An Analysis of Triangular Intuitionistic Fuzzy Quadratic Programming Problem

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Abstract: In this paper, a two-stage simplex technique is developed to obtain the fuzzy optimal solution for a special kind of fuzzy quadratic programming problem. The fuzzy quadratic objective function is the product of linear factors and all decision parameters, cost coefficients, constraint coefficients and right hand side are taken to be triangular intuitionistic fuzzy numbers. A numerical example is illustrated to authenticate the proposed methodology.

Keywords: Intuitionistic triangular fuzzy numbers, fuzzy quadratic programming, two-stage simple procedure, fuzzy optimal solution.
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1.INTRODUCTION

Intuitionistic fuzzy set is an extension of general fuzzy sets which was developed by Atanossov [1]. Nagoorgani [5] proposed a new approach for solving linear programming problem in intuitionistic fuzzy environment. Sujeet Singh and Shiv Prasad Yadav [6] investigated on intuitionistic fuzzy nonlinear programming problem in modeling and optimization in

manufacturing systems. Many researchers proposed different techniques for analyzing linear and nonlinear programming problems involving intuitionistic fuzzy parameters. In this paper, a new two-stage intuitionistic fuzzy simplex procedure is developed to solve fuzzy quadratic programming problems whose quadratic objective function is assumed to be product of fuzzy linear factors and all decision parameters are taken to be triangular intuitionistic fuzzy numbers (TIFN). Furthermore, an average ranking method is introduced to identify the entering and leaving fuzzy variables. To validate the effectiveness of proposed method, a numerical example is illustrated.

2. PRELIMINARIES

We review the basic results and definitions which are applied to this study.

Definition 2.1 Fuzzy Number

A fuzzy set $\tilde{A}$ defined on the real numbers R is said to be a fuzzy number if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following characteristics:

(i) $\tilde{A}(x)$ is convex. ie., $\tilde{A}(\lambda x_1 + (1-\lambda)x_2) \geq$

$\min\left[\tilde{A}(x_1), \tilde{A}(x_2)\right], \lambda \in [0,1] \forall x_1, x_2 \in R$

(ii) $\tilde{A}$ is normal .i.e., there exists an $x \in R$ such that $\tilde{A}(x) = 1$.

(iii) $\tilde{A}$ is upper semi continuous.

(iv) Supp $(\tilde{A})$ is bounded in R .

Definition 2.2 Intuitionistic fuzzy number

An intuitionistic fuzzy number $\tilde{A}^I$ is

(i) $\tilde{A}^I$ is an intuitionistic fuzzy subset of the real line .

(ii) $\tilde{A}^I$ is normal. i.e., there is any $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0) = 1$.

(iii) $\tilde{A}^I$ is convex for the membership function and concave for the non membership function.

$$\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2)\right) \ \forall x_1, x_2 \in R, \lambda \in [0,1].$$

$$\vartheta_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \leq \max\left(\vartheta_{\tilde{A}^I}(x_1), \vartheta_{\tilde{A}^I}(x_2)\right) \ \forall x_1, x_2 \in R, \lambda \in [0,1].$$

Definition 2.3 Triangular intuitionistic fuzzy number (TIFN)

A triangular intuitionistic fuzzy number **(TIFN)** $\tilde{A}^I = \langle (a,b,c); (a',b,c') \rangle$ is an intuitionistic fuzzy set in R with the following membership and non-membership function.

$$\mu_{\tilde{A}^I}(x) = \begin{cases} \frac{x-a}{b-a} & ; a \leq x \leq b \\ \frac{c-x}{c-b} & ; b \leq x \leq c \\ 0 & ; \text{otherwise} \end{cases}$$

$$\vartheta_{\tilde{A}^I}(x) = \begin{cases} \frac{b-x}{b-a'} & ; a' \leq x \leq b \\ \frac{x-b}{c'-b} & ; b \leq x \leq c' \\ 1 & ; \text{otherwise} \end{cases}$$

where $a' \leq a \leq b \leq c \leq c'$; $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) = 1$ or $\mu_{\tilde{A}^I}(x) = \vartheta_{\tilde{A}^I}(x)$; $\forall x \in R$.

Definition 2.3 Arithmetic operations of TIFN

Let $\tilde{A}^I = \langle (a,b,c); (a',b,c') \rangle$ and

$\tilde{B}^I = \langle (d,e,f); (d',e,f') \rangle$ be any two triangular intuitionistic fuzzy numbers. Then

(i) $\widetilde{A}^I + \widetilde{B}^I = \langle (a+d, b+e, c+f); (a'+d', b+e, c'+f') \rangle$
(ii) $\widetilde{A}^I - \widetilde{B}^I = \langle (a-f, b-e, c-d); (a'-f', b-e, c'-d') \rangle$
(iii) $\widetilde{A}^I * \widetilde{B}^I = \langle (ad, be, cf); (a'd', be, c'f') \rangle$
(iv) $\frac{\widetilde{A}^I}{\widetilde{B}^I} = \left\langle \left(\frac{a}{f}, \frac{b}{e}, \frac{c}{d}\right); \left(\frac{a'}{f'}, \frac{b}{e}, \frac{c'}{d'}\right) \right\rangle$
(v) $\lambda\widetilde{A}^I = \langle (\lambda a, \lambda b, \lambda c); (\lambda a', \lambda b, \lambda c') \rangle\ ;\ \lambda > 0$
(vi) $\lambda\widetilde{A}^I = \langle (\lambda c, \lambda b, \lambda a); (\lambda c', \lambda b, \lambda a') \rangle\ ;\ \lambda < 0$

