Shock Model with Multiple Change Points Having Exponential Threshold

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Abstract

Cumulative damage process is related to shock models in reliability theory. The threshold or withstanding capacity of the system can be considered to be any one of the constant level, cumulative shock random threshold level, maximum shock random threshold etc. In this paper, cumulative shock random threshold level is considered wherein a system undergoes a shock and encounters random amount of damage but the system survives with the damages. Successive shocks at random epochs lead to the cumulative damages and when the cumulative damage crosses the threshold level of the system, the system fails. Assuming threshold level undergoes a change in the form of distribution after the change points are taken to be exponential random variable, the distribution function for the threshold level is obtained by taking exponential distribution as the threshold levels are before, in between and after the change points respectively. Using this distribution function for the threshold level, the expected time to the breakdown of the system and its variance are obtained by using shock model and cumulative damage process approach.

Keywords: Shock Model, Cumulative damage Process, Threshold, Change Points, Laplace transforms

Introduction

When the device can be exposed to a shock, it undergoes damages. After damage we can do repair or replacing a new device. Every device has withstanding capacity, which is termed as threshold level. The system survives if the damage is less than threshold level on the other hand, the device fails. Esary, Marshall and Proschan (1973) discussed about shock model and cumulative damage process, in which a device undergoes random amount of damage at the epoch of a shock and successive shocks at random epochs leads to the cumulative damage and when the cumulative damage crosses the threshold level, the device fails. Esary and Marshall (1973) the shocks occurred to the system are managed by homogeneous Poisson process and also the properties of the survivor function of the life time of the device are discussed. Gopal and Suresh Kumar (2006) has discussed about a shock model when the threshold has a change of distribution after a change point.

In this paper, considering the threshold has an exponential distribution after two change points and distribution function for the threshold level is obtained when the change points are random variables. The threshold level is distributed as exponential before τ_1 , τ_1 to τ_2 and after τ_2. This says that the withstanding capacity of the system becomes weaker after a course of time. Due to the fact that the distribution as same in the threshold distribution at any stage, here τ_1 to τ_2 is considered to be random variables (r.v) The expected time to the breakdown of the system and its variance are obtained by using the shock model approach when two change points.

Notations:

The notations used in this paper are given as follows:

X_i =continuous random variable (r.v) representing the random amount of the damage due to i^{th} shock, i = 1,2,….k. Also X_i's are i.i.d.

Its probability density function (p.d.f) is g(x) and cumulative distribution function (c.d.f) is G(x).

Y: continuous r.v. denotes the threshold level whose p.d.f. is h(y) and c.d.f. is H(y).

U_i : continuous r.v denoting the inter-arrival times between successive shocks, i =1,2,….k, U_i's are i.i.d f(u) is its p.d.f and F(u) is its c.d.f.

g_k, f_k and F_k are k fold convolutions of g, f and F respectively.

T: continuous r.v representing the time to the breakdown of the system. Its p.d.f. is l(t) and c.d.f. is L(t).

*: Laplace Transform.

2. Threshold Distribution having multiple Change Points

Y is a threshold r.v and it means that the withstanding ability of the system is distributed according to a probability law.

Let τ_1 ,$\tau_2$are change points .Y falls below τ_1 then Y is distributed according to the probability law $h_1(y;\theta_1)$,if Y falls in between τ_1 , τ_2 then it is distributed to $h_2(y;\theta_2)$ and Y falls after τ_2 this can be distributed to $h_3(y;\theta_3)$

The r.v Y has the p.d.f $h_1(y)$ before $\tau_1$and its c.d.f $H_1(y)$ and it undergoes a change in the form of p.d.f $h_2(y)$ in between (τ_1 , τ_2) the c.d.f $H_2(y)$and the next change in the form of p.d.f $h_3(y)$ after τ_2 then the c.d.f is $H_3(y)$

When $y \leq \tau_1$ the c.d.f is $H_1(y)$

When $\tau_1 \leq y \leq \tau_2$ the c.d.f is$H_1(\tau_1)+H_2(\tau_2-\tau_1) - H_1(\tau_1)H_2(\tau_2-\tau_1)$

