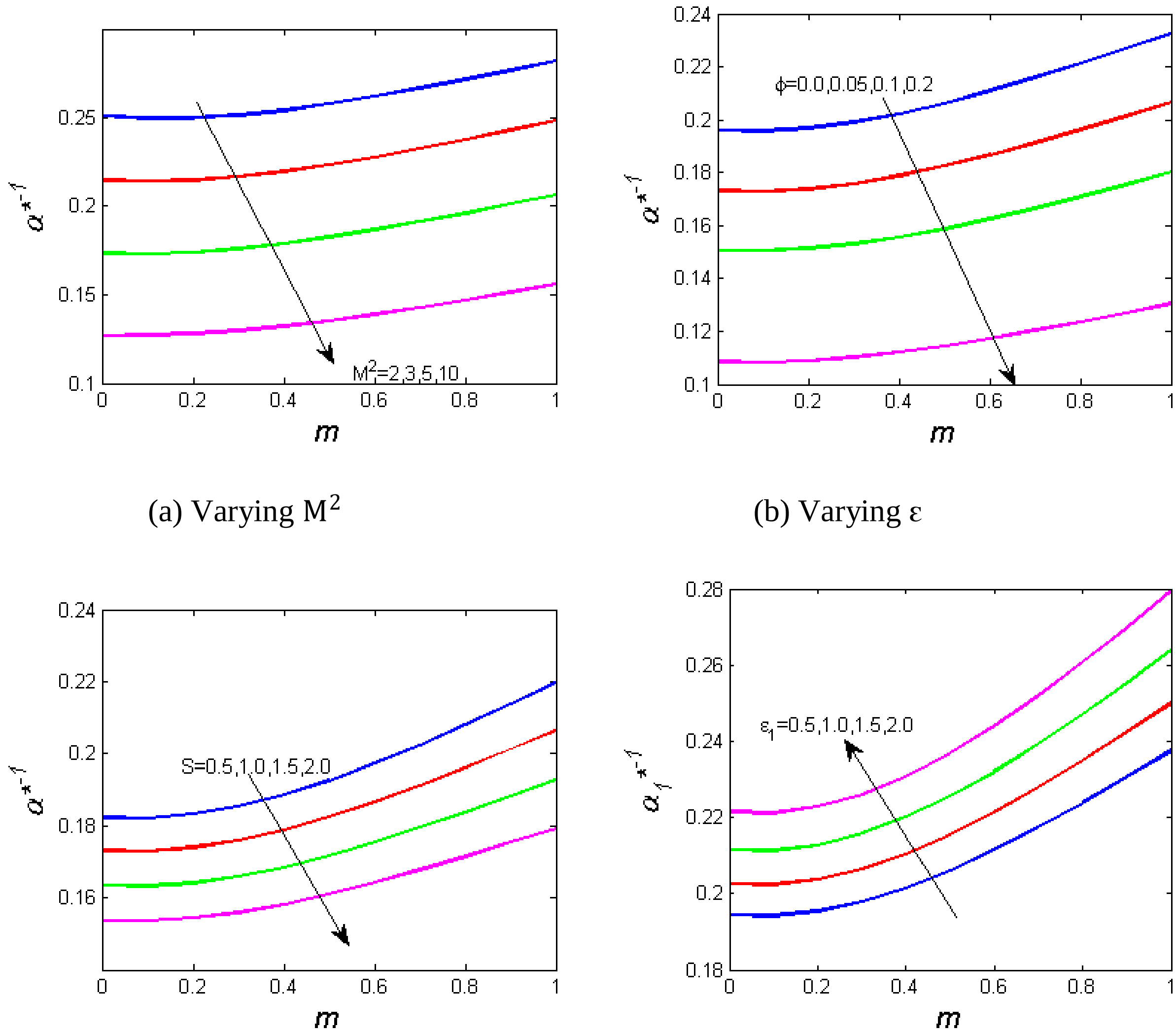
and$M^2 = \frac{\sigma_f B_0^2}{(\rho_f\Omega)}$ is the magnetic parameter which represents the ratio of the magnetic force to the viscous force and $\varepsilon = \frac{\omega_0}{\sqrt{\Omega v f}}$the suction/blowing parameter. The boundary conditions for F (η) are

