Estimation of the Parameters of the Wiener Degradation Process for the Reliability Analysis Using the Amoebaalgorithm

Mostafa Abdul-Jabbar Dawod[a] , Entsar Arebe Fadam[b]

a. College of Administration and Economics, Department of Statistics, University of Baghdad, Iraq.
mostafa.abduljabar1201a@coadec.uobaghdad.edu.iq

b. College of Administration and Economics, Department of Statistics, University of Baghdad, Iraq.
entsar_arebe@coadec.uobaghdad.edu.iq

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Abstract

Deterioration models have provided accuracy in estimating reliability, including theWiener process-based degradation model, researchers have introduced several methods for estimating the parameters of the degradation process. In this research, the researcher proposed the Amoeba algorithm to estimate the parameters of the Wiener degradation process and it was compared with the method of maximum likelihood estimations, this was applied through Monte Carlo simulation, the results were that the Amoeba method is best based on the MSE comparison criterion, then the reliability was estimated based on the inverse Gaussian distribution according to the characteristics of the Wiener process.

Keywords: The Wiener process, Amoeba algorithm, Inverse Gaussian distribution, Reliability.

1.Introduction

Due to the variety of failure patterns of most products, more attention has been paid to assessing product reliability, which has largely drawn attention in the literature. Reliability is the ability of products to perform their required functions under specific conditions for a specific period of time. Conventional methods of estimating reliability rely heavily on time-to-fail data. However, it is usually difficult to collect sufficient failure time data to estimate reliability, and a reliability technique based on performance degradation has been robustly developed and widely used. Degradation data is a decrease in product performance over time without reaching a state of complete failure. The degradation processes can be observed by using condition

monitoringtechnologies,the observed condition based data are often referred to as the degradation data, which are closely associated with the underlying physical degradation process. The Wiener process, due to its simple structure and rich research results, has become the most widely used model in degradation modeling[6,7]. AWiener processes are more suitable for modeling non-monotonic degradation processes,this type of stochastic process has been widely applied to describe the evolution of the degradation process over time[9], the majority of current work on reliability estimation based on the Wiener process, it is generally assumed that the random effect of the degradation model follows a normal distribution, that is, the normal distribution is used to describe the unit-to-unit variance of a set of systems. Such an assumption leaves something to be desired because the drift parameter that is frequently encountered in practice can always be positive about some engineering systems, such as the deterioration of lasers, train wheels, etc. Compared with the normal distribution, the inverse Gaussian distribution is more suitable for unit-to-unit variable characterization, which can avoid the influence of negative drift coefficients on the reliability estimate[10].Several literatures mentioned the Wiener degradation process and its importance in estimating reliability as well as methods for estimating parameters of the degradation process can be found in references[4,8,11,12].In this paper ,the researcher suggested used the Amoeba algorithm to estimated parameter of Wiener process to estimate the reliability. The remainder of this paper is organized as follows: In section 2, clarification of the degradation model and estimation of reliability. In Section 3,application of a case study through Monte Carlo simulation to give results for illustrate the proposed method. Finally, conclusion is presented in Section 4.

2.Methodology

2.1.Wiener process

Thedegradation data can be described using the Wiener process if it has the normality distribution. The Wiener process is as follows[12]:

$$X_i(t) = \mu_i t + \sigma_i B(t) \qquad ,i = 1,2,\dots,k \quad (1)$$

Where$X_i(t)$is represents the amount of degradation at timet ,μ is drift parameter, σ isdiffusion parameter, $B(.)$ is Wiener standard process subject to $N(0,t)$.The characteristics of the Wiener process $\{X_i(t); t \geq 0\}$ asfollows:

1. $(X(t);\ t \geq 0)$have independent increments, that is $((X(t_2) - X(t_1))$independent from$((X(t_3) - X(t_4))$ for $0 \leq t_1 \leq t_2 \leq t_3 \leq t_4$.

2. Increase in the path of degradation ΔX (t) = X (t + Δt) - X (t) follows a normal distribution$\{\Delta X(t) \sim N\ (\mu\Delta t, \sigma^2\Delta t)\}$.