When $y \geq \tau_2$the c.d.f is$H_1(\tau_1)+H_2(\tau_2-\tau_1)+H_3(y-\tau_3) - H_1(\tau_1)H_2(\tau_2-\tau_1) - H_2(\tau_2-\tau_1)H_3(y-\tau_3)$ $-H_1(\tau_1)H_3(y-\tau_3)+H_1(\tau_1)H_2(\tau_2-\tau_1)H_3(y-\tau_3)$

i.e

$$H(y) = \{ \quad H_1(y) \;\; if \;\; y \leq \tau_2 \; H_1(\tau_1) + H_2(\tau_2 - \tau_1) - H_1(\tau_1)H_2(\tau_2 - \tau_1) \;\; if \;\; \tau_1 \leq y \leq \tau_2 \; H_!(\tau_1) + H_2(\tau_2 - \tau_1) + H_3(y - \tau_2) - H_1(\tau_1)H_2(\tau_2 - \tau_1) - H_2(\tau_2 - \tau_1)H_3(y - \tau_2) - H_3(y - \tau_2)H_!(\tau_1) + H_1(\tau_1)H_2(\tau_2 - \tau_1)H_3(y - \tau_2) \;\; if \; y \geq \tau_2 \tag{2.1}$$

It can be verified that h(y) is a proper p.d.f.

Assume $h_1(y)$ follows exponential distribution with parameter θ_1 , $h_2(\tau_2 - \tau_1)$ follows exponential with parameter θ_2 and $h_2(y - \tau_2)$ follows exponential with parameter θ_3.

h(y) ={$e^{-\theta 1\tau_1} \theta_1 e^{-\theta 2\,(y-\tau_1)}$ $\quad if\ \tau_1 \le y \le \tau_2$ $\theta_1 e^{-\theta 1\, y}$ $\; if\ \ 0 \le y \le \tau_1$ $\; e^{-\theta 1 \tau_1}\, e^{-\theta 2\,(\tau_2-\tau_1)} \theta_3\, e^{-\theta 3\,(y-\tau_2)}$ $\; if\ y \ge \tau_2$

(2.2)

It can shown that H(0) = 0 and H(∞) = 1

The survivor function of Y is $\underline{H}$ (y) = 1 – H(y)

$$\underline{H}(y) = e^{-\theta 1\, y} + e^{-\theta 1 \tau_1}\, e^{-\theta 2\,(y-\tau_1)} + e^{-\theta 1 \tau_1}\, e^{-\theta 2\,(\tau_2-\tau_1)} e^{-\theta 3\,(y-\tau_2)} \qquad (2.3)$$

3. Shock Model and Cumulative Damage Process

Assumptions: The following are the assumptions underlying in the model developed here.

(i) The damage occurs to the system is linear and cumulative.

(ii) The damages occur to the system are independent of the threshold level.

(iii) The threshold level of the system is assumed to be r.v. Also, the change points $\tau_1 and\ \tau_2$ are comsidered to be r.v.

S(t) is the survivor function which gives the probability that the system will fail after time 'T' .

Using the idea of shock model and cumulative damage process, the survivor function can be given as

S(t) =$\sum_{k=0}^{\infty}$ Probability that there are exactly k shocks in(0,t) × probability that the cumulative damages do not cross its threshold levels.It is known from renewal theory that

Pr[exactly k policy decisions in(0,t)] =$F_k(t) - F_{k+1}(t)$

Probabilty that the cumulative damages do not cross its threshold level is given by

$$P\left[\sum_{i=1}^{k} X_i < y\right] = \int_0^{\infty} g_k(x)\underline{H}(x)dx \qquad (3.1)$$

$$S(t) = \sum_{k=0}^{\infty} (F_K(t) - (F_{K+1}(t)) P\left[\sum_{i=1}^{K} x_i < y\right] \qquad (3.2)$$

$$P\left[\sum X_i < Y\right] = \int_0^{\infty} g_k(x)[1 - H(x)]dx \qquad (3.3)$$

Substitute the value of 1 –H(x) value in equation (3.3)