3. Proposed Average value and Ranking method for TIFN

3.1 Proposed Average Value of TIFN

Let $\widetilde{A}^I = \langle (a, b, c); (a', b, c') \rangle$ be a TIFN. Then its combined average value of both membership and non-membership function is defined by

$$Av(\widetilde{A}^I) = \frac{a+b+c+a'+c'}{5}$$

3.2 Ranking of TIFN using Average Value

Let $\widetilde{A}^I = \langle (a, b, c); (a', b, c') \rangle$ and $\widetilde{B}^I = \langle (d, e, f); (d', e, f') \rangle$ be any two triangular intuitionistic fuzzy numbers and let their average values are $Av(\widetilde{A}^I)$ and $Av(\widetilde{B}^I)$ respectively.Then

(i) $\widetilde{A}^I < \widetilde{B}^I$ if $Av(\widetilde{A}^I) < Av(\widetilde{B}^I)$
(ii) $\widetilde{A}^I > \widetilde{B}^I$ if $Av(\widetilde{A}^I) > Av(\widetilde{B}^I)$

4.1 Proposed two-stage Simple procedure for solving special kind of intuitionistic fuzzy quadratic programming problems.

Step 1: Let the mathematical formulation of special kind of Intuitionistic fuzzy quadratic programming problem (IFQPP) be

Maximize $\tilde{Z}^I = (\tilde{c}^I\tilde{x}^I + \tilde{\alpha}^I)(\tilde{d}^I\tilde{x}^I + \tilde{\beta}^I)$

subject to the constraints

$\tilde{a}_{i1}{}^I\tilde{x}^I{}_1 + \tilde{a}_{i2}{}^I\tilde{x}^I{}_2 + \tilde{a}_{i3}{}^I\tilde{x}^I{}_3 + \cdots \tilde{a}_{in}{}^I\tilde{x}^I{}_n \leq \tilde{b}_i{}^I\ ;\ i = 1,2,3, \ldots .m.$

$\tilde{x}^I{}_j \geq 0.$

where $\tilde{c}^I, \tilde{\alpha}^I, \tilde{d}^I, \tilde{\beta}^I, \tilde{a}_{ij}{}^I, \tilde{x}^I{}_j$ are TIFNs.

Step 2: Covert the IFQPP into its standard form by introducing intuitionistic fuzzy slack variables we get,

Maximize $\tilde{Z}^I = (\tilde{c}^I\tilde{x}^I + \tilde{\alpha}^I)(\tilde{d}^I\tilde{x}^I + \tilde{\beta}^I)$

Subject to the constraints

$\tilde{a}_{i1}{}^{I}\tilde{x}^{I}{}_{1} + \tilde{a}_{i2}{}^{I}\tilde{x}^{I}{}_{2} + \tilde{a}_{i3}{}^{I}\tilde{x}^{I}{}_{3} + \cdots \tilde{a}_{in}{}^{I}\tilde{x}^{I}{}_{n} + \tilde{S}_{i}{}^{I} = \tilde{b}_{i}{}^{I}\,;\ i = 1,2,3, \ldots . m\,; \tilde{x}^{I}{}_{j} \geq 0.$

where $\tilde{c}^{I}, \tilde{\alpha}^{I}, \tilde{d}^{I}, \tilde{\beta}^{I}, \tilde{a}_{ij}{}^{I}, \tilde{x}^{I}{}_{j}$ are TIFNs and $\tilde{S}_{i}{}^{I} = \langle(1,1,1); (1,1,1)\rangle\,;\ \forall i$

Step 3: Let $\tilde{X}_{B}{}^{I} = \tilde{B}^{I\,-1}\tilde{b}_{i}{}^{I}$ be an initial intuitionistic fuzzy basic feasible solution of IFQPP where $\tilde{B}^{I}$ the triangular intuitionistic fuzzy unit matrix as is follows.

$$\tilde{B}^{I} = \begin{bmatrix} \langle(1,1,1); (1,1,1)\rangle & \langle(0,0,0); (0,0,0)\rangle & \langle(0,0,0); (0,0,0)\rangle \\ \langle(0,0,0); (0,0,0)\rangle & \langle(1,1,1); (1,1,1)\rangle & \langle(0,0,0); (0,0,0)\rangle \\ \langle(0,0,0); (0,0,0)\rangle & \langle(0,0,0); (0,0,0)\rangle & \langle(1,1,1); (1,1,1)\rangle \end{bmatrix}$$

Step 4: Let the Objective function of the IFQPP split into two sub-objective functions such that

$$\tilde{Z}^{I^1} = \tilde{c}^{I}\tilde{x}^{I} + \tilde{\alpha}^{I}$$

$$\tilde{Z}^{I^2} = \tilde{d}^{I}\tilde{x}^{I} + \tilde{\beta}^{I}$$

$$\tilde{Z}^{I} = \tilde{Z}^{I^1} * \tilde{Z}^{I^2}$$

Step 5: Determine the net evaluations

$\tilde{P}_{j}{}^{I} = \left(\tilde{c}_{j}{}^{I} - \tilde{z}_{j}{}^{I^1}\right)\left(\tilde{d}_{j}{}^{I} - \tilde{z}_{j}{}^{I^2}\right)$ and

$$\tilde{Q}_{j}{}^{I} = \tilde{z}_{j}{}^{I^2}\left(\tilde{c}_{j}{}^{I} - \tilde{z}_{j}{}^{I^1}\right) + \tilde{z}_{j}{}^{I^1}\left(\tilde{d}_{j}{}^{I} - \tilde{z}_{j}{}^{I^2}\right)$$

Step 5: Compute $\tilde{\theta}_{j}{}^{I}$ and $\tilde{R}_{j}{}^{I}$ such that

$\tilde{\theta}_{j}{}^{I} = \text{minimum}\left[\frac{\tilde{x}_{B_i}{}^{I}}{\tilde{y}_{u_i}{}^{I}}; \tilde{y}_{u_i}{}^{I} \text{ is positive}\right]$ and $\tilde{R}_{j}{}^{I} = \tilde{Q}_{j}{}^{I} + \left(\tilde{P}_{j}{}^{I} * \tilde{\theta}_{j}{}^{I}\right)$

If $\tilde{R}_{j}{}^{I} \leq 0\,; \forall j$ then $\tilde{X}_{B}{}^{I}$ is the required optimal solution of IFQPP. At least one $\tilde{R}_{j}{}^{I} > 0$ then move onto step 6.