F(0) = 1 and F → 0 as η→ ∞

Solution of (2.24) subject to boundary conditions (2.25) can easily be obtained and on using (2.22), we get

$\frac{f}{\Omega l} = 1 - e^{-\alpha^*\eta}\cos\beta\eta$(2.26)

$\frac{g}{\Omega l} = e^{-\alpha^*\eta}\sin\beta\eta$ (2.27)



(a) Varying M^2 (b) Varying ε

(c) Varying φ(d) Varying ε_1

Fig.2. Variations of α^{*-1} and α_1^{*-1} for increasing values of embedded parameter.

Where

$\alpha, \beta = \frac{1}{2\sqrt{2}\,x_2}\left[(a_1^2 + b_1^2)^{\frac{1}{2}} \pm a\right]^{\frac{1}{2}}, \alpha^* = \left(\frac{\varepsilon x_1}{2x_2} + \alpha\right),$

$a = \left(\varepsilon^2 x_1^2 + \frac{4x_2 x_3 M^2}{1+m^2}\right), b = 4x_2\left(x_1 + \frac{mM^2 x_3}{1+m^2}\right)$ (2.28)

It is seen that the solution given by (2.26) and (2.27) are valid for both suction (ε>0) and blowing (ε< 0) at the disk.Equations (2.26) and (2.27) show that the velocity distribution is in the from an Ekman spiral representing the flow past aporous rotating disk.The flow exhibits aboundary layer behaviour with themomentum boundry layer thickness of order $O(\alpha^{*-1})$. Itis evident from Figures 2(a)–(c) that thisthickness decreases with an increase ineither magnetic parameter M^2 or suction parameter ε or volume fraction φ of nanoparticles whileit increases with an increase in Hall parameter m.

In case of blowing at the disk (ε< 0), an asymptotic solution similar to above is also possible. In this case we take $\varepsilon = -\varepsilon_1$ with $\varepsilon_1 > 0$. The solution is now given from (2.26) and (2.27) as

$$\frac{f}{\Omega l} = 1- e^{-\alpha_1^* \eta} \cos \beta_1 \eta \quad (2.29)$$

$$\frac{g}{\Omega l} = e^{-\alpha_1^* \eta} \sin \beta_1 \eta \quad (2.30)$$

where

$$\alpha_1, \beta_1, = \frac{1}{2\sqrt{2}\,x_2}[(a_1^2 + b_1^2)^{\frac{1}{2}} \pm a]^{\frac{1}{2}}, \alpha_1^* = \left(-\frac{\varepsilon_1 x_1}{2x_2} + \alpha_1\right),$$

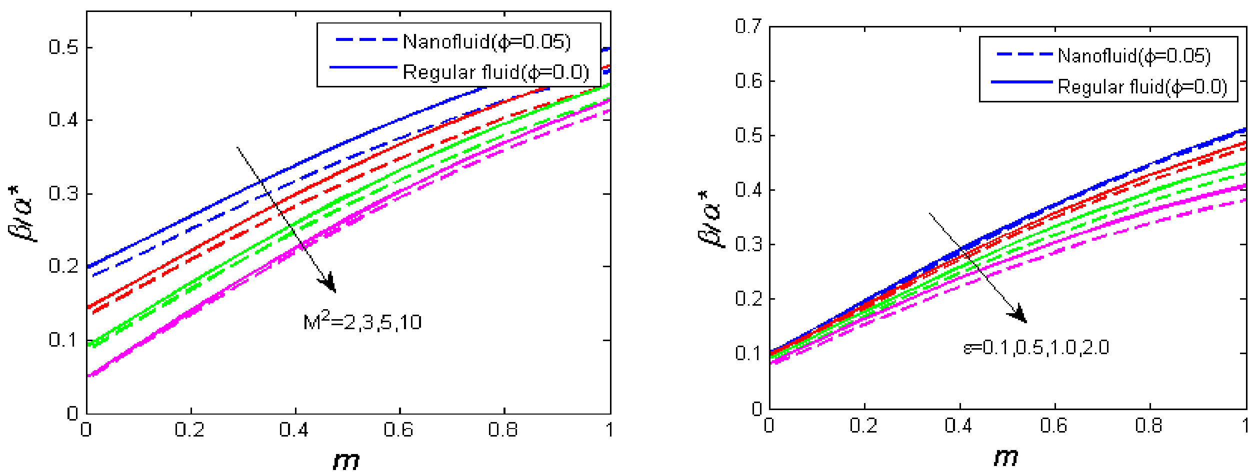
$$a_1 = \left(\varepsilon_1^2 x_1^2 + \frac{4x_2 x_{3M^2}}{1+m^2}\right), \qquad b_1 = 4x_2\left(x_1 + \frac{mM^2 x_3}{1+m^2}\right) \quad (2.31)$$

