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Illation for the Groups SL(2,11^2) and SL(2,13^2)

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Abstract
The Artin indicator for the groupsSL(2,11^2) and SL(2,13^2) compute for each group in this work from the character table of rational representations and the induced characters table.

Keywords: Artin indicator, rationalcharacter table, induced characters table.

1. Introduction

Searchers in [1] define the representation of the group, the group of all matrices of determinant 1 is $\mathcal{SL}(n,\mathrm{F})$, [2] and [3].The Artinindicator for the groups *SL(2,11^2)* and *SL(2,13^2)*obtained in this work.

2. The Fundamentals

Theorem2.1:[2] $\left|\ \mathcal{SL}(2,q^n)\right| = q^n\,(q^{2n}-1)$.

Definition 2.4: [4]Let H be a cyclic subgroup of a group G, and ϕ be a class function of H. Then

$$\phi\uparrow^{G}(g)=\frac{\left|C_G(g)\right|}{\left|C_H(g)\right|}\sum_{i=1}^{m}\phi(x_i)$$

The character induced from the principal character of cyclic subgroups of G is Artin character.

Definition 2.5: [5]The character induced from the principal character of a cyclic subgroups of G is called Artin character.

Definition 2.6: [5]Let G be a finite group and let χ be any rational valued character on G. The smallest positive number n such that,

$$n\chi = \sum_c a_c \phi_c$$

where $a_c \in Z$ and ϕ_c is Artin character, is called the Artin exponent of G and denoted by A(G).

3. The Results

Authors in [4] and [7-8] study the character table of rational representations for the group $\mathcal{SL}(2,p)$ we apply that idea and compute the character table of rational representations for the groups SL(2,11^2) and SL(2,13^2). Also we apply the same idea in [4] and [6] to compute the Artinindicator for the same groups.

3.1. The results for the group $\mathcal{SL}(2,11^2)$

The character table of rational representations for the group $\mathcal{SL}(2,11^2)$ is

C_g	1	z	$c=d$	$zc=zd$	a	a^2	a^3	a^4	a^5	a^6
$\lvert C_g \rvert$	1	1	7320	7320	14762	14762	14762	14762	14762	14762
$\lvert C_G(g) \rvert$	1771440	1771440	242	242	120	120	120	120	120	120
1_G	1	1	1	1	1	1	1	1	1	1
Ψ	121	121	0	0	1	1	1	1	1	1
χ_1	3904	-3904	32	-32	0	0	0	-128	0	0
χ_2	976	976	8	8	0	-16	0	16	0	32
χ_3	1952	-1952	16	-16	0	0	0	64	0	128
χ_4	488	488	4	4	-4	4	8	0	16	-8
χ_5	976	-976	8	-8	0	0	0	16	-16	0
χ_6	488	488	4	4	0	8	0	-16	0	-16
χ_8	488	488	4	4	4	32	-8	0	-16	-16
χ_{10}	244	244	2	2	0	4	0	-4	-8	-8
χ_{12}	244	244	2	2	2	-2	-4	-4	-8	-8
χ_{15}	488	-488	4	-4	0	0	-8	-16	-16	16
χ_{20}	122	122	1	1	1	-1	-2	-2	2	2
χ_{24}	244	244	2	2	-2	-4	-8	8	8	8
χ_{30}	122	122	1	1	0	-2	2	2	2	2
χ_{40}	122	122	1	1	-1	-2	2	2	2	2
θ_1	7200	-7200	-60	60	0	0	0	0	0	0
θ_2	3600	3600	-30	-30	0	0	0	0	0	0
ξ	122	122	1	1	-2	2	-2	2	-2	2
η	240	-240	-1	1	0	0	0	0	0	0

C_g	a^8	a^{10}	a^{12}	a^{15}	a^{20}	a^{24}	a^{30}	a^{40}	b^1	b^2
$\lvert C_g \rvert$	14762	14762	14762	14762	14762	14762	14762	14762	14520	14520
$\lvert C_G(g) \rvert$	120	120	120	120	120	120	120	120	122	122
1_G	1	1	1	1	1	1	1	1	1	1
Ψ	1	1	1	1	1	1	1	1	-1	-1
χ_1	128	0	256	0	512	-256	0	-512	0	0
χ_2	0	64	-32	0	64	-64	-128	-128	0	0
χ_3	-64	0	-128	-128	-256	-256	-256	256	0	0
χ_4	0	-16	-16	-32	-32	-32	32	32	0	0
χ_5	-32	-32	-64	-64	-64	-64	64	64	0	0
χ_6	-16	-32	-32	32	32	32	32	32	0	0
χ_8	-32	-32	32	32	32	32	32	32	0	0
χ_{10}	-8	8	8	8	8	8	8	8	0	0
χ_{12}	8	8	8	8	8	8	8	8	0	0