Step 6: (i) To identify the entering variable:

Compute the average value of $\tilde{R}_{j}{}^{I}$; $\forall j$ and let $Av\left(\tilde{R}_{j}{}^{I}\right) = R_{j}{}^{*}$. The triangular intuitionistic fuzzy variable $\tilde{y}_{u_k}{}^{I}$corresponding to maximum of $R_{j}{}^{*}$ is the new entering variable to the basis vector.

(ii) To identify the leaving variable:

Calculate the ratio $\tilde{r}_{j}{}^{I} = \min\left[\frac{\tilde{x}_{B_i}{}^{I}}{\tilde{y}_{u_k}{}^{I}}; \tilde{y}_{u_k}{}^{I} \text{ is positive}\right]$ and find $Av(\tilde{r}_{j}{}^{I}) = r_{j}{}^{*}$. The triangular intuitionistic fuzzy variable $\tilde{X}_{B_i}{}^{I}$ corresponding to minimum of $r_{j}{}^{*}$ is the outgoing variable from the basis vector.

The intersection of (i) and (ii) is the TIF pivotal element. Divide the row by its TIF pivotal element and find other new entries as it is by regular simple procedure.

Step 7: Repeat Step (5) and compute the net evaluations in anticipation of an optimal solution or an unbounded solution.

4.2 Theorem: Let $\tilde{X}_B{}^I$ be an initial intuitionistic fuzzy basic feasible solution of IFQPP then $\widehat{\tilde{X}_B}{}^I$ be the improved intuitionistic fuzzy basic feasible solution if $\hat{\hat{\tilde{Z}}}{}^I = \hat{\tilde{Z}}{}^{I^1} * \hat{\tilde{Z}}{}^{I^2}$.

Proof:

Let $\tilde{X}_B{}^I$ be an initial intuitionistic fuzzy basic feasible solution of IFQPP

Maximize $\tilde{Z}^I = (\tilde{c}^I\tilde{x}^I + \tilde{\alpha}^I)(\tilde{d}^I\tilde{x}^I + \tilde{\beta}^I)$

Subject to the constraints

$\tilde{a}_{i1}{}^I\tilde{x}^I{}_1 + \tilde{a}_{i2}{}^I\tilde{x}^I{}_2 + \tilde{a}_{i3}{}^I\tilde{x}^I{}_3 + \cdots \tilde{a}_{in}{}^I\tilde{x}^I{}_n + \tilde{S}_i{}^I = \tilde{b}_i{}^I;\ i = 1,2,3,\ldots.m.$

$\tilde{x}^I{}_j \geq 0.$

Let $\widehat{\tilde{X}_B}{}^I$ be another intuitionistic fuzzy feasible solution obtained by replacing $\tilde{b}_r{}^I$ by $\tilde{a}_u{}^I$. The improved values of the intuitionistic fuzzy variables are given by

$$\widehat{\tilde{X}_{B_\imath}}{}^I = \tilde{X}_{B_i}{}^I - \tilde{X}_{B_\gamma}{}^I\left(\frac{\tilde{y}_{iu}{}^I}{\tilde{y}_{ru}{}^I};\ i \neq r\right)$$

$$\widehat{\tilde{X}_{B_r}}{}^I = \frac{\tilde{x}_{B_r}{}^I}{\tilde{y}_{ru}{}^I} = \tilde{\theta}_u{}^I \text{ where } \tilde{a}_u{}^I = \sum_{i=1}^m \tilde{y}_{iu}{}^I\tilde{b}_i{}^I$$

Stage: 1 The new improved intuitionistic fuzzy optimizing value $\hat{\tilde{Z}}{}^{I^1}$ is

$$\hat{\tilde{Z}}{}^{I^1} = \sum_{i=1}^m \widehat{\tilde{C}_{B_\imath}}{}^I\widehat{\tilde{X}_{B_\imath}}{}^I + \tilde{\alpha}^I = \sum_{i=1}^m \widehat{\tilde{C}_{B_\imath}}{}^I\widehat{\tilde{X}_{B_\imath}}{}^I + \widehat{\tilde{C}_{B_\gamma}}{}^I\widehat{\tilde{X}_{B_\gamma}}{}^I + \tilde{\alpha}^I$$

Let us assume $\widehat{\tilde{C}_{B_\imath}}{}^I = \tilde{C}_{B_i}{}^I$ and $\widehat{\tilde{C}_{B_\gamma}}{}^I = \tilde{C}_u{}^I$

$$\hat{\tilde{Z}}{}^{I^1} = \sum_{i=1}^m \tilde{C}_{B_i}{}^I\widehat{\tilde{X}_{B_\imath}}{}^I + \tilde{C}_u{}^I\widehat{\tilde{X}_{B_\gamma}}{}^I + \tilde{\alpha}^I$$

$$\hat{\tilde{Z}}{}^{I^1} = \sum_{i=1}^m \tilde{C}_{B_i}{}^I\left[\tilde{X}_{B_i}{}^I - \tilde{X}_{B_\gamma}{}^I\left(\frac{\tilde{y}_{iu}{}^I}{\tilde{y}_{ru}{}^I}\right)\right] + \tilde{C}_u{}^I\frac{\tilde{x}_{B_r}{}^I}{\tilde{y}_{ru}{}^I} + \tilde{\alpha}^I$$