$$= \int_0^{\infty} g_k(x)\left[e^{-\theta_1 x}\right]dx + \int_0^{\infty} g_k(x) e^{-\theta_1 \tau_1} e^{-\theta_2(-\tau_1)} dx + \int_0^{\infty} g_k(x) e^{-\theta_1 \tau_1} e^{-\theta_2(\tau_2-\tau_1)} e^{-\theta_3(x-\tau_2)} dx$$

$$= \left[g^*(\theta_1)\right]^k + e^{(\theta_2-\theta_1)\tau_1}\left[g^*(\theta_2)\right]^k + e^{(\theta_2-\theta_1)\tau_1} e^{(\theta_3-\theta_2)\tau_2}\left[g_\phi^*(\theta_3)\right]^k \qquad (3.4)$$

Then,

$$S(t)=\sum_{K=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left\{\left[g^*(\theta_1)\right]^k+e^{(\theta_2-\theta_1)\tau_1}\left[g^*(\theta_2)\right]^k+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\left[g^*(\theta_3)\right]^k\right\}$$

$$=\sum_{K=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left[g^*(\theta_1)\right]^k+e^{(\theta_2-\theta_1)\tau_1}\sum_{K=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left[g^*(\theta_2)\right]^k+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\sum_{K=0}^{\infty}\left[F_k(t)-F_{k+1}(t)\right]\left[g^*(\theta_3)\right]$$

$$=\left[1-(1-g^*(\theta_1)\right]\sum_{K=0}^{\infty}\left[F_k(t)(g^*(\theta_1)^{k-1}\right]+e^{(\theta_2-\theta_1)\tau_1}\left[1-(1-g^*(\theta_2)\right]\sum_{K=1}^{\infty}\left[F_k(t)(g^*(\theta_2)^{k-1}\right]+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\left[1-(1-g^*(\theta_3)\right]\sum_{K=1}^{\infty}\left[F_k(t)\right]\left[g^*(\theta_3)^{k-1}\right]$$

$$H(t)=1-S(t)$$

$$=1-\left\{\left[1-(1-g^*(\theta_1)\right]\sum_{K=1}^{\infty}\left[F_k(t)(g^*(\theta_1)^{k-1}\right]+e^{(\theta_2-\theta_1)\tau_1}\left[1-(1-g^*(\theta_2)\right]\sum_{K=1}^{\infty}\left[F_k(t)(g^*(\theta_2)^{k-1}\right]+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\left[1-(1-g^*(\theta_3)\right]\sum_{K=1}^{\infty}\left[F_k(t)\right]\left[g^*(\theta_3)^{k-1}\right]\right\}$$

$$H(t)=1-1+(1-g^*(\theta_1))\sum_{k=1}^{\infty}F_k(t)(g^*(\theta_1))^{k-1}-e^{(\theta_2-\theta_1)\tau_1}\sum_{k=1}^{\infty}F_k(t)(g^*(\theta_2))^{k-1}+e^{(\theta_2-\theta_1)\tau_1}\sum_{k=1}^{\infty}F_k(t)(g^*(\theta_2))^{k-1}-e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\sum_{k=1}^{\infty}F_k(t)(g^*(\theta_3))^{k-1}+(1-g^*(\theta_3)\sum_{k=1}^{\infty}F_k(t)(g^*(\theta_2))^{k-1}e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}$$

(3.5)

Laplace Transform H(t) is $\underline{H}(s)$,

$$\overline{H}(s)=(1-g^*(\theta_1)\sum_{k=1}^{\infty}F_k(t)(g^*(\theta_1))^{k-1}\int_0^{\infty}e^{-st}F_k(t)dt-e^{(\theta_2-\theta_1)\tau_1}(g^*(\theta_2))^{k-1}\int_0^{\infty}e^{-st}F_k(t)dt+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\sum_{k=1}^{\infty}(g^*(\theta_2))^{k-1}\int_0^{\infty}e^{-st}F_k(t)dt+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\sum_{k=1}^{\infty}(g^*(\theta_3))^{k-1}(1-g^*(\theta_3)\int_0^{\infty}e^{-st}F_k(t)dt$$