χ_{15}	16	16	16	16	16	16	16	16	0	0
χ_{20}	2	2	2	2	2	2	2	2	0	0
χ_{24}	8	8	8	8	8	8	8	8	0	0
χ_{30}	2	2	2	2	2	2	2	2	0	0
χ_{40}	2	2	2	2	2	2	2	2	0	0
θ_1	0	0	0	0	0	0	0	0	-60	60
θ_2	0	0	0	0	0	0	0	0	30	60
ξ	2	2	2	-2	2	2	2	2	0	0
η	0	0	0	0	0	0	0	0	4	-4

The Artin character table for the group $\mathcal{SL}(2,11^2)$ is

C_g	**1**	***z***	***c=d***	***zc=zd***	***a***	a^2	a^3	a^4	a^5	a^6
$\lvert C_g \rvert$	**1**	**1**	**7320**	**7320**	**14762**	**14762**	**14762**	**14762**	**14762**	**14762**
$\lvert C_G(g) \rvert$	**1771440**	**1771440**	**242**	**242**	**120**	**120**	**120**	**120**	**120**	**120**
Φ_1	1771440	0	0	0	0	0	0	0	0	0
Φ_2	885720	885720	0	0	0	0	0	0	0	0
Φ_3	14640	0	2	0	0	0	0	0	0	0
Φ_4	14640	7320	2	2	0	0	0	0	0	0
Φ_5	14762	29524	0	0	2	0	0	0	0	0
Φ_6	7320	0	0	0	0	4	0	0	0	0
Φ_7	4880	9760	0	0	0	0	6	0	0	0
Φ_8	3660	0	0	0	0	0	0	8	0	0
Φ_9	2928	5856	0	0	0	0	0	0	10	0
Φ_{10}	2440	0	0	0	0	0	0	0	0	12
Φ_{11}	1830	0	0	0	0	0	0	0	0	0
Φ_{12}	1464	0	0	0	0	0	0	0	0	0
Φ_{13}	1220	0	0	0	0	0	0	0	0	0
Φ_{14}	976	1952	0	0	0	0	0	0	0	0
Φ_{15}	732	0	0	0	0	0	0	0	0	0
Φ_{16}	610	0	0	0	0	0	0	0	0	0
Φ_{17}	488	0	0	0	0	0	0	0	0	0
Φ_{18}	366	0	0	0	0	0	0	0	0	0
Φ_{19}	14520	29040	0	0	0	0	0	0	0	0
Φ_{20}	7320	0	0	0	0	0	0	0	0	0

C_g	a^8	a^{10}	a^{12}	a^{15}	a^{20}	a^{24}	a^{30}	a^{40}	b^1	b^2
$\lvert C_g \rvert$	**14762**	**14762**	**14762**	**14762**	**14762**	**14762**	**14762**	**14762**	**14520**	**14520**
$\lvert C_G(g) \rvert$	**120**	**120**	**120**	**120**	**120**	**120**	**120**	**120**	**122**	**122**
Φ_1	0	0	0	0	0	0	0	0	0	0
Φ_2	0	0	0	0	0	0	0	0	0	0
Φ_3	0	0	0	0	0	0	0	0	0	0
Φ_4	0	0	0	0	0	0	0	0	0	0
Φ_5	0	0	0	0	0	0	0	0	0	0
Φ_6	0	0	0	0	0	0	0	0	0	0
Φ_7	0	0	0	0	0	0	0	0	0	0
Φ_8	0	0	0	0	0	0	0	0	0	0
Φ_9	0	0	0	0	0	0	0	0	0	0
Φ_{10}	0	0	0	0	0	0	0	0	0	0
Φ_{11}	16	0	0	0	0	0	0	0	0	0
Φ_{12}	0	20	0	0	0	0	0	0	0	0
Φ_{13}	0	0	24	0	0	0	0	0	0	0

Φ_{14}	0	0	0	30	0	0	0	0	0	0
Φ_{15}	0	0	0	0	40	0	0	0	0	0
Φ_{16}	0	0	0	0	0	48	0	0	0	0
Φ_{17}	0	0	0	0	0	0	60	0	0	0
Φ_{18}	0	0	0	0	0	0	0	80	0	0
Φ_{19}	0	0	0	0	0	0	0	0	2	0
Φ_{20}	0	0	0	0	0	0	0	0	0	2

Hence, we written the rational valued characters in the first tables as a linear combination of induced characters in the second table

$$1=\frac{1}{2}\Phi_{20}+\frac{1}{2}\Phi_{19}+\frac{1}{80}\Phi_{18}+\frac{1}{60}\Phi_{17}+\frac{1}{48}\Phi_{16}+\frac{1}{40}\Phi_{15}+\frac{1}{30}\Phi_{14}+\frac{1}{24}\Phi_{13}+\frac{1}{20}\Phi_{12}+\frac{1}{16}\Phi_{11}+\frac{1}{12}\Phi_{10}+\frac{1}{10}\Phi_9+\frac{1}{8}\Phi_8+\frac{1}{6}\Phi_7+\frac{1}{4}\Phi_6+\frac{1}{2}\Phi_5+\frac{1}{2}\Phi_4-0.039762377877132\Phi_2+0.003199492126933\Phi_1$$

