New Vogel's Approximation Method (NVAM) to Determine Better Feasible Solution of Transportation Problem

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Abstract

Nowadays, getting substantial results for problems in operations research is crucial. The transportation problem can be solved very effectively with Vogel's Approximation Method (VAM) and discover a workable solution that is closer to the ideal solution. The basic principle of VAM is to allocate as many resources as possible to the cell with the smallest cost in the column or row with the greatest penalty. This cell will receive the maximum number of resources. A problem arises when the magnitudes of the least cost and the next-least cost are identical. Then, we devised a new method called the "New Vogel's Approximation Method (NVAM)" to provide a workable solution to the transportation problem that may then be used to determine the optimal solution. Transporting capacity to the required location is done by three distinct companies. The supply chain's participants seek to move their goods from warehouse to distribution centre, from plant to warehouse, and from distribution centre to retail store. In this study, the VAM method and the NVAM approach are used to determine the optimal transportation cost. These techniques are examined to provide potential supply chain management techniques.

Keywords — Transportation Problem (TP), VAM, NVAM, Feasible Solution, Penalty Cost

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1. Introduction

Real-world transportation problems occur when goods are moved from factories to retail stores with the goal of reducing overall transportation costs [1]. In operations research, the VAM is regarded as the most effective process to tackle the special class of linear programming problems known as TP [2]. In [3-6], Manu authors create an effective algorithm for locating a first-level, workable solution to a transportation problem (TP). In logistics and supply chains, transportation models are crucial. The main challenge is figuring out how much it will cost to convey a single product from several sources to various destinations [7]. The main purpose is to keep shipping costs to a minimum such that each shipping point operates within its capacity and the needs of each arriving region are met [8]. Figure 1 [9] depicts a network representation of the transportation problem. By utilizing the

capabilities of m supply points, it attempts to determine the most effective manner to satisfy the need of n demand points.
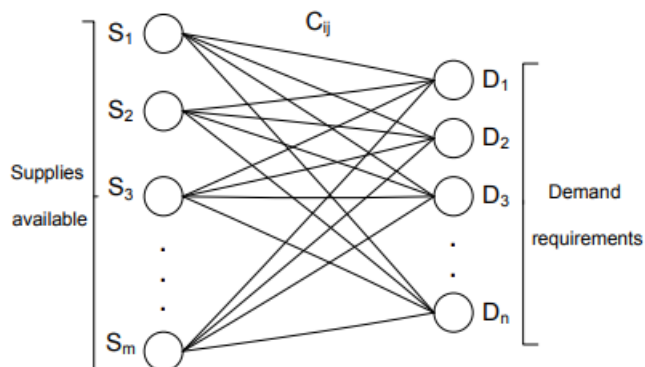

Figure 1. The transportation problem's network model

The entire opportunity cost matrix in [10] is subjected to VAM. Moreover to this approach, [11] improved VAM (IVAM) calculates alternative allocation costs while taking into account the highest three penalty costs. Following that, it chooses at least one of them. Therefore, our proposed approach is more effective than other ways like the Least Cost Method (LCM), North West Corner Rule (NWC), etc. related to the notion of penalty cost However, in some situations, the method for calculating the penalty cost is illogical. We explored the same kind of circumstance in [-,-] and created a new algorithm with a notion that provides a workable solution which is almost as good as the ideal answer and is less expensive than VAM. Hence Limitations are solved in [-,-].

Take into consideration a TP with n destinations and m sources, where C_{ij} represents the unit transportation expense from source i^{th} to destination j^{th}. Let D_j be the amount of demand at destination j^{th} and S_i be the amount of supply at source i^{th}. To reduce overall transportation costs, we need to determine the quantity of commodity x_{ij} that is transported. The aforementioned LPP problem is given as:

Minimize

$$\text{Total cost} = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

$$\sum_{i=1}^{m} x_{ij} \geq D_j \text{ for } j = 1,2,\dots n \text{ (Demand constraints)}$$

$$\sum_{j=1}^{n} x_{ij} \leq S_i \text{ for } i = 1,2,\dots m \text{ (Supply constraints)}$$

$$\sum_{j=1}^{n} C_{ij} x_{ij}$$

$$x_{ij} \geq 0 \qquad \forall \, i,j$$

To discover a workable solution, This LPP must be converted into the subsequent mathematical model and apply transportation techniques.

2. Vogel's Approximation Method (VAM)- Existing Algorithm:

A basic feasible solution to a TP can be determined using the iterative Vogel Approximation approach. The Least Cost Strategy and the North West Corner Rule are not favoured over this method. The following is the VAM algorithm:

Step-1:

a. Find the cells in each row that have the lowest and next-lowest transportation costs, and then note the difference (Penalty) against the corresponding row on the table's side.

b. Find the cells in each column that have the lowest and next-lowest transportation costs, and then note the difference (Penalty) next to the corresponding column on the table's side.

Step-2: If the least cost appears twice or more in a column or row, choose that cost as the least, and the next-to-least penalty and cost will be zero.