$$=(1-g^*(\theta_1)\sum_{k=1}^{\infty}(g^*(\theta_1))^{k-1}\overline{F_k(s)}-e^{(\theta_2-\theta_1)\tau_1}\sum_{k=1}^{\infty}(g^*(\theta_2))^{k-1}\overline{F_k(s)}+e^{(\theta_2-\theta_1)\tau_1}(1-g^*(\theta_2)\sum_{k=1}^{\infty}(g^*(\theta_2))^{k-1}\overline{F_k(s)}-e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\sum_{k=1}^{\infty}(g^*(\theta_3))^{k-1}\overline{F_k(s)}+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(1-g^*(\theta_3)\sum_{k=1}^{\infty}(g^*(\theta_3))^{k-1}\overline{F_k(s)}$$

(3.6)

$$H^*(s)=(1-g^*(\theta_1)\sum_{k=1}^{\infty}(g^*(\theta_1))^{k-1}\left(f^*(s)\right)^k-e^{(\theta_2-\theta_1)\tau_1}\sum_{k=1}^{\infty}(g^*(\theta_2))^{k-1}\left(f^*(s)\right)^k+e^{(\theta_2-\theta_1)\tau_1}(1-g^*(\theta_2)\sum_{k=1}^{\infty}(g^*(\theta_2))^{k-1}\left(f^*(s)\right)^k-e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}\sum_{k=1}^{\infty}(g^*(\theta_3))^{k-1}\left(f^*(s)\right)^k+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(1-g^*(\theta_3)\sum_{k=1}^{\infty}(g^*(\theta_3))^{k-1}\left(f^*(s)\right)^k$$

$$=(1-g^*(\theta_1)f^*(s)\sum_{k=1}^{\infty}(g^*(\theta_1)f^*(s)^{k-1}-e^{(\theta_2-\theta_1)\tau_1}f^*(s)\sum_{k=1}^{\infty}(g^*(\theta_2)f^*(s)^{k-1}+e^{(\theta_2-\theta_1)\tau_1}(1-g^*(\theta_2)f^*(s)\sum_{k=1}^{\infty}(g^*(\theta_2)f^*(s)^{k-1}-e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}f^*(s)\sum_{k=1}^{\infty}(g^*(\theta_3)f^*(s)^{k-1}+e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(1-g^*(\theta_3)f^*(s)\sum_{k=1}^{\infty}(g^*(\theta_3)f^*(s)^{k-1}$$

$$l^*(s)=(1-g^*(\theta_1)f^*(s)(1-g^*(\theta_1)f^*(s)^{-1}-\frac{e^{(\theta_2-\theta_1)\tau_1}f^*(s)}{(1-g^*(\theta_2)f^*(s))}+\frac{e^{(\theta_2-\theta_1)\tau_1}(1-g^*(\theta_2)f^*(s)}{(1-g^*(\theta_2)f^*(s))}-\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}f^*(s)}{(1-g^*(\theta_3)f^*(s))}+\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(1-g^*(\theta_3)f^*(s)}{(1-g^*(\theta_3)f^*(s))}$$

(3.7)

$f^*(s)=\dfrac{c}{c+s}$ in equation (3.7) we get,

$$l^*(s)=\frac{c(1-g^*(\theta_1)}{c+s-cg^*(\theta_1)}-\frac{ce^{(\theta_2-\theta_1)\tau_1}}{c+s-cg^*(\theta_2)}+\frac{e^{(\theta_2-\theta_1)\tau_1}c(1-g^*(\theta_2)}{c+s-cg^*(\theta_2)}-\frac{ce^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c+s-cg^*(\theta_3)}+\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(1-g^*(\theta_3)}{c+s-cg^*(\theta_3)}$$

$$E(T)=\left[\frac{-dl^*(s)}{ds}\right]_{s=0}$$

$$=\frac{1}{c(1-g^*(\theta_1)}-\frac{e^{(\theta_2-\theta_1)\tau_1}}{c(1-g^*(\theta_2)^2}+\frac{e^{(\theta_2-\theta_1)\tau_1}}{c(1-g^*(\theta_2))^2}-\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c(1-g^*(\theta_3))^2}+\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c(1-g^*(\theta_3))} \quad (3.8)$$