$$\Psi=-\frac{1}{2}\Phi_{20}-\frac{1}{2}\Phi_{19}+\frac{1}{80}\Phi_{18}+\frac{1}{60}\Phi_{17}+\frac{1}{48}\Phi_{16}+\frac{1}{40}\Phi_{15}+\frac{1}{30}\Phi_{14}+\frac{1}{24}\Phi_{13}+\frac{1}{20}\Phi_{12}+\frac{1}{16}\Phi_{11}+\frac{1}{12}\Phi_{10}+\frac{1}{10}\Phi_9+\frac{1}{8}\Phi_8+\frac{1}{6}\Phi_7+\frac{1}{4}\Phi_6+\frac{1}{2}\Phi_5+\frac{1}{2}\Phi_4-0.002707778229388\Phi_2+0.001470108122958\Phi_1$$

$$\chi_1=-6.4\Phi_{18}-5.33333333\Phi_{16}+12.8\Phi_{15}+10.6666667\Phi_{13}+8\Phi_{11}-16\Phi_8-16\Phi_4+32\Phi_3+0.13663912\Phi_2-0.11471093\Phi_1$$

$$\chi_2=-1.6\Phi_{18}-2.13333\Phi_{17}-1.33333\Phi_{16}+1.6\Phi_{15}-1.33333\Phi_{13}+3.2\Phi_{12}+2.66667\Phi_{10}+2\Phi_8-4\Phi_6+4\Phi_4-0.03196\Phi_2-0.02629\Phi_1$$

$$\chi_3=3.2\Phi_{18}-4.26667\Phi_{17}-5.33333\Phi_{16}-6.4\Phi_{15}-4.26667\Phi_{14}-5.33333\Phi_{13}-4\Phi_{11}+10.66667\Phi_{10}+8\Phi_8-8\Phi_4+16\Phi_3-0.00535\Phi_2-0.08108\Phi_1$$

$$\chi_4=0.4\Phi_{18}+0.53333\Phi_{17}-0.66667\Phi_{16}-0.8\Phi_{15}-1.06667\Phi_{14}-0.66667\Phi_{13}-0.8\Phi_{12}-0.66667\Phi_{10}+1.6\Phi_9+1.33333\Phi_7+\Phi_6-2\Phi_5+2\Phi_4+0.02777\Phi_2-0.007099\Phi_1$$

$$\chi_5=0.8\Phi_{18}+1.06667\Phi_{17}-1.33333\Phi_{16}-1.6\Phi_{15}-2.13333\Phi_{14}-2.66667\Phi_{13}-1.6\Phi_{12}-2\Phi_{11}-1.6\Phi_9+2\Phi_8-4\Phi_4+8\Phi_3+0.04724\Phi_2-0.02693\Phi_1$$

$$\chi_6=0.4\Phi_{18}+0.53333\Phi_{17}+0.66667\Phi_{16}+0.8\Phi_{15}+1.06667\Phi_{14}-1.33333\Phi_{13}-1.6\Phi_{12}-\Phi_{11}-1.33333\Phi_{10}-2\Phi_8+2\Phi_6+2\Phi_4-0.01833\Phi_2-0.01665\Phi_1$$

$$\chi_8=0.4\Phi_{18}+0.53333\Phi_{17}+0.66667\Phi_{16}+0.8\Phi_{15}+1.06667\Phi_{14}+1.33333\Phi_{13}-1.6\Phi_{12}-2\Phi_{11}-1.33333\Phi_{10}+1.6\Phi_9-1.33333\Phi_7+8\Phi_6+2\Phi_5+2\Phi_4-0.08088\Phi_2-0.062021\Phi_1$$

$$\chi_{10}=0.1\Phi_{18}+0.13333\Phi_{17}+0.16667\Phi_{16}+0.2\Phi_{15}+0.26667\Phi_{14}+0.33333\Phi_{13}+0.4\Phi_{12}-0.5\Phi_{11}-0.66667\Phi_{10}-0.8\Phi_9-0.5\Phi_8+\Phi_6+2\Phi_5+\Phi_4-0.00327\Phi_2-0.00937\Phi_1$$

$$\chi_{12}=0.1\Phi_{18}+0.13333\Phi_{17}+0.16667\Phi_{16}+0.2\Phi_{15}+0.26667\Phi_{14}+0.33333\Phi_{13}+0.4\Phi_{12}+0.5\Phi_{11}-0.66667\Phi_{10}-0.8\Phi_9-0.5\Phi_8-0.66667\Phi_7-0.5\Phi_6+\Phi_5+\Phi_4-0.09275\Phi_2-0.010705\Phi_1$$

$$\chi_{15}=0.2\Phi_{18}+0.26667\Phi_{17}+0.33333\Phi_{16}+0.4\Phi_{15}+0.53333\Phi_{14}+0.66667\Phi_{13}+0.8\Phi_{12}+\Phi_{11}+1.33333\Phi_{10}-1.6\Phi_9-2\Phi_8-\Phi_7-2\Phi_4+4\Phi_3-0.000514\Phi_2-0.0114004\Phi_1$$

$$\chi_{20}=0.025\Phi_{18}+0.03333\Phi_{17}+0.04167\Phi_{16}+0.05\Phi_{15}+0.06667\Phi_{14}+0.08333\Phi_{13}+0.1\Phi_{12}+0.125\Phi_{11}+0.16667\Phi_{10}+0.2\Phi_9-0.25\Phi_8-0.33333\Phi_7-0.25\Phi_6-0.5\Phi_5+0.5\Phi_4-0.018457\Phi_2-0.00668\Phi_1$$