It follows from (2.29) and (2.30) that the flow has a boundary layer structure even in the presence of blowing at the disk with boundary layer thickness of order$O(\alpha^{*-1})$.

Figure 2(d) shows that the variation of $\alpha_1^{*-1}$with blowing. It is observed that this thickness increases monotonically with an increase in blowing parameterε_1, which is consistent with the fact that blowing tends to thicken the boundary layer.

For impermeable disk (ε = 0), solutions (2.26) and (2.27) are identical with the Eqs.(2.26) and (2.27) of Das and Jana in the absence of Hall currents (m = 0) and nanoparticles (φ= 0). The velocity components given by (2.26) and (2.27) reduce to those given by Erdogan in the absence of a magnetic field (M^2= 0) and nanoparticles (φ = 0). The present study is identical with Kanch and Jana when φ = 0. It is seen from (2.26) and (2.27) that very near the disk z = 0,

$$\frac{f}{\Omega l} \cong \alpha^* \eta \quad , \frac{g}{\Omega l} \cong \beta \eta \qquad (2.32)$$



(a) Varying M^2 (b) Varying ε

Fig.3.Variation of $\frac{\beta}{\alpha^*}$ for increasing values of embedded parameters.

and hence the velocity near z = 0 is inclined at an angle arctan$\left(\frac{\beta}{\alpha^*}\right)$in an anticlockwise direction from the y-axis. The values of $\frac{\beta}{\alpha^*}$ have been plotted against m for several values of $M^2$and ε in Figure 3. It is observed that $\frac{\beta}{\alpha^*}$decreases with increase in either M^2or ε while it increases with increase in m. The velocity components given by (2.32) reduce to the result (2.27) of Kanch and Jana when φ= 0 the suction/blowing parameter. The boundary conditions for F are

Substitute the derivation (2.22)

$F = e^{\alpha\eta}[A \cos \beta\eta + i\, B \sin \beta\eta]$ [η = 0 , A = 1 , η = ∞ , B = 1]

F = 0

For eqn(2.22) we

$1\ \Omega l + \Omega l = e^{\alpha\eta}[\cos\beta\eta + i\sin\beta\eta]$

Eqn real &Img.we get

$$\frac{f}{\Omega l} = 1 - e^{\alpha\eta} \cos \beta\eta$$

$\frac{g}{\Omega l} = e^{\alpha\eta} \sin \beta\eta$

Remaining effects are special cases.