2.2.Reliability estimation based on Wiener process

The failure of the product occurs when the deterioration process represented here by the Wiener process $X_i(t)$of the product *i* reaches the failurethreshold W (which is the first time the product reaches failure) at time t, where the failure time T can be defined as[3]:

$$T = \inf\{t \geq 0\ ;\ X(t) \geq w\} \qquad (2)$$

According to the characteristics of the Wiener process, the product failure time T follows the inverse Gaussian distribution(IG):

$$T \sim IG(^{w}/_{\mu}\ ,\ ^{w^2}/_{\sigma^2}) \qquad (3)$$

Then, the probability density function of T as:

$$f_T(t)) = \frac{w}{\sqrt{2\pi\sigma^2 t^3}} \exp\ (\frac{-(w-\mu t)^2}{2\sigma^2 t})(4)$$

and the reliability function is given as:

$$R(t) = \Phi\left(\frac{w\ -\ \mu t}{\sigma\sqrt{t}}\right) - exp\left(\frac{2\mu w}{\sigma^2}\right)\Phi\left(\frac{-\mu t - w}{\sigma\sqrt{t}}\right) \qquad (5)$$

2.3. Parameters estimation of the model

We assume thatnis the number of products and the performance of each product is measuredk times at the same time for all products.By the characteristics of the Wiener process, the increase in deterioration follows a normal distribution[4]:

$$\Delta X_{ij} \sim N\left(\mu_i \Delta t_{ij}\ ,\ \sigma_i^2 \Delta t_{ij}\right)(6)$$

Where $i = 1\ ,2\ ,3, \dots, n\ \ and\ \ j = 1\ ,2\ ,3, \dots, k$. And that the distribution ofΔX_{ij}as follows:

$$f\left(\Delta X_{ij}\right) = \frac{1}{\sqrt{2\pi\sigma_i^2 \Delta t_{ij}}} exp\left(\frac{-(\Delta X_{ij} - \mu_i \Delta t_{ij})^2}{2\sigma_i^2 \Delta t_{ij}}\right) \qquad (7)$$

Where

(8)$\Delta X_{ij} = X_{ij} - X_{ij-1}$

and

(9)$\Delta t_{ij} = t_{ij} - t_{ij-1}$

In the next section, methods for estimating Wiener process parameters will be explained.

2.3.1.Maximum likelihood estimatorsmethod

Using the maximum likelihood estimators(MLE) method, the parameters of the Wiener process can be estimated by the probability density function of the incremental performance degradation, the function of maximum likelihood as follows[2]:

$$L(\mu_i, {\sigma_i}^2) = \prod_{j=1}^{K} \frac{1}{\sqrt{2\pi\sigma_i^2 \Delta t_{ij}}} exp\left(\frac{-(\Delta X_{ij} - \mu_i \Delta t_{ij})^2}{2\sigma_i^2 \Delta t_{ij}}\right)$$

$$= \left(2\pi\sigma_i^2 \Delta t_{ij}\right)^{-\frac{K}{2}} exp\left(\frac{-\sum_{j=1}^{K}(\Delta X_{ij} - \mu_i \Delta t_{ij})^2}{2\sigma_i^2 \Delta t_{ij}}\right) \quad (10)$$

taking the logarithm of both sides of the Eq.(10), and find the partial derivatives of μ_i and σ_i respectively to obtain the estimated parameters of the model:

$$\hat{\mu}_i = \frac{\sum_{j=1}^{K} \Delta X_{ij}}{\sum_{j=1}^{K} \Delta t_{ij}} \quad (11)$$

$${\hat{\sigma}_i}^2 = \frac{1}{K}\left[\sum_{j=1}^{K} \frac{(\Delta X_{ij})^2}{\Delta t_{ij}} - \frac{\left(\sum_{j=1}^{K} \Delta X_{ij}\right)^2}{\sum_{j=1}^{K} \Delta t_{ij}}\right] \quad (12)$$

2.3.2. Amoebaalgorithm

The Amoeba algorithm was proposed by Roger Mead & John Nelder in 1965, also called Nealder-Mead or Downhill Simplex, which is a numerical iterative optimization algorithm that uses geometric relations to obtain the lower bound of the objective function (the mathematical problem under consideration that consists of several variables) in a multidimensional space, characterized by not requiring derivatives, but only evaluation of a number of points for each variable in the function, it has been applied in many areas that require numerical optimization techniques. This algorithm depends in its work on the concept of Simplex, which is a geometric shape in z of dimensions and z + 1 of vertices points, exists in several geometric shapes straight, tetrahedron,

polygon and the triangle, and the most common and well-known is the triangle shape, as it generates a new test position by extrapolating the behavior of the objective function measured when arranging each test point, and the Simplex vertices are represented by $A_1, A_2, \ldots, A_{z+1}$.