$\chi_{24} =$
$0.1\Phi_{18} + 0.3333\Phi_{17} + 0.13333\Phi_{16} + 0.2\Phi_{15} + 0.26667\Phi_{14} + 0.33333\Phi_{13} + 0.4\Phi_{12} + 0.5\Phi_{11} +$
$0.66667\Phi_{10} + 0.8\Phi_9 + \Phi_8 - 1.33333\Phi_7 - \Phi_6 - \Phi_5 + \Phi_4 + 0.03416\Phi_2 + 0.00228\Phi_1$
$\chi_{30} = 0.025\Phi_{18} + 0.03333\Phi_{17} + 0.04167\Phi_{16} + 0.05\Phi_{15} + 0.06667\Phi_{14} + 0.08333\Phi_{13} +$
$0.1\Phi_{12} + 0.125\Phi_{11} + 0.16667\Phi_{10} + 0.2\Phi_9 + 0.25\Phi_8 + 0.33333\Phi_7 - 0.5\Phi_6 + 0.5\Phi_4 -$
$0.009137\Phi_2 - 0.004612\Phi_1$
$\chi_{40} = 0.025\Phi_{18} + 0.03333\Phi_{17} + 0.04167\Phi_{16} + 0.05\Phi_{15} + 0.06667\Phi_{14} + 0.08333\Phi_{13} +$
$0.1\Phi_{12} + 0.125\Phi_{11} + 0.16667\Phi_{10} + 0.2\Phi_9 + 0.25\Phi_8 + 0.33333\Phi_7 - 0.5\Phi_6 - 0.5\Phi_5 + 0.5\Phi_4 -$
$0.00753\Phi_2 - 0.0001807\Phi_1$
$\theta_1 = 30\Phi_{20} - 30\Phi_{19} + 30\Phi_4 - 60\Phi_3 + 0.72754\Phi_2 + 0.37393\Phi_1$
$\theta_2 = 30\Phi_{20} + 15\Phi_{19} - 15\Phi_4 - 0.36377\,\Phi_2 - 0.120918\Phi_1$
$\zeta = 0.025\Phi_{18} + 0.03333\Phi_{17} + 0.04167\Phi_{16} 0.05\Phi_{15} - 0.06667\Phi_{14} + 0.08333\Phi_{13} +$
$0.1\Phi_{12} + 0.125\Phi_{11} + 0.16667\Phi_{10} - 0.2\Phi_9 + 0.25\Phi_8 - 0.33333\Phi_7 + 0.5\Phi_6 - \Phi_5 + 0.5\Phi_4 +$
$0.04275\Phi_2 - 0.00591\Phi_1$
$\eta = -2\Phi_{20} + 2\Phi_{19} + 0.5\Phi_4 - \Phi_3 - 0.06998\Phi_2 - 0.00386\Phi_1$

Therefore $\mathcal{A}(\mathcal{SL}(2, 11^2)) = 1771440\chi_1$

3.2. The illation for the group $\mathcal{SL}(2, 13^2)$

The character table of rational representations for the group $\mathcal{SL}(2, 13^2)$ is

C_g	1	z	c=d	zc=zd	a	a^2	a^3	a^4	a^6	a^7	a^8	a^{12}	a^{14}
$\lvert C_g \rvert$	1	1	14280	14280	28730	28730	28730	28730	28730	28730	28730	28730	28730
$\lvert C_G(g) \rvert$	4826640	4826640	338	338	168	168	168	168	168	168	168	168	168
1_G	1	1	1	1	1	1	1	1	1	1	1	1	1
Ψ	169	169	0	0	1	1	1	1	1	1	1	1	1
χ_1	8160	-8160	48	48	0	0	0	0	0	0	0	0	0
χ_2	2040	2040	12	12	0	0	0	24	48	0	72	-48	0
χ_3	4080	-4080	24	24	0	0	96	48	0	0	-48	-144	0
χ_4	1020	1020	6	6	-6	6	12	18	-12	36	-18	0	-36
χ_6	1020	1020	6	6	0	12	0	-12	-18	0	0	-72	-72
χ_7	1360	-1360	8	-8	0	0	0	32	0	-32	-32	-64	-64
χ_8	1020	1020	6	6	6	18	-12	-18	0	-36	-36	-72	-72
χ_{12}	510	510	3	3	3	-3	-9	0	-18	-18	-18	18	18
χ_{14}	340	340	2	2	0	4	0	-4	-8	-8	-8	8	8
χ_{21}	680	-680	4	4	0	0	-8	-16	16	16	16	16	16
χ_{24}	510	510	3	3	-3	0	-18	-18	-18	18	18	18	18
χ_{28}	170	170	1	1	1	-1	-2	-2	2	2	2	2	2
χ_{42}	170	170	1	1	0	-2	2	2	2	2	2	2	2
χ_{56}	170	170	1	1	-1	-2	2	2	2	2	2	2	2
θ_1	10752	-10752	-64	64	0	0	0	0	0	0	0	0	0
θ_2	5376	5376	-32	-32	0	0	0	0	0	0	0	0	0
θ_5	2688	-2688	-16	16	0	0	0	0	0	0	0	0	0
θ_{10}	1344	1344	-8	-8	0	0	0	0	0	0	0	0	0
θ_{17}	672	-672	-4	4	0	0	0	0	0	0	0	0	0
θ_{34}	336	336	-2	-2	0	0	0	0	0	0	0	0	0
ξ	170	170	-1	-1	-2	2	-2	2	2	-2	2	2	2
η	336	-336	1	-1	0	0	0	0	0	0	0	0	0