$$\hat{\tilde{Z}}{}^{I^1} = \sum_{i=1}^m \tilde{C}_{B_i}{}^I\tilde{X}_{B_i}{}^I + \tilde{\alpha}^I - \tilde{C}_{B_i}{}^I\tilde{X}_{B_\gamma}{}^I\left(\frac{\tilde{y}_{iu}{}^I}{\tilde{y}_{ru}{}^I}\right) + \tilde{C}_u{}^I\frac{\tilde{x}_{B_r}{}^I}{\tilde{y}_{ru}{}^I}$$

$$\hat{\tilde{Z}}{}^{I^1} = \sum_{i=1}^m \tilde{C}_{B_i}{}^I\tilde{X}_{B_i}{}^I + \tilde{\alpha}^I - \tilde{C}_{B_i}{}^I\tilde{y}_{iu}{}^I\left(\frac{\tilde{x}_{B_\gamma}{}^I}{\tilde{y}_{ru}{}^I}\right) + \tilde{C}_u{}^I\frac{\tilde{x}_{B_r}{}^I}{\tilde{y}_{ru}{}^I}$$

$$\widehat{\tilde{Z}^I}^1 = \tilde{Z}^{I^1} + \sum_{i=1}^{m}\left[\tilde{C}_u{}^I - \tilde{C}_{B_i}{}^I \tilde{y}_{iu}{}^I\right]\left(\frac{\tilde{X}_{B_\gamma}{}^I}{\tilde{y}_{ru}{}^I}\right)$$

$$\widehat{\tilde{Z}^I}^1 = \tilde{Z}^{I^1} + \left[\tilde{C}_u{}^I - \tilde{Z}_u{}^{1^I}\right]\widehat{\tilde{X}_{B_r}}^I$$

Stage: 2 The new improved intuitionistic fuzzy optimizing value $\widehat{\tilde{Z}^I}^1$ is

$$\widehat{\tilde{Z}^I}^2 = \sum_{i=1}^{m}\widehat{\tilde{d}_{B_i}}^I\widehat{\tilde{X}_{B_i}}^I + \tilde{\beta}^I = \sum_{i=1}^{m}\widehat{\tilde{d}_{B_i}}^I\widehat{\tilde{X}_{B_i}}^I + \widehat{\tilde{d}_{B_\gamma}}^I\widehat{\tilde{X}_{B_\gamma}}^I + \tilde{\beta}^I$$

Let us assume $\widehat{\tilde{d}_{B_i}}^I = \tilde{d}_{B_i}{}^I$ *and* $\widehat{\tilde{d}_{B_\gamma}}^I = \tilde{d}_u{}^I$

$$\widehat{\tilde{Z}^I}^2 = \sum_{i=1}^{m}\tilde{d}_{B_i}{}^I\widehat{\tilde{X}_{B_i}}^I + \tilde{d}_u{}^I\widehat{\tilde{X}_{B_\gamma}}^I + \tilde{\beta}^I$$

$$\widehat{\tilde{Z}^I}^2 = \sum_{i=1}^{m}\tilde{d}_{B_i}{}^I\left[\tilde{X}_{B_i}{}^I - \tilde{X}_{B_\gamma}{}^I\left(\frac{\tilde{y}_{iu}{}^I}{\tilde{y}_{ru}{}^I}\right)\right] + \tilde{d}_u{}^I\frac{\tilde{X}_{B_r}{}^I}{\tilde{y}_{ru}{}^I} + \tilde{\beta}^I$$

$$\widehat{\tilde{Z}^I}^2 = \sum_{i=1}^{m}\tilde{d}_{B_i}{}^I\tilde{X}_{B_i}{}^I + \tilde{\beta}^I - \tilde{d}_{B_i}{}^I\tilde{X}_{B_\gamma}{}^I\left(\frac{\tilde{y}_{iu}{}^I}{\tilde{y}_{ru}{}^I}\right) + \tilde{d}_u{}^I\frac{\tilde{X}_{B_r}{}^I}{\tilde{y}_{ru}{}^I}$$

$$\widehat{\tilde{Z}^I}^2 = \sum_{i=1}^{m}\tilde{d}_{B_i}{}^I\tilde{X}_{B_i}{}^I + \tilde{\beta}^I - \tilde{d}_{B_i}{}^I\tilde{y}_{iu}{}^I\left(\frac{\tilde{X}_{B_\gamma}{}^I}{\tilde{y}_{ru}{}^I}\right) + \tilde{d}_u{}^I\frac{\tilde{X}_{B_r}{}^I}{\tilde{y}_{ru}{}^I}$$

$$\widehat{\tilde{Z}^I}^2 = \tilde{Z}^{I^2} + \sum_{i=1}^{m}\left[\tilde{d}_u{}^I - \tilde{d}_{B_i}{}^I\tilde{y}_{iu}{}^I\right]\left(\frac{\tilde{X}_{B_\gamma}{}^I}{\tilde{y}_{ru}{}^I}\right)$$

$$\widehat{\tilde{Z}^I}^2 = \tilde{Z}^{I^2} + \left[\tilde{d}_u{}^I - \tilde{Z}_u{}^{2^I}\right]\widehat{\tilde{X}_{B_r}}^I$$

Hence,

$$\widehat{\tilde{Z}^I} = \widehat{\tilde{Z}^I}^1 * \widehat{\tilde{Z}^I}^2 = \left[\tilde{Z}^{I^1} + \left(\tilde{C}_u{}^I - \tilde{Z}_u{}^{1^I}\right)\widehat{\tilde{X}_{B_r}}^I\right] * \left[\tilde{Z}^{I^2} + \left(\tilde{d}_u{}^I - \tilde{Z}_u{}^{2^I}\right)\widehat{\tilde{X}_{B_r}}^I\right]$$

4.3 Theorem: A sufficient condition for a intuitionistic fuzzy basic feasible solution to the IFQPP to be optimum is that $\tilde{R}_j{}^I \leq 0$; $\forall j$.