These vertices (points) represent the objective function that is arranged at each test position:

$$S(A_1) < S(A_2) < S(A_3) < \ldots < S(A_{z+1})$$

Where A_{z+1} is the worst point, and A_1 is the best point. These points are tested by continuous improvement process in an iterative manner by updating the worst point through the operations of reflection (r), expansion (e), contraction (c), shrink (sh).Thus, this algorithm performs the process of optimization, as after it finds the initial Simplex shape that forms from the initial point[1,5]. The idea of working the Amoeba algorithm in estimating the parameters of the Wiener process is shown in the following algorithm.

Algorithm of Amoeba to estimating the parameters of the Wiener process

1. Define the objective function which includes parameters (μ, σ^2)Here, the objective function is the negative logarithm of the maximum likelihood function of Eq.(10):
   $S(A) = -LnL(A), where\ \mathrm{A} = (\mu, \sigma^2)$(13)
2. Determine the parameters of the Amoeba algorithm, that the standard values of these parameters are:
   $$\tau = 1, \delta = 2, \beta = \alpha = 0.5$$
   $\tau > 0,\ \delta > 1,\ 0 < \beta < 1,\ 0 < \alpha < 1$
3. Generating the initial solution space, the matrix V with dimensions z × 2, as the rows i represent the number of solutions and the columns j represent the parameters to find values for it, that each row in V represents solution:
   $$V = \begin{bmatrix} m_1 & \sigma_1^2 \\ \vdots & \vdots \\ m_z & \sigma_z^2 \end{bmatrix} (14)$$
   where i = 1, 2, . . . , z and j = 1, 2
   and each row in the matrix V can be expressed by (A), which represents the requiredparameters, so the matrix becomes as follows:
   $$V = \begin{bmatrix} A_1 \\ \vdots \\ A_z \end{bmatrix}, where\ A = (\mu, \sigma^2) \quad (15)$$

4. Compute the objective function for each row in V.
5. Arrange the solutions in V according to the value of the objective function and the highest value is the one with the worst solution.
6. Compute the mean of the matrix, which is the centroid:

$$\bar{V} = \frac{\sum_{i=1}^{z} A_i}{z} \quad , \quad where \ i = 1,2,3, \dots, z \qquad (16)$$

7. Generate a new checkpoint represent the reflection (r):
   $A_r = \bar{V} + \tau\,(\bar{V} - A_z)$ (17)
8. Compute the values of the objective function according to the point (r):
   ˆ If $S(A_1) < S(A_r) < S(A_z)$make $A_r = A_z$.
   ˆ If $S(A_r) \leq S(A_1)$, go to step (9)
9. Generate a new checkpoint that represents expansion (e)

$A_e = \bar{V} + \delta(A_r - \bar{V})$(18)

10. Calculate the values of the objective function according to (e): ˆ
    If $S(A_e) < S(A_r)$make $A_e = A_z$.
    If $S(A_e) \geq S(A_r)$ go to step (11).
11. Generate a new test point representing the contraction (c) as follows:

(19)$A_c = \bar{V} + \beta(A_z - \bar{V})$

12. If $S(A_c) < S(A_z)$make$\mathrm{A_c} = \mathrm{A_z}$otherwise, go to step (13).
13. Generate a new test point representing the shrinkage (Sh), where the contraction is done towards the best candidate for the solution:
    $A_{shi} = A_1 + \alpha(A_i - A_1)$(20)
14. If the algorithm stop condition is met, the best solution that was found, that achieves the lowest value of the objective function is printed, i.e. iterations stop when it reaches:

$$\left|\frac{\max(S) - \min(S)}{\max(S)}\right| < \in \qquad (21)$$

where ε is a very small number, then, go to step (15), otherwise, go back to step (7).

15. Print the best solutions that makes the objective function as minimal as possible

16. End.

3. Simulation study

A Monte Carlo simulation study was conducted to verify that the proposed Amoeba approach is more effective for the Wiener degradation model, compared with MLE. The samples size of tested units chosen to be n=5 and n=10 ; each unit was inspected k times, as k= 0, 50, 100, 150, 200, 250.