Step-3:

a. By arbitrarily breaking ties, determine the column and the row with the biggest penalty. The variable in the chosen column or row should receive as much of the total allocation as possible. Eliminate the satisfied column or row and adjust the demand and supply. Crossing one of them and the remaining column or row is given a zero demand or supply if a column or row fulfil concurrently.

b. Choose any one of the penalty costs if more than two of them have the same greatest magnitude (or extreme left column or select most top row).

Step-4:

a. Stop if exactly one column or row has neither zero demand nor zero supply marked out.

b. If there is still just one uncrossed-out column or row with positive demand or supply, calculate the fundamental variables in the column or row using the least-cost method.

c. The Least-Cost Method should be used to determine the zero fundamental variables if there is no demand or supply in any uncrossed-out columns or rows. Stop.

d. If not, proceed to Step 1.

3. New Vogel's Approximation Method (NVAM)- Proposed Algorithm:

In this section, we presented the "New Vogel's Approximation Method (NVAM)," a new algorithm. In our suggested algorithm (NVAM), the penalty is defined by the ratio of two smallest costs, taking one of them as a minimum and the following least cost other than equal lowest costs as a next to minimum. This applies when the least cost appears two or more times in a column or row. As an illustration, if a column or row's costs are 3, 10, 3, 7, and 9, choose 3 as the least cost, then choose 7 as the next-smallest cost in place of 3 once more, then the penalty will be 0.42. The NVAM algorithm is provided as follows:

Step-1: Set the supply amount (S_i), demand amount (D_j), and unit transportation cost (C_{ij}) for the source (i) and destination (j), respectively.

Verify: Stop if $S_i < 0$ and $D_j < 0$.

Step-2:

a.

$$\sum_i S_i > \sum_j D_j$$

or if

$$\sum_i S_i < \sum_j D_j$$

then adjust the supply or demand to balance the transportation problem.

b. Make all dummy columns or rows $C_{ij} = 0$.

Step-3:

a. Determine the lowest and next-lowest cost for each column and row, and then determine the ratio between them, known as the penalty.

Row and column penalties are set using P_i and P_j, respectively.

$P_i = \frac{C_{ih}}{C_{ik}}$ and $P_j = \frac{C_{hj}}{C_{kj}}$

where $C_{ih} \leq C_{ik}$ and $C_{hj} \leq C_{kj}$.

b. If the smallest cost appears twice or more in a column or row, choose the smallest cost that is not the same as the next-smallest cost.

c. Choose the least and next to least costs as equal, and the penalty will be one, if there are no additional costs beyond the smallest costs that are equal.

Step-4:

a. Choose $\max(P_i, P_j)$. Set x_{ij}: Quantity of the positive coming from source (i) to destination (j); Choose the cost that has the lowest penalty in the column or row, and then allot the maximum amount x_{ij}, or $\min(S_i, D_j)$. Choose whichever cell in the column or row has the lowest price if it appears in two or more cells there.

b. Choose any one of the penalty costs if two or more of them have the same highest magnitude (or select extreme column or top most row).

Step-5: Eliminate the satisfied column or row and adjust the demand and supply. If a column and a row are both satisfied at once, cross out one of them and set zero demand or supply in the other.

Step-6:

a. Stop if exactly one column or one row still has no demand or supply.

b. Identify the fundamental variables in the column or row if there is just one column or row with positive demand or supply that has not been crossed out.

c. Determine the zero fundamental variables if there is zero demand or supply in every uncrossed-out column or row. Stop.

d. If not, proceed to Step 3.

4. Numerical Simulation

Considering some particular sorts of TP where the least cost appears in two or more columns or rows and solving them using the NVAM and compare the results to the VAM.

4.1. Example-1

A Company has three Distribution centres at location S1, S2, S3 ,which supply to Retail stores placed at D1, D2, D3, D4 , D5 .Weekly Distribution centrest capacities are 100,120,120 units respectively. Weekly Retail stores e requirements are 40,50,70,90,90 units respectively .Unit transportation cost (in rupees) given below:

Source	Destination					Supply
	D1	D2	D3	D4	D5	
S1	4	1	2	6	9	100
S2	6	4	3	5	7	120
S3	5	2	6	4	8	120
Demand	40	50	70	90	90	

Now, use NVAM and VAM, respectively, to solve the following problem:

4.1.1 Solution of Example-1 Using NVAM:

Table-1

	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	4	1	2	6	9x	**100**	½=0.5
S_2	6	4	3	5	7(90) x	**120**	¾=0.75
S_3	5	2	6	4	8 x	**120**	2/4=0.5
Demand	**40**	**50**	**70**	**90**	**90**		
Column Penalty	4/5=0.8	½=0.5	2/3=0.7	4/5=0.8	7/8=0.9		

Column D5 represents the maximum penalty, which is 0.9.

This row's minimum C_{ij} is C_{25}=7.

This cell's maximum allocation is min (90,120) = **90**.

It satisfies D5's demand and reduces S2's supply from 120 to 30 (120 - 90=30).

Table-2

	D_1	D_2	D_3	D_4	D_5	**Supply**	**Row Penalty**