g(.) ~ exp(α)

$$=\frac{1}{c(1-\frac{\alpha}{\alpha+\theta_1})}-\frac{e^{(\theta_2-\theta_1)\tau_1}}{c(1-\frac{\alpha}{\alpha+\theta_2})^2}+\frac{e^{(\theta_2-\theta_1)\tau_1}}{c(1-\frac{\alpha}{\alpha+\theta_2})^2}-\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c(1-\frac{\alpha}{\alpha+\theta_3})^2}+\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c(1-\frac{\alpha}{\alpha+\theta_3})}$$

$$E(T)=\frac{\alpha+\theta_1}{c\theta_1}-\frac{e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_2)^2}{c{\theta_2}^2}+\frac{e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_2)^2}{c\theta_2}-\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^2}{c{\theta_3}^2}+\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)}{c\theta_3}$$

$$=\frac{\alpha+\theta_1}{c\theta_1}+\frac{e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_2)}{c\theta_2}\left(1-\frac{\alpha+\theta_2}{c\theta_2}\right)+\frac{e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^2}{c\theta_3}\left(1-\frac{\alpha+\theta_3}{c\theta_3}\right) \quad (3.9)$$

$$E(T^2)=\frac{d^2}{ds^2}l^*(s)\Big/_{s=0}$$

$$=\frac{d}{ds}\left[\frac{d}{ds}l^*(s)\right]$$

$$\frac{d^2}{ds^2}(l^*(s))_{s=0}=\frac{2c(1-g^*(\theta_1))}{c^4(1-g^*(\theta_1))^4}-\frac{2ce^{(\theta_2-\theta_1)\tau_1}}{c^4(1-g^*(\theta_2))^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}c(1-g^*(\theta_2))}{c^4(1-g^*(\theta_2))^4}-\frac{2ce^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c^4(1-g^*(\theta_3))^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}c(1-g^*(\theta_3))}{c^4(1-g^*(\theta_3))^4}$$

$$=\frac{2}{c^3(1-g^*(\theta_1))^3}-\frac{2ce^{(\theta_2-\theta_1)\tau_1}}{c^3(1-g^*(\theta_2))^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}}{c^3(1-g^*(\theta_2))^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c^3(1-g^*(\theta_3))^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c^3(1-g^*(\theta_3))^3}$$

$$=\frac{2}{c^3\left[1-\frac{\alpha}{\alpha+\theta_1}\right]^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}}{c^3\left[1-\frac{\alpha}{\alpha+\theta_2}\right]^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}}{c^3\left[1-\frac{\alpha}{\alpha+\theta_2}\right]^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c^3\left[1-\frac{\alpha}{\alpha+\theta_3}\right]^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}}{c^3\left[1-\frac{\alpha}{\alpha+\theta_3}\right]^3}$$

$$E(T^2)=\frac{2(\alpha+\theta_1)^3}{c^3(\theta_1)^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_1)^4}{c^3(\theta_1)^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_2)^3}{c^3(\theta_2)^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^4}{c^3(\theta_3)^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^3}{c^3(\theta_3)^3} \quad (3.10)$$

$$V(T)=\frac{2(\alpha+\theta_1)^3}{c^3(\theta_1)^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_1)^4}{c^3(\theta_1)^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_2)^3}{c^3(\theta_2)^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^4}{c^3(\theta_3)^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^3}{c^3(\theta_3)^3}$$

$$=\frac{2(\alpha+\theta_1)^3}{c^3(\theta_1)^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_1)^4}{c^3(\theta_1)^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_2)^3}{c^3(\theta_2)^3}-\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^4}{c^3\theta^4}+\frac{2e^{(\theta_2-\theta_1)\tau_1}e^{(\theta_3-\theta_2)\tau_2}(\alpha+\theta_3)^3}{c^3(\theta_3)^3}-\frac{(\alpha+\theta_1)^2}{c^2\theta_1^2}-\frac{2e^{(\theta_2-\theta_1)\tau_1}(\alpha+\theta_1)(c\theta^2-\alpha-\theta_2)^2}{c^4(\theta_2)^4} \quad (3.11)$$