Proof:

The objective function value will improve if $\widehat{\tilde{Z}^I} > \tilde{Z}^I$

$$\Rightarrow\left[\tilde{Z}^{I^1} + \left(\tilde{C}_j{}^I - \tilde{Z}_j{}^{1^I}\right)\widehat{\tilde{X}_{B_r}}^I\right] * \left[\tilde{Z}^{I^2} + \left(\tilde{d}_j{}^I - \tilde{Z}_j{}^{2^I}\right)\widehat{\tilde{X}_{B_r}}^I\right] > \tilde{Z}^{I^1} * \tilde{Z}^{I^2}$$

$$\Rightarrow \tilde{Z}^{I^1}\tilde{Z}^{I^2} + \tilde{Z}^{I^1}\left(\tilde{d}_j{}^I - \tilde{Z}_j{}^{2^I}\right)\widehat{\tilde{X}_{B_r}}^I + \tilde{Z}^{I^2}\left(\tilde{C}_j{}^I - \tilde{Z}_j{}^{1^I}\right)\widehat{\tilde{X}_{B_r}}^I + \left[\left(\tilde{C}_j{}^I - \tilde{Z}_j{}^{1^I}\right)\widehat{\tilde{X}_{B_r}}^I * \left(\tilde{d}_j{}^I - \tilde{Z}_j{}^{2^I}\right)\widehat{\tilde{X}_{B_r}}^I\right] > \tilde{Z}^{I^1} * \tilde{Z}^{I^2}$$

$$\Rightarrow \tilde{Z}^{I^1}\left(\tilde{d}_j{}^I - \tilde{Z}_j{}^{2^I}\right)\widehat{\tilde{X}_{B_r}}^I + \tilde{Z}^{I^2}\left(\tilde{C}_j{}^I - \tilde{Z}_j{}^{1^I}\right)\widehat{\tilde{X}_{B_r}}^I + \tilde{\theta}_j{}^I\left(\tilde{C}_j{}^I - \tilde{Z}_j{}^{1^I}\right) * \left(\tilde{d}_j{}^I - \tilde{Z}_j{}^{2^I}\right) > 0$$

$$\Rightarrow \tilde{R}_j{}^I > 0.$$

Hence, the solution can be enhanced until $\tilde{R}_j{}^I \leq 0\ ;\ \forall j$.

5. Numerical example

Consider the IFQPP

Maximize $\tilde{Z}^I = \left(\tilde{2}^I\tilde{x}_1{}^I + \tilde{4}^I\tilde{x}_2{}^I + \tilde{x}_3{}^I + \tilde{1}^I\right)\left(\tilde{1}^I\tilde{x}_1{}^I + \tilde{1}^I\tilde{x}_2{}^I + \tilde{2}^I\tilde{x}_3{}^I + \tilde{2}^I\right)$

subject to the constraints

$\tilde{1}^I\tilde{x}_1{}^I + \tilde{3}^I\tilde{x}_2{}^I \leq \tilde{4}^I$

$\tilde{2}^I\tilde{x}_1{}^I + \tilde{1}^I\tilde{x}_2{}^I \leq \tilde{3}^I$

$\tilde{1}^I\tilde{x}_2{}^I + \tilde{4}^I\tilde{x}_3{}^I \leq \tilde{3}^I$

$\tilde{x}^I{}_j \geq 0\ ;\ j = 1,2,3.$

where the intuitionistic fuzzy numbers are taken as follows:

Cost coefficients

$\tilde{2}^I = \langle(1.5,2,4); (1,2,4.5)\rangle$

$\tilde{4}^I = \langle(3.6,4,4.2); (3,4,5.1)\rangle$

$\tilde{1}^I = \langle(0.8,1,2.1); (0.6,1,2.3)\rangle$

$\tilde{1}^I = \langle(0.5,1,2.5); (0.3,1,2.6)\rangle$

$\tilde{1}^I = \langle(0.6,1,1.8); (0.5,1,2.0)\rangle$

$\tilde{1}^I = \langle(0.4,1,2); (0.3,1,2)\rangle$

$\tilde{2}^I = \langle(1.6,2,3.4); (1.4,2,3.6)\rangle$

$\tilde{2}^I = \langle(1.8,2,3.2); (1.6,2,3.3)\rangle$

Constraint 1-Ccoefficients

$\tilde{1}^I = \langle(0.4,1,1.5); (0.3,1,1.5)\rangle$

$\tilde{3}^I = \langle(2.5,3,3.2); (2,3,3.2)\rangle$

$\tilde{4}^I = \langle(2,4,5);(1.8,4,5.2)\rangle$

Constraint 2-Ccoefficients

$\tilde{2}^I = \langle(1,2,3);(1,2,4)\rangle$

$\tilde{1}^I = \langle(0.7,1,1.6);(0.5,1,1.8)\rangle$

$\tilde{3}^I = \langle(2.7,3,3.3);(2.6,3,3.4)\rangle$

Constraint 3-Ccoefficients

$\tilde{1}^I = \langle(0.3,1,2.3);(0.6,1,2.4)\rangle$

			$\tilde{c}_j{}^I$	(1.5,2,4); (1,2,4.5)	(3.6,4,4.2); (3,4,5.1)	(0.8,1,2.1); (0.6,1,2.3)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$		
			$\tilde{d}_j{}^I$	(0.6,1,1.8); (0.5,1,2)	(0.4,1,2); (0.3,1,2)	(1.6,2,3.4); (1.4,2,3.6)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$		
$\tilde{c}_B{}^I$	$\tilde{d}_B{}^I$	$\tilde{y}_B{}^I$	$\tilde{X}_B{}^I$	$\tilde{x}_1{}^I$	$\tilde{x}_2{}^I$	$\tilde{x}_3{}^I$	$\tilde{s}_1{}^I$	$\tilde{s}_2{}^I$	$\tilde{s}_3{}^I$	$\tilde{r}^I$	$Av(\tilde{r}^I)$
$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{s}_1{}^I$	(2,4,5); (1.8,4,5.2)	(0.4,1,1.5); (0.3,1,1.5)	(2.5,3,3.2); (2,3,3.2)	$\tilde{0}^I$	$\tilde{1}^I$	$\tilde{0}^I$	$\tilde{0}^I$	(0.63,1.33,2); (0.56,1,33,2.6)	1.42
$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{s}_2{}^I$	(2.7,3,3.3); (2.6,3,3.4)	(1,2,3); (1,2,4)	(0.7,1,1.6); (0.5,1,1.8)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{1}^I$	$\tilde{0}^I$	(1.69,3.4.71)	3.53
$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{s}_3{}^I$	(2.4,3,3.4); (2.3,3,3.5)	$\tilde{0}^I$	(0.3,1,2.3); (0.6,1,2.4)	(3.2,4,5.5); (3,4,6)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{1}^I$	(1.04,3,11.33); (0.96,3,5.83)	6.43
$\tilde{P}_j{}^I = \left(\tilde{c}_j{}^I - \tilde{z}_j{}^{I1}\right) * \left(d^I - \tilde{z}_j{}^{I2}\right)$				(0.9,2,7.2); (0.5,2,9)	(1.44,4,8.4); (0.9,4,10.2)	(1.28,2,7.14); (0.84,2,8.28)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$		
$\tilde{Q}_j{}^I = \tilde{z}_j{}^{I2}\left(\tilde{c}_j{}^I - \tilde{z}_j{}^{I1}\right) + \tilde{z}_j{}^{I1}\left(d^I - \tilde{z}_j{}^{I2}\right)$				(3,5,17.3); (1.75,5,20.05)	(6.68,9,18.44); (4.89,9,22.03)	(2.24,4,15.22); (1.38,4,17.86)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$		
$\tilde{\theta}_j{}^I$				(0.9,1.5,3.3); (0.65,1.5,3.4)	(0.63,1.33,2); (0.56,1.33,2.6)	(0.44,0.75,1.06); (0.38,0.75,1.17)	-	-	-		
$\tilde{R}_j{}^I$				(3.81,8,41.6); (2.08,8,50.65)	(7.59,14.32,35.24); (5.39,14.32,48.55)	(2.8,5.5,22.79); (1.7,5.5,27.55)	-	-	-		
$Av(\tilde{R}_j{}^I) = R_j{}^*$				12.3	22.22	12.07	-	-	-		
$\widetilde{Z^1}^I$=(0.5,1,2.5); (0.3,1,2.6)					$\widetilde{Z^2}^I$=(1.8,2,3.2); (1.6,2,3.3)				$\tilde{Z}^I$=(0.9,2,8); (0.4,2,8.58)		

$\tilde{4}^I = \langle(3.2,4,5.5);(3,4,6)\rangle$

$\tilde{3}^I = \langle(2.4,3,3.4);(2.3,3,3.5)\rangle$

Using proposed two-stage intuitionistic fuzzy simplex procedure the solution of given IFQPP are given in the following.