For each experiment setting (n, k) was generate random observation from random effects Wiener degradation model under the two parameters ($\hat{\mu}$,$\hat{\sigma}^2$). The methods referred to in Section (2) were applied to obtain the estimated parameters($\hat{\mu}$,$\hat{\sigma}^2$) of the parameters (μ, σ^2)in an iterative manner based on the simulated degradation data. And then, mean squared error (MSE) of these estimators was calculate. Table 1 presents the parameter results for (n, k) combinations using MLE and Amoeba methods, in addition to presenting the MSE for each parameter based on 1000 iterations through Matlab application.

Table 1: Estimated parameter values of the Wiener process and MSE for each parameter.

n	Units	MLE method		Amoeba method		MSE under MLE method		MSE under Amoeba method	
		$\hat{\mu}$	$\hat{\sigma}^2$	$\hat{\mu}$	$\hat{\sigma}^2$	$\hat{\mu}$	$\hat{\sigma}^2$	$\hat{\mu}$	$\hat{\sigma}^2$
n=5	1	7.90	4.98	9.79	7.59	3.6218563	6.856857	8.87E-05	5.85E-05
	2	8.07	5.09	9.90	7.90	3.3665627	7.870923	7.49E-06	3.73E-06
	3	8.09	5.10	10.10	7.70	4.0549267	6.775225	1.12E-05	6.59E-07
	4	8.53	5.36	10.91	7.79	5.6370735	5.949966	0.000201	0.000122
	5	9.27	5.82	12.00	8.28	7.4370137	6.130608	5.18E-06	0.000248
n=10	1	7.9	4.98	9.8	7.6	3.6218563	6.856857	3.78E-07	6.02E-07
	2	8.07	5.09	9.9	7.9	3.3665627	7.8709229	2.23E-07	2.99E-06
	3	8.09	5.1	10.1	7.7	4.0549267	6.7752252	9.39E-08	4.75E-08
	4	8.53	5.36	10.9	7.8	5.6370735	5.949966	1.38E-06	2.33E-05
	5	9.19	5.77	12	8.09	7.873379	5.4274067	7.20E-07	2.60E-05

	6	8.59	5.41	12	8.09	4.866234	7.2179583	1.4420366	3.62E-05
	7	8.06	5.07	10.2	7.5	4.5905213	5.8982526	2.94E-07	7.30E-06
	8	8.58	5.39	10.9	7.84	5.8769987	5.805784	0.0097668	0.001556
	9	7.6	4.84	8.9	7.97	1.6780271	10.009256	2.71E-07	0.0006709
	10	8.82	5.56	11.1	8.29	5.1851337	7.5248622	3.80E-06	4.59E-05

From Table 1, it can be seen that the decrease in MSE for each parameter with the increase in the degradation data. In addition, when comparing the MSE of the parameters estimated by the Amoeba algorithmat the sample size of n=5 and n=10, we note that it is less than the estimated by MLE method, this means that the Amoeba method is the bestfor estimating the parameters.
Based on the interpretation of the results of Table1 about the best estimation method, the parameters estimated by Amoeba method will be adopted in the degradation model Eq. (1) to model the degradation data, to clarify this, the generated with n = 10 will be used, as shown in Table2 and Figure1.

Table 2:Amount of degradation $X_i(t)$in the data generated after modeling using the Wiener process whose parameters were estimated by the Amoeba method with n=10 at 6 degradation times (0,50,150,200,250)

Time	Units									
	1	2	3	4	5	6	7	8	9	10
0	9.8	9.9	10.1	11	12	10.8	10.2	11	8.9	11.99
50	20.24	20.8	20.84	22.32	24.66	22.49	20.72	22.38	19.37	24.14
100	27.06	27.71	27.99	30.36	33.95	30.4	27.99	30.53	25.14	32.39
150	40.55	41.84	41.83	44.85	50.08	45.44	41.5	45.07	38.9	48
200	42.83	44.58	43.97	46.28	51.19	47.54	43.15	46.39	42.96	50.15
250	48.93	50.57	50.48	54.04	60.4	54.87	50.01	54.32	47.16	57.81