Mathematical Illustration

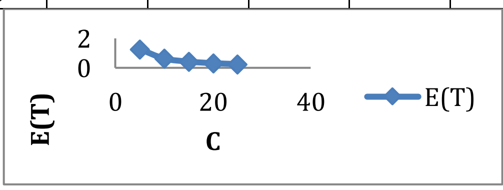
The above equations can be illustrated using a set of values and its respective graphical representations are presented as follows:

The following parameters are taken into consideration. $\theta_1, \theta_{2,}\theta_3, \tau_1, \tau_2$ and a Constant C.

Case 1 a:Here,

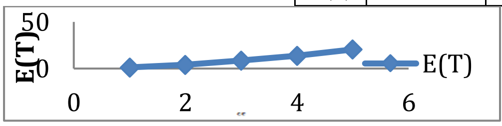
$\theta_1, \theta_{2,}\theta_3, \tau_1, \tau_2$ *is constant and C values are increases the inter arrival times between successive shocks* $E(T)$ *decreases for fig* (1) *and* $V(T)$ *is also decreases it shown in fig* (2)

C	5	10	15	20	25
E(T)	1.271	0.635	0.423	0.317	0.254



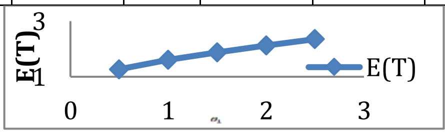
Case 1 b: For the values of C increases V(T) decreases its shown in Table 2.

C	1	2	3	4	5
V(T)	60043.706	7093.618	1979.784	783.741	374.917



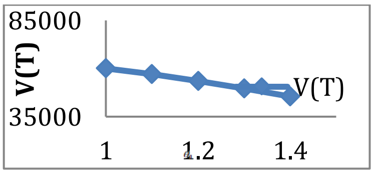
Case 2 a:$C, \theta_2, \theta_3, \tau_1, \tau_2$ *is constant and* θ_1 *increases we get* $E(T)$ *is increases for fig* (3) *but* $V(T)$ *was decreases it shown in fig*(4)

θ_1	0.5	1	1.5	2	2.5
E(T)	1.271	1.6	1.878	2.123	2.351



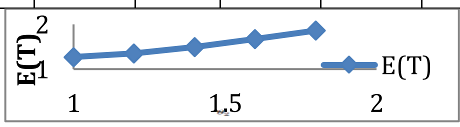
θ_1	1	1.1	1.2	1.3	1.4
V(T)	60043.7062	57229.751	53600.982	49598.7087	45466.4203

Case 2 b: If θ_1 increases Variance of T decreases are given in Table 4.



Case 3 a :$C, \theta_1, \theta_3, \tau_1, \tau_2$ *is constant and* θ_2 *increases then* $E(T)$ *increases in fig* (5) and variance of T decreases in fig (6)

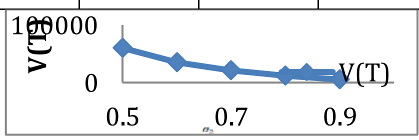
θ_2	1	1.2	1.4	1.6	1.8
E(T)	1.271	1.35	1.497	1.674	1.861



Case 3 b :*The curve was increasing direction,* $V(T)$ *in fig*(6)*the curve declines.*

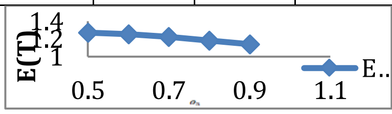
Variance of T for the varying values of θ_2 are given in Table 6.

θ_2	0.5	0.6	0.7	0.8	0.9
V(T)	60043.706	34999.952	20813.964	11678.061	5232.62



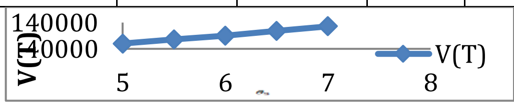
Case 4 a: $C, \theta_1, \theta_2, \tau_1, \tau_2$ *is constant and* θ_3 *values are increases* $E(T)$ was decreases *and fig*(7) *and* $V(T)$*the curve increases in fig*(8).