C_g	a^{21}	a^{24}	a^{28}	a^{42}	a^{56}	b	b^2	b^5	b^{10}	b^{17}	b^{34}

$\|C_g\|$	28730	28730	28730	28730	28730	28392	28392	28392	28392	28392	28392
$\|C_G(g)\|$	168	168	168	168	168	170	170	170	170	170	170
1_G	1	1	1	1	1	1	1	1	1	1	1
Ψ	1	1	1	1	1	-1	-1	-1	-1	-1	-1
χ_1	0	-384	1152	0	-1152	0	0	0	0	0	0
χ_2	0	0	-144	-288	-288	0	0	0	0	0	0
χ_3	-288	-576	-576	576	576	0	0	0	0	0	0
χ_4	-72	-72	-72	72	72	0	0	0	0	0	0
χ_6	72	-72	72	72	72	0	0	0	0	0	0
χ_7	64	64	64	64	64	0	0	0	0	0	0
χ_8	72	72	72	72	72	0	0	0	0	0	0
χ_{12}	18	18	18	18	18	0	0	0	0	0	0
χ_{14}	8	8	8	8	8	0	0	0	0	0	0
χ_{21}	16	16	16	16	16	0	0	0	0	0	0
χ_{24}	18	18	18	18	18	0	0	0	0	0	0
χ_{28}	2	2	2	2	2	0	0	0	0	0	0
χ_{42}	2	2	2	2	2	0	0	0	0	0	0
χ_{56}	2	2	2	2	2	0	0	0	0	0	0
θ_1	0	0	0	0	0	64	-64	-256	256	-1024	1024
θ_2	0	0	0	0	0	-32	0	128	0	512	1024
θ_5	0	0	0	0	0	-16	16	-128	128	-256	-256
θ_{10}	0	0	0	0	0	-8	0	64	128	-128	-128
θ_{17}	0	0	0	0	0	-4	4	8	-16	-16	-16
θ_{34}	0	0	0	0	0	2	4	-8	-8	-8	-8
ξ	-2	2	2	2	2	0	0	0	0	0	0
η	0	0	0	0	0	4	-4	4	-4	4	-4

The Artin character table for the group $\mathcal{SL}(2,3^6)$ is

C_g	1	z	c=d	zc=zd	a	a^2	a^3	a^4	a^6	a^7	a^8	a^{12}
$\|C_g\|$	1	1	14280	14280	28730	28730	28730	28730	28730	28730	28730	28730
$\|C_G(g)\|$	4826640	4826640	338	338	168	168	168	168	168	168	168	168
Φ_1	4826640	0	0	0	0	0	0	0	0	0	0	0
Φ_2	2413320	2413320	0	0	0	0	0	0	0	0	0	0
Φ_3	28560	0	2	0	0	0	0	0	0	0	0	0
Φ_4	28560	14280	2	2	0	0	0	0	0	0	0	0
Φ_5	28730	57460	0	0	2	0	0	0	0	0	0	0
Φ_6	14280	0	0	0	0	4	0	0	0	0	0	0
Φ_7	9520	19040	0	0	0	0	6	0	0	0	0	0
Φ_8	7140	0	0	0	0	0	0	8	0	0	0	0
Φ_9	4760	0	0	0	6	0	0	0	12	0	0	0
Φ_{10}	4080	8160	0	0	0	0	0	0	0	14	0	0
Φ_{11}	3570	0	0	0	0	0	0	0	0	0	16	0
Φ_{12}	2380	0	0	0	0	0	0	0	0	0	0	24
Φ_{13}	2040	0	0	0	0	0	0	0	0	0	0	0
Φ_{14}	1360	2720	0	0	0	0	0	0	0	0	0	0
Φ_{15}	1190	0	0	0	0	0	0	0	0	0	0	0
Φ_{16}	1020	0	0	0	0	0	0	0	0	0	0	0
Φ_{17}	680	0	0	0	0	0	0	0	0	0	0	0
Φ_{18}	510	0	0	0	0	0	0	0	0	0	0	0
Φ_{19}	28392	56784	0	0	0	0	0	0	0	0	0	0
Φ_{20}	14280	0	0	0	0	0	0	0	0	0	0	0
Φ_{21}	5712	11424	0	0	0	0	0	0	0	0	0	0
Φ_{22}	2856	0	0	0	0	0	0	0	0	0	0	0