Matlab Coding:

```
z=linspace(0,1,100);
phi=0;
rho_s=997.1;
rho_f=8933;
rho=rho_s/rho_f;
sigma_s=0.0000055;
sigma_f=59600000;
sigma=sigma_s/sigma_f;
ε =1;
M=5;
m=0.2;
x1=(1-phi)+phi*(rho);
x2=1/((1-phi)^(2.5));
x3=(1+((3*(sigma-1)*phi)/((sigma+2)-(sigma-1)*phi)));
a= ε ^2*x1^2+((4*x2*x3*M)/(1+m^2));
b=4*x2*(x1+((m*M*x3)/(1+m^2)));
p=(1/2*(sqrt(2))*x2)*(((a^2+b^2)^(1/2))+a)^(1/2);
q=(1/2*(sqrt(2))*x2)*(((a^2+b^2)^(1/2))-a)^(1/2);
r=(( ε *x1)/(2*x2))+p;
for i = 1 : length(z)
F(i,:)=1-(exp(-r*z(i)))*cos(q*z(i));
G(i,:)=(exp(-r*z(i)))*sin(q*z(i));
end
```

```
plot(z,F)
figure
plot(z,G)
```

3.Result and discussion:

The profiles of real part and imaginary Part of the velocity profile and shear stress is with respect to the rotation disk are computed for various values of governing parameters M^2, m, S, volume fraction of φ of nano particles E_c using the software package MATLAB 12.0 for all the assumed combinations of boundary condition and the obtained values are tabulated.

The numerical procedure adopted is validated by taking the value of the for the base fluid is kept as Pr=6.2 and the effect of solid volume of nanoparticle is investigated in the range of $0 \leq \varphi \leq 0.2$. It is well in agreement with Das S et al.,[54].

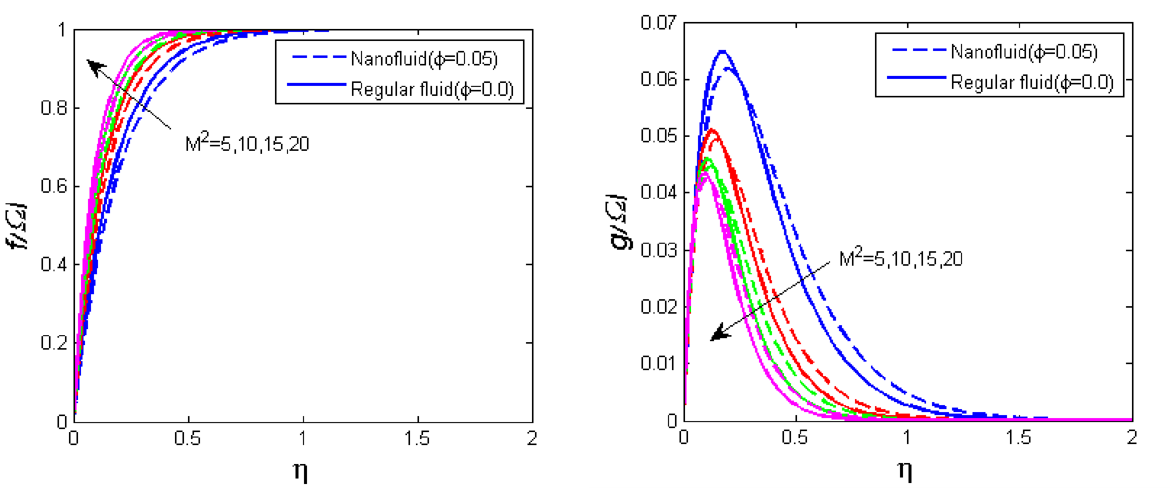
3.1 .Effects of parameter on velocity profiles:

The velocity component f/(Ωl) (the velocity component in the direction normal to the plane containing the axis of rotation of the disk and that of the fluid at infinity) and the magnitude of the velocity component g/(Ωl) (the velocity component in the transverse direction parallel to the plane of the disk) are presented in Figure 4 for the cases of both regular fluid (φ = 0) and nanofluid (φ = 0.1). The velocity components f/(Ωl) and g/(Ωl) retard with an increase in magnetic parameter