θ_3	0.5	0.6	0.7	0.8	0.9
E(T)	1.271	1.253	1.221	1.182	1.141



Case 4 b : Variance of T for the varying values of θ_3 are given in Table 8

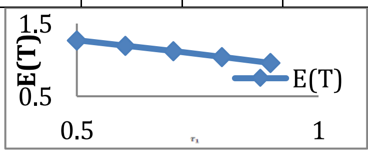
θ_3	5	5.5	6	6.5	7
V(T)	60043.706	74886.145	90756.7348	107647	125557.059



Case 5 a :

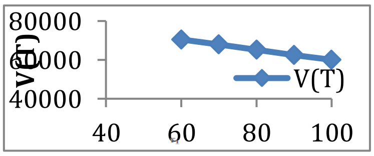
$C, \theta_1, \theta_2, \theta_3, \tau_2$ *is constant and* τ_1 *values are increases* $E(T)$ *decreases for fig* (9) *and* $V(T)$ *is also decreasing in direction in fig*(10)

τ_1	0.5	0.6	0.7	0.8	0.9
E(T)	1.271	1.198	1.121	1.042	0.959



Case 5 b : Variance of T for the varying values of τ_1 are given in Table 10.

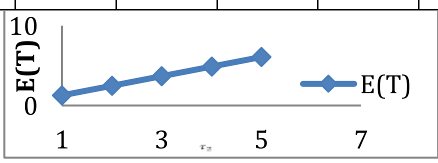
τ_1	60	70	80	90	100
V(T)	70505.740	67811.504	65169.753	62580.487	60043.7062



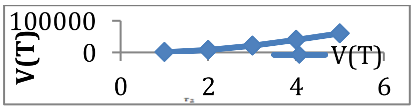
Case 6 a:

$C, \theta_1, \theta_2, \theta_3, \tau_1$ *is constant and* τ_2 *increases we get* $E(T)$ *increases for fig* (11) , *and* $V(T)$ *the curve increases in fig*(12) .

τ_2	1	2	3	4	5
E(T)	1.271	2.471	3.671	4.871	6.071



Case 6 b:Variance of T for the varying values of τ_2 are given in Table 12.

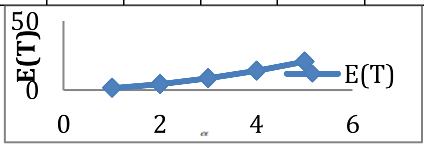


τ_2	1	2	3	4	5
V(T)	630.832	7548.660	20821	38851.538	60043.7062

Case 7 a:

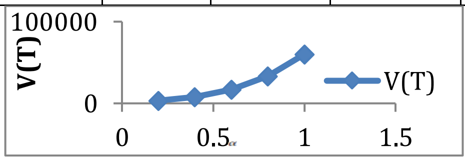
When α increases we get E(T) increases in fig(13) and V(T) is also increases in fig (!4).

α	1	2	3	4	5
E(T)	1.271	4.271	8.471	13.871	20.471



Case 7 b: for the varying value of α

α	0.2	0.4	0.6	0.8	1
V(T)	3348.6960	8004.787	17120.521	33287.3273	60043.706



CONCLUSION

Manpower planning has made use of the shock model and the cumulative damage process. Frequent requirements are not recommended from a cost aspect. Consider an organisation where the exit of manpower occurs at the time of a policy announcement and, despite the loss of manpower, the organisation continues to operate successfully with the existing manpower available. Subsequent policy announcements at random time epochs result in a cumulative loss of manpower. When the cumulative loss of manpower exceeds a certain threshold, the organisation reaches a breaking point, requiring immediate recruitment. The threshold level is the maximum allowable loss of manpower beyond which the organisation reaches a breaking point. Because the existence of a competitive market environment causes an organisation to have varying work loads at different stages, it is reasonable to assume that the threshold is exponential. Having the approach suggested in section 3, one can find the expected time to recruitment and its variance when there is a shock model with multiple change points having exponential threshold.

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