Φ_{23}	1680	3360	0	0	0	0	0	0	0	0	0	0
Φ_{24}	1840	0	0	0	0	0	0	0	0	0	0	0

Hence, we written the rational valued characters in the first tables as a linear combination of induced characters in the second table

1=0.01471 $\Phi_{24} + 0.02941\ \Phi_{23}$+0.05 Φ_{22}+0.1 $\Phi_{21} + 0.5\Phi_{20} + 0.5\Phi_{19} + 0.00893\Phi_{18} + 0.01191\Phi_{17} + 0.01786\Phi_{16} + 0.02083\ \Phi_{15}$+0.02381 Φ_{14} +0.03571 $\Phi_{13} + 0.041667\ \Phi_{12} + 0.625\Phi_{11} + 0{,}07143\Phi_{10} + 0.08333\Phi_9 + 0.125\Phi_8 + 0.16667\Phi_7 + 0.25\Phi_6 + 0.5\Phi_5 + 0.5\Phi_4 - 0.02873\Phi_2 - 0.01209\Phi_1$

Ψ=− 0.01471 $\Phi_{24} - 0.02941\ \Phi_{23}$ − 0.05 Φ_{22} −0.1 $\Phi_{21} - 0.5\Phi_{20} - 0.5\Phi_{19} + 0.00893\Phi_{18} + 0.01191\Phi_{17} + 0.01786\Phi_{16} + 0.02083\ \Phi_{15}$+0.02381 Φ_{14} +0.03571 $\Phi_{13} + 0.041667\ \Phi_{12} + 0.625\Phi_{11} + 0{,}07143\Phi_{10} + 0.08333\Phi_9 + 0.125\Phi_8 + 0.16667\Phi_7 + 0.25\Phi_6 + 0.5\Phi_5 + 0.5\Phi_4 - 0.001139\Phi_2 + 0.000149\Phi_1$

$$\chi_1 = -10.28571\Phi_{18} + 20.57143\Phi_{16} - 8\Phi_{15} + 24\Phi_4 - 0.14539\Phi_2 - 0.17728\Phi_1$$

$$\chi_2 == -2.57143\Phi_{18} - 3.42857\ \Phi_{17} - 2.57143\Phi_{16} - 2\Phi_{12} + 4.5\ \Phi_{11} + 4\Phi_9 + 3\ \Phi_8 + 24\Phi_4 - 0.03466\Phi_2 - 0.044507\Phi_1$$

$$\chi_3 == -5.14286\Phi_{18} + 6.85714\ \Phi_{17} - 10.28571\Phi_{16} - 6\ \Phi_{15} - 5.14286\ \Phi_{13} - 2\Phi_{12} + 4\Phi_9 + 12\ \Phi_8 + 12\Phi_4 - 0.0726972\Phi_2 - 0.08655\Phi_1$$

$$\chi_4 == 0.64286\Phi_{18} + 0.85714\ \Phi_{17} - 1.2857\Phi_{16} - 1.5\ \Phi_{15} - 1.71429\ \Phi_{14} - 5.14286\ \Phi_{13} + 2.57143\Phi_{10} - \Phi_9 + 2.25\ \Phi_8 + 2\ \Phi_7 + 1.5\Phi_6 - 3\Phi_5 + 2\Phi_4 + 0.04537\Phi_2 - 0.02042\Phi_1$$

$$\chi_6 = 0.64286\Phi_{18} + 0.85714\ \Phi_{17} + 1.2857\Phi_{16} - 1.5\ \Phi_{15} + 1.71429\ \Phi_{14} - 2.57143\Phi_{13} - 3\Phi_{12} - 1.5\ \Phi_9 - 1.5\ \Phi_8 + 3\Phi_6 + 3\Phi_4 - 0.01926\Phi_2 - 0.02217\Phi_1$$

$$\chi_7 = 0.57143\Phi_{18} + 0.76191\ \Phi_{17} + 1.14286\Phi_{16} - 1.33333\ \Phi_{15} + 1.52381\ \Phi_{14} - 2.57143\Phi_{13} - 2.66667\Phi_{12} - 2\ \Phi_{11} - 2.28571\ \Phi_{10} + 4\Phi_8 - 4\ \Phi_4 + 8\ \Phi_3 + 0.2912\Phi_2 - 0.24779\Phi_1$$