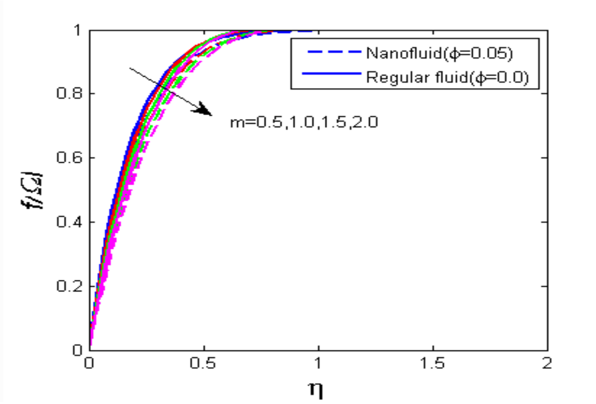
M^2as shown in Figures 4(a) and (b). The magnetic field has a significant influence on the velocity components. It is clearly seen from these figures that the velocity components and the boundary layer thickness decrease if M^2 increases. This is not a surprise as the transverse magnetic field produces a resisting force (Lorentz force) that is similar to the drag force which tends to oppose the flow and to depress the velocities in magnitude and it exerts a very dramatic effect on the flow. Figures 4(c) and (d) display the effects of Hall parameter m on the velocity components. The velocity component f/(Ωl) decreases with increase in Hall parameter m, as seen in Figure 4(c). The influence of Hall current on the velocity component g/(Ωl) is shown in Figure 4(d). It is seen that the velocity component g/(Ωl) increases with m and it is equal to zero when m becomes very large. This is because the effective term

$1/(1+m^2)$ decreases as m increases, and hence the resistive effect of the magnetic field is diminished. Since the magnetic field is strong, the electromagnetic force becomes very large, which results in the occurrence of Hall currents. The y-component of the velocity is totally dependent on Hall currents; therefore it can be manipulated by changing the Hall parameter.

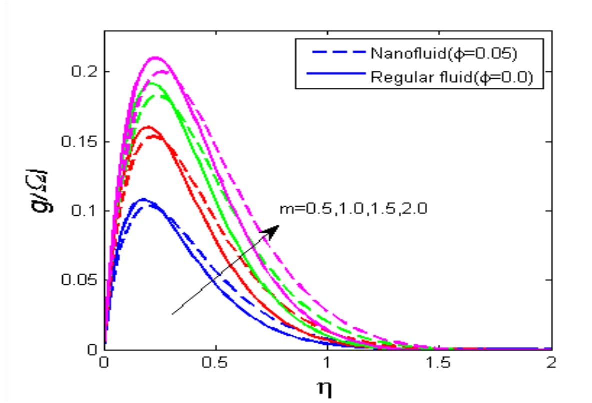
The influence of suction parameter ε on the velocity components is elucidated from Figures 4(e) and (f). It can be observed that the velocity components f /(Ωl) and g/(Ωl) increase for increasing values of S. Physically, because the fluid is sucked away near the disk due to suction. Consequently the velocity components increase. There is a rise in the velocity component f/(Ωl) due to the enlargement of φ which is elucidated from Figure 4(g). Figure 4(h) reveals that an increase in φ leads to first increase in the velocity component g/(Ωl) and then decrease. This shows that the addition of nanoparticles presents extra resistance to the flow away from the disk and hence results in lowering the velocity component g/(Ωl) in that region. However, an increase in φ near the disk prevents the reflux phenomenon. The results show that the presence of solid nanoparticles leads to further thinning of the velocity boundary layer.