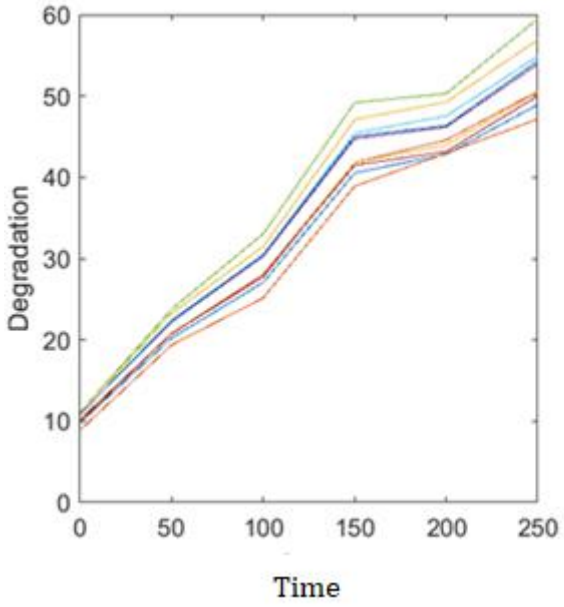

Figure1:Plot of the degradation data for a samplesize n=10 measured at 6 degradation times for each unit with Amoeba method

And then, the reliability function R(t) was evaluation based on Eq. (5) according to the Amoeba method of estimating the parameters, the results of R(t) are as shown in Table 3 and Figure 2.

Table 3: Reliability functionwith n=10through estimating the parameters by Amoeba methodat 6 degradation times (0,50,150,200,250)

R(t)	**Units**									
	1	2	3	4	5	6	7	8	9	10
R(0)	1	1	1	1	1	1	1	1	1	1
R(50)	0.6497	0.8318	0.9324	0.8514	0.8733	0.6909	0.9169	0.9819	0.4392	0.6869
R(100)	0.4748	0.7029	0.7659	0.8106	0.6873	0.6691	0.5817	0.9097	0.4154	0.5383
R(150)	0.4096	0.4159	0.7346	0.6854	0.6622	0.5392	0.5693	0.6934	0.3156	0.5338
R(200)	0.1044	0.4004	0.2187	0.5332	0.6082	0.2887	0.4153	0.3806	0.2795	0.4951
R(250)	0.0399	0.2546	0.0997	0.3244	0.2746	0.1280	0.1995	0.3720	0.2732	0.1435

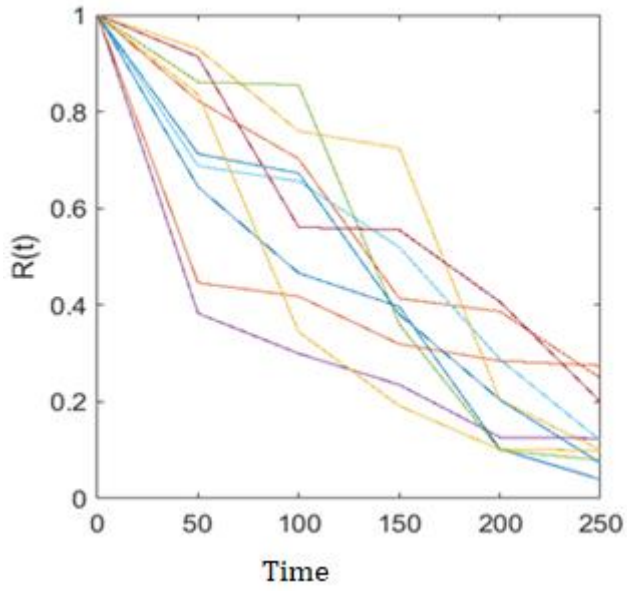

Figure2: Plot of the reliability function for each units of a sample of size n=10 with parameters estimated by theAmoeba method

From Table 3, we note that the reliability of each product decreases over time and this is according to the theory of reliability when testing the product at all times until it stops working.However, when the failure quantity (shown in the Table 2) reaches the failure threshold here (w = 30), the product will have failed completely by the time it exceeds the failure as in Table 4, it also shows the corresponding reliability of each product at the time of failure.

Table 4: Product failure time at the failure threshold with the corresponding reliability estimated R(t) by the Ameba method

Units	Failure time	R(t)
1	150	0.4096
2	150	0.4159
3	150	0.7346
4	100	0.8106
5	100	0.6873
6	100	0.6691
7	150	0.5693
8	150	0.6934

9	150	0.3156
10	100	0.5383

5.Conclusions

1. Through the results of the comparison of the MSE values of the parameters estimatedby using Monte Carlo simulation, the proposed Amoeba method to estimate parameters is the best because its estimators have the lowest values of the MSE compared to the method of MLE.
2. Degradation data showed the gradient of product failure and thus it is possible to estimate the reliability of the product at the time when the product actually fails without waiting for a complete stop when comparing the amount of deterioration that the product reaches to the failure threshold that is set previously.

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