Initial iteration

Iteration 1

			$\tilde{c}_j^{\ I}$	(1.5,2,4); (1,2,4.5)	(3.6,4,4.2); (3,4,5.1)	(0.8,1,2.1); (0.6,1,2.3)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$
			$\tilde{d}_j^{\ I}$	(0.6,1,1.8); (0.5,1,2)	(0.4,1,2); (0.3,1,2)	(1.6,2,3.4); (1.4,2,3.6)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$
$\tilde{c}_B^{\ I}$	$\tilde{d}_B^{\ I}$	$\tilde{y}_B^{\ I}$	$\tilde{X}_B^{\ I}$	$\tilde{x}_1^{\ I}$	$\tilde{x}_2^{\ I}$	$\tilde{x}_3^{\ I}$	$\tilde{s}_1^{\ I}$	$\tilde{s}_2^{\ I}$	$\tilde{s}_3^{\ I}$
(3.6,4,4.2); (3,4,5.1)	(0.4,1,2); (0.3,1,2)	$\tilde{x}_2^{\ I}$	(0.63,1.33,2); (0.56,1,33,2.6)	(0.13,0.33,0.6); (0.09,0.33,0.75)	(0.78,1,1.28); (0.63,1,1.6)	$\tilde{0}^I$	(0.31,0.33,0.40); (0.31,0.33,0.40)	$\tilde{0}^I$	$\tilde{0}^I$
$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{s}_2^{\ I}$	(0.7,1.67,2.67); (0.0,1.67,2.84)	(0.4,1.67,2.87); (0.25,1.67,3.91)	(-0.58,0,0.82); (-1.1,0,1.17)	$\tilde{0}^I$	-{(0.31,0.33,0.40); (0.31,0.33,0.40)}	$\tilde{1}^I$	$\tilde{0}^I$
$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{s}_3^{\ I}$	(0.4,1.67,2.77); (-0.3,1.67,2.94)	-{(0.13,0.33,0.6); (0.09,0.33,0.75)}	(-0.98,0,1.52); -1,0,1.17)	(3.2,4,5.5); (3,4,6)	-{(0.31,0.33,0.40); (0.31,0.33,0.40)}	$\tilde{0}^I$	$\tilde{1}^I$
$\tilde{P}_j^{\ I} = \left(\tilde{c}_j^{\ I} - \tilde{z}_j^{\ I^1}\right) * \left(d^I - \tilde{z}_j^{\ I^2}\right)$				(0,0.46,5.9); (0.71,0.46,8.08)	-	(1.28,2,7.14); (0.84,2,8.28)	(0.35,0.44,0.67); (0.29,0.44,0.82)	$\tilde{0}^I$	$\tilde{0}^I$
$\tilde{Q}_j^{\ I} = \tilde{z}_j^{\ I^2}\left(\tilde{c}_j^{\ I} - \tilde{z}_j^{\ I^1}\right) + \tilde{z}_j^{\ I^1}\left(d^I - \tilde{z}_j^{\ I^2}\right)$				(-2.09,6.49,43.62); (-5.51,6.49,66.25)	(-6.09,0,23.31); (-11.7,0,49.02)	(6.07,15.97,52.18); (3.83,15.97,76.65)	-{(3.16,6.49,16.46); (2.26,6.49,23.68)}	$\tilde{0}^I$	$\tilde{0}^I$
$\tilde{\theta}_j^{\ I}$				-	-	-	(0.07,0.42,0.87); (-0.05,0.42,0.98)	(1.58,4.03,6.45) (1.4,4.03,8.39)	-
$\tilde{R}_j^{\ I}$				-	-	-	(6.16,16.81,58.39); (3.79,16.81,84.76)	-{(-1.16,4.72,15.91); (-4.62,4.72,23.27)}	-
$Av(\tilde{R}_j^{\ I}) = R_j^*$				-	-	-	33.92	-7.62	-
$\widetilde{Z^1}^I$=(2.77,6.32,10.9); (1.98,6.32,15.86)					$\widetilde{Z^2}^I$=(2.05,3.33,7.2); (1.77,3.33,8.5)			$\tilde{Z}^I$=(5.68,21.05,78.48); (3.5,21.05,134.81)	

Iteration 2

			$\tilde{c}_j^{\ I}$	(1.5,2,4); (1,2,4.5)	(3.6,4,4.2); (3,4,5.1)	(0.8,1,2.1); (0.6,1,2.3)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$
			$\tilde{d}_j^{\ I}$	(0.6,1,1.8); (0.5,1,2)	(0.4,1,2); (0.3,1,2)	(1.6,2,3.4); (1.4,2,3.6)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$
$\tilde{c}_B^{\ I}$	$\tilde{d}_B^{\ I}$	$\tilde{y}_B^{\ I}$	$\tilde{X}_B^{\ I}$	$\tilde{x}_1^{\ I}$	$\tilde{x}_2^{\ I}$	$\tilde{x}_3^{\ I}$	$\tilde{s}_1^{\ I}$	$\tilde{s}_2^{\ I}$	$\tilde{s}_3^{\ I}$
(3.6,4, 4.2); (3,4,5. 1)	(0.4,1, 2); (0.3,1, 2)	$\tilde{x}_2^{\ I}$	(0.63,1.33, 2); (0.56,1,33, 2.6)	(0.13,0.33,0 .6); (0.09,0.33,0 .75)	(0.78,1,1. 28); (0.63,1,1. 6)	$\tilde{0}^I$	(0.31,0.33,0 .40); (0.31,0.33,0 .40)	$\tilde{0}^I$	$\tilde{0}^I$
$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{s}_2^{\ I}$	(0.7,1.67,2 .67); (0.0,1.67,2 .84)	(0.4,1.67,2. 87); (0.25,1.67,3 .91)	(- 0.58,0,0.8 2); (- 1.1,0,1.17)	$\tilde{0}^I$	- {(0.31,0.33, 0.40); (0.31,0.33,0 .40)}	$\tilde{1}^I$	$\tilde{0}^I$
(0.8,1, 2.1); (0.6,1, 2.3)	(1.6,2, 3.4); (1.4,2, 3.6)	$\tilde{x}_3^{\ I^I}$	(0.07,0.42, 0.87); (- 0.05,0.42,0 .980	- {(0.02,0.08, 0.19); (0.02,0.08,0 .25)}	(- 0.18,0,0.4 8); (- 0.17,0,0.5	(0.58,1,1. 72); (0.5,1,2)	- {(0.06,0.08, 0.13); (0.05,0.08,0 .13)}	$\tilde{0}^I$	(0.18,0.25,0.3 (0.17,0.25,0.3