$$\chi_8 = 0.64286\Phi_{18} + 0.85714\ \Phi_{17} + 1.2857\Phi_{16} + 1.5\ \Phi_{15} + 1.71429\ \Phi_{14} - 2.57143\Phi_{13} - 3\Phi_{12} - 2.25\ \Phi_{11} - 2.27143\ \Phi_{10} - 2.25\ \Phi_8 - 2\ \Phi_7 + 4.5\ \Phi_6 + 3\ \Phi_5 + 3\Phi_4 + 0.06723\Phi_2 - 0.036103\Phi_1$$

$$\chi_{12} = 0.160714\Phi_{18} + 0.21429\ \Phi_{17} + 0.32142\Phi_{16} + 0.375\ \Phi_{15} + 0.42857\ \Phi_{14} - 0.64286\ \Phi_{13} - 0.64286\Phi_{12} - 0.75\ \Phi_{11} - 1.125\ \Phi_{10} - 1.28571\Phi_9 - 1.5\ \Phi_7 + 0.75\ \Phi_6 + 1.5\ \Phi_5 + 1.5\ \Phi_4 - 0.03047\Phi_2 - 0.010574\Phi_1$$

$$\chi_{14} = 0.07143\Phi_{18} + 0.09524\ \Phi_{17} + 0.14286\Phi_{16} + 0.16667\ \Phi_{15} + 0.19048\ \Phi_{14} - 0.28571\ \Phi_{13} - 0.33333\Phi_{12} - 0.5\ \Phi_{11} - 0.57143\ \Phi_{10} - 0.66667\ \Phi_9 - 0.5\ \Phi_8 + \Phi_6 + \Phi_4 - 0.004059\Phi_2 - 0.006984\Phi_1$$

$$\chi_{21} = 0.14286\Phi_{18} + 0.19048\ \Phi_{17} + 0.28571\Phi_{16} + 0.33333\ \Phi_{15} + 0.38095\ \Phi_{14} + 0.57143\ \Phi_{13} + 0.66667\Phi_{12} + \Phi_{11} + 1.14286\ \Phi_{10} + 1.33333\ \Phi_9 - 2\ \Phi_8 - 1.3333\ \Phi_7 + 2\ \Phi_4 - 0.005944\Phi_2 - 0.0009988\Phi_1$$

$$\chi_{24} = 0.07143\Phi_{18} + 0.09524\ \Phi_{17} + 0.14286\Phi_{16} + 0.16667\ \Phi_{15} + 0.19048\ \Phi_{14} + 0.28571\ \Phi_{13} + 0.33333\Phi_{12} + 0.5\ \Phi_{11} + 0.57143\ \Phi_{10} - 0.66667\ \Phi_9 - 0.25\ \Phi_8 - 3\ \Phi_7 - 1.5\ \Phi_5 + 1.5\ \Phi_4 - 0.04857\Phi_2 - 0.0087756\Phi_1$$

$$\chi_{28} = 0.01786\Phi_{18} + 0.0238095\ \Phi_{17} + 0.03571\Phi_{16} + 0.04167\ \Phi_{15} + 0.04762\ \Phi_{14} + 0.07143\ \Phi_{13} + 0.08333\Phi_{12} + 0.125\ \Phi_{11} + 0.14286\ \Phi_{10} + 1.6667\ \Phi_9 - 0.25\ \Phi_8 - 0.3333\ \Phi_7 - 0.25\ \Phi_6 - 0.25\ \Phi_5 + 0.5\ \Phi_4 - 0.004413\Phi_2 - 0.00461796\Phi_1$$

$$\chi_{42} = 0.01786\Phi_{18} + 0.0238095\ \Phi_{17} + 0.03571\Phi_{16} + 0.04167\ \Phi_{15} + 0.04762\ \Phi_{14} + 0.07143\ \Phi_{13} + 0.08333\Phi_{12} + 0.125\ \Phi_{11} + 0.14286\ \Phi_{10} + 0.6667\ \Phi_9 + 0.25\ \Phi_8 + 0.3333\ \Phi_7 - 0.5\ \Phi_6 + 0.5\ \Phi_4 - 0.015434\Phi_2 - 0.000297\Phi_1$$

$$\chi_{56} = 0.01786\Phi_{18} + 0.0238095\ \Phi_{17} + 0.03571\Phi_{16} + 0.04167\ \Phi_{15} + 0.04762\ \Phi_{14} + 0.07143\ \Phi_{13} + 0.08333\Phi_{12} + 0.125\ \Phi_{11} + 0.14286\ \Phi_{10} + 0.6667\ \Phi_9 + 0.25\ \Phi_8 + 0.3333\ \Phi_7 - 0.5\ \Phi_6 + 0.5\ \Phi_4 - 0.000346\Phi_2 + 0.002622\Phi_1$$