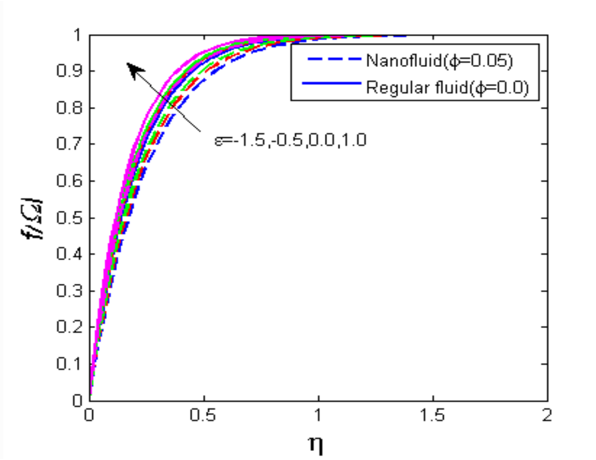
(a)Varying M^2 (b) Varying M^2



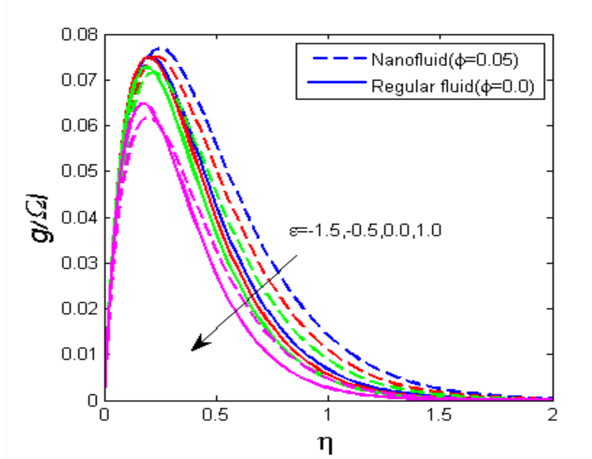
(c) Varying m



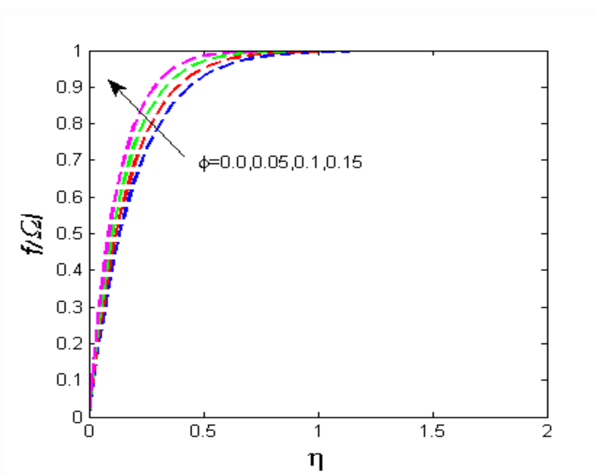
(d) Varying m



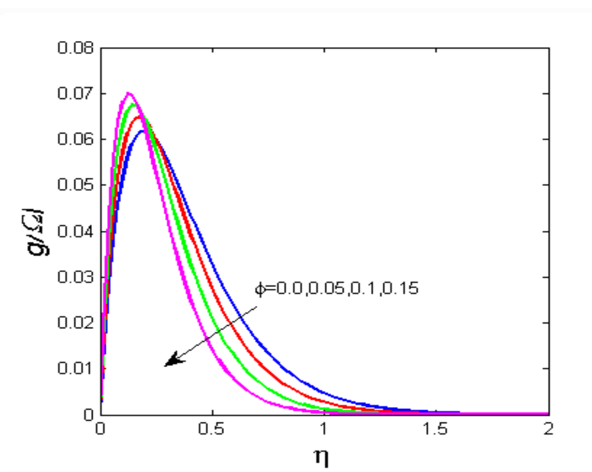
(e) Varying time ε



(f) Varying ε



(g) Varying time φ



(h) Varying φ

Fig 4.Variations off/(Ωl) andg/(Ωl) for increasing values of embedded parameters.

Conclusion :

In this study, we have obtained an exact solution for the governing momentum and energy equations corresponding to the magnetohydrodynamic flow of nanofludis due to non-co-axial rotations of an electrically non-conducting porous disk and a fluid at infinity. The effects of the pertinent parameters on the flow fields and the rate of heat and force exerted by the fluid on the disk have been analyzed. The important results of the present study can be listed as,the velocity components are suppressed as the strength of the magnetic field enhances, the velocity components are significantly influenced by Hall parameter,

The presence of nanoparticle volume fraction is to reduce the rate of heat transfer at the disk.

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