		9)				
$\tilde{P}_j^{\,I} = \left(\tilde{c}_j^{\,I} - \tilde{z}_j^{\,I^1}\right) * \left(d^I - \tilde{z}_j^{\,I^2}\right)$	(-0.03,0.63,9.12); (0.62,0.63,13.52)	(2.31,0,7); (5.33,0,22.30))	(4.05,0,11.94); (5.8,0,23.20)	(-0.11,0.21,0.49); (-0.04,0.21,0.48)	$\tilde{0}^I$	(0.04,0.13,0.68); (0.02,0.13,0.9)
$\tilde{Q}_j^{\,I} = \tilde{z}_j^{\,I^2}\left(\tilde{c}_j^{\,I} - \tilde{z}_j^{\,I^1}\right) + \tilde{z}_j^{\,I^1}\left(d^I - \tilde{z}_j^{\,I^2}\right)$	(-2.08,8.76,69.46); (-5.22,8.76,108.75)	((-13.13,0,34.76); (-17.75,0,68.98)	(-18.1,0,48.1); (-18.11,0,76.58)	(-{(-2.21,6.32,20.38); (-1.21,6.32,28.53)}	$\tilde{0}^I$	(-{1.12,4.41,19.97); (0.64,4.41,30.69)}
$\tilde{\theta}_j^{\,I}$	(0.24,1,6.68); (0,1,11.36)	-	-	(1.58,4.03,6.45); (1.4,4.03,8.39)	-	-
$\tilde{R}_j^{\,I}$	(-2.09,9.39,130.38); (-5.22,939,262.34)	-	-	-{{-5.37,5.47,20.55); (-5.24,5.47,28.59)}		-
$Av(\tilde{R}_j^{\,I}) = R_j^*$	78.56	-	-	-8.8	-	-
$\widetilde{Z^1}^I$=(2.83,6.74,12.73); (1.95,6.74,18.11)	$\widetilde{Z^2}^I$=(2.16,4.17,10.16); (1,7,4.17,12.03			$\tilde{Z}^I$=(6.11,28.11,129.34); (3.32,28.11,217.86)		

Iteration 3

			$\tilde{c}_j^I$	(1.5,2,4); (1,2,4.5)	(3.6,4,4.2); (3,4,5.1)	(0.8,1,2.1); (0.6,1,2.3)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$
			$\tilde{a}_j^I$	(0.6,1,1.8); (0.5,1,2)	(0.4,1,2); (0.3,1,2)	(1.6,2,3.4); (1.4,2,3.6)	$\tilde{0}^I$	$\tilde{0}^I$	$\tilde{0}^I$
$\tilde{c}_B^I$	$\tilde{a}_B^I$	$\tilde{y}_B^I$	$\tilde{X}_B^I$	$\tilde{x}_1^I$	$\tilde{x}_2^I$	$\tilde{x}_3^I$	$\tilde{s}_1^I$	$\tilde{s}_2^I$	$\tilde{s}_3^I$
(3.6,4,4.2); (3,4,5.1)	(0.4,1,2); (0.3,1,2)	$\tilde{x}_2^I$	(3.38,1,1.97); (-7.96,1,2.6)	(-4.18,0,0.58); (-11.44,0,0.74)	(-0.45,1,1.31); (-2.88,1,1.63)	$\tilde{0}^I$	(0.32,0.4,1); (0.32,0.4,1.6)	≡{(0.05,0.2,1.5); (0.02,0.2,3)}	$\tilde{0}^I$
(1,2,4); (1.5,2,4.5)	(0.6,1,1.8); (0.5,1,2)	$\tilde{x}_1^I$	(0.24,1,6.68); (0,1,11.36)	(0.14,1,7.18); (0.06,1,15.64)	(-0.2,0,2.05); (-0.28,0,4.68)	$\tilde{0}^I$	≡{(0.11,0.2,1); (0.08,0.2,1.6)}	(0.35,0.6,2.5); (0.26,0.6,4)	$\tilde{0}^I$
(0.8,1,2.1); (0.6,1,2.3)	(1.6,2,3.4); (1.4,2,3.6)	$\tilde{x}_3^I$	(0.07,0.5,2.14); (-0.05,0.5,3.82)	(-0.19,0,1.34); (-0.25,0,3.89)	(-0.18,0,0.87); (-0.18,0,1.76)	(0.58,1,1.72); (0.5,1,2)	≡{(0.06,0.1,0.52); (0.05,0.1,0.53)}	(0.01,0.05,0.98); (0.01,0.05,1)	(0.18,0.25,0.31); (0.17,0.25,0.33)
$\tilde{P}_j^I = \left(\tilde{c}_j^I - \tilde{z}_j^{I1}\right) * \left(d^I - \tilde{z}_j^{I2}\right)$				-	-	-	(0.54,0,7.55); (0.57,0,32.31)	(0.01,0.23,57.01); (-1.15,0.23,216.26)	(0.04,0.13,0.68); (0.02,0.13,0.9)
$\tilde{Q}_j^I = \tilde{z}_j^{I2}\left(\tilde{c}_j^I - \tilde{z}_j^{I1}\right) + \tilde{z}_j^{I1}\left(d^I - \tilde{z}_j^{I2}\right)$				-	-	-	≡{(102.6,5.45,66.12); (-287.06,5.45,287.46)}	≡{(-0.36,6.05,503.39); (3.22,6.05,1716.87)}	≡{(-3.2,5,61.27); (-5.92,5,124.35)}
$\tilde{\theta}_j^I$				-	-	-	(-3.38,2.5,6.16); (-4.98,2.5,8.13)	(0.1,1.67,19.09); (0,1.67,43.69)	(0.23,2,11.89); (-0.15,2,22.47)
$\tilde{R}_j^I$				-	-	-	≡{(-142.23,5.45,65.68); (-529.33,5.45,28.96)}	≡{(-1.88.9,5.67,503.39); (-9445.2,5.67,1716.9)}	≡{(-11.29,4.74,61.26); (-26.14,4.74,124.35)}
$\tilde{Z}^{1I}$ ≡(-11.37,7.5,41.98); (-23.61,7.5,75.77)				$\tilde{Z}^{2I}$ ≡(0.7,5,26.44); (-2.46,5,44.97)			$\tilde{Z}^I$ ≡(7.96,37.5,1109.95); (58.08,37.5,3407.38)		

Conclusion:

We proposed a new solution technique to determine an intuitionistic fuzzy optimal solution for fuzzy quadratic programming problem in which the objective function has to be factorized This technique provides a favorable fuzzy solution without using clumsy of constraints. Moreover, the developed algorithm is based on familiar simplex procedure we obtain the fuzzy solution in less number of iterations.

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