C_g	a^{14}	a^{21}	a^{24}	a^{28}	a^{42}	a^{56}	b	b^2	b^5	b^{10}	b^{17}	b^{34}
$\lvert C_g \rvert$	28730	28730	28730	28730	28730	28730	28392	28392	28392	28392	28392	28392
$\lvert C_G(g) \rvert$	168	168	168	168	168	168	170	170	170	170	170	170
Φ_1	0	0	0	0	0	0	0	0	0	0	0	0
Φ_2	0	0	1	0	1	0	0	0	0	0	0	0
Φ_3	0	0	0	0	0	0	0	0	0	0	0	0
Φ_4	0	0	0	0	0	0	0	0	0	0	0	0
Φ_5	0	0	0	0	0	0	0	0	0	0	0	0
Φ_6	0	0	0	0	0	0	0	0	0	0	0	0
Φ_7	0	0	0	0	0	0	0	0	0	0	0	0
Φ_8	0	0	0	0	0	0	0	0	0	0	0	0
Φ_9	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{10}	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{11}	0	8	0	0	0	0	0	0	0	0	0	0
Φ_{12}	0	0	0	0	0	0	0	0	0	0	0	0
Φ_{13}	28	0	0	0	0	0	0	0	0	0	0	0
Φ_{14}	0	42	0	0	0	0	0	0	0	0	0	0
Φ_{15}	0	0	48	0	0	0	0	0	0	0	0	0
Φ_{16}	0	0	0	56	0	0	0	0	0	0	0	0
Φ_{17}	0	0	0	0	84	0	0	0	0	0	0	0
Φ_{18}	0	0	0	0	0	112	0	0	0	0	0	0
Φ_{19}	0	0	0	0	0	0	2	0	0	0	0	0
Φ_{20}	0	0	0	0	0	0	0	2	0	0	0	0
Φ_{21}	0	0	0	0	0	0	0	0	10	0	0	0
Φ_{22}	0	0	0	0	0	0	0	0	0	20	0	0
Φ_{23}	0	0	0	0	0	0	0	0	0	0	34	0
Φ_{24}	0	0	0	0	0	0	0	0	0	0	0	68

$$\theta_1 = 15.05882\ \Phi_{24} - 30.11765\ \Phi_{23} + 12.8\ \Phi_{22} - 25.6\ \Phi_{21} - 32\ \Phi_{20} + 32\ \Phi_{19} + 32\ \Phi_4 - 64\ \Phi_3 - 1.0115195\ \Phi_2 + 0.27393\ \Phi_1$$

$$\theta_2 = 15.05882 + 15.05882\ \Phi_{23} - 12.8\ \Phi_{21} - 16\ \Phi_{19} + 16\ \Phi_4 + 0.32365\ \Phi_2 + 0.004723\ \Phi_1$$

$$\theta_5 = -3.76471\ \Phi_{24} - 7.5294\ \Phi_{23} + 6.4\ \Phi_{22} + 12.8\ \Phi_{21} + 8\ \Phi_{20} - 8\ \Phi_{19} + 8\ \Phi_4 - 16\ \Phi_3 + 0.06774\ \Phi_2 + 0.0864\ \Phi_1$$

$$\theta_{10} = -1.88235 - 3.76471\ \Phi_{23} + 6.4\ \Phi_{22} + 6.4\ \Phi_{21} - 4\ \Phi_{19} - 4\ \Phi_4 + 0.093315\ \Phi_2 + 0.0340876\ \Phi_1$$

$$\theta_{17} = -0.23529\ \Phi_{24} - 0.47059\ \Phi_{23} - 0.8\ \Phi_{22} + 0.8\ \Phi_{21} + 2\ \Phi_{20} - 2\ \Phi_{19} + 2\ \Phi_4 - 4\ \Phi_3 + 0.0554\ \Phi_2 + 0.017601\ \Phi_1$$

$$\theta_{34} = -0.11765\ \Phi_{24} - 0.23529\ \Phi_{23} - 0.4\ \Phi_{22} - 0.8\ \Phi_{21} + 2\ \Phi_{20} + \Phi_{19} - \Phi_4 -$$

$0.013359\ \Phi_2 - 0.004503\ \Phi_1$
ζ=$0.01786\Phi_{18} + 0.0238095\ \Phi_{17} + 0.03571\Phi_{16} + 0.04167\ \Phi_{15} - 0.04762\ \Phi_{14} + 0.07143\ \Phi_{13} + 0.08333\Phi_{12} + 0.125\ \Phi_{11} - 0.14286\ \Phi_{10} + 0.6667\ \Phi_9 + 0.25\ \Phi_8 - 0.3333\ \Phi_7 + 0.5\ \Phi_6 - \Phi_5 - 0.5\ \Phi_4 + 0.03829\Phi_2 + 0.007538\Phi_1$
η=$-0.05882\ \Phi_{24} + 136\ \Phi_{23} - 0.2\ \Phi_{22} + 0.4\ \Phi_{21} - 2\ \Phi_{20} + 2\ \Phi_{19} - 0.5\ \Phi_4 + \Phi_3 - 0.231695\ \Phi_2 + 0.055341\ \Phi_1$
Therefore$\mathcal{A}(\mathcal{SL}(2,13^2))$=4826640$\chi_1$.

4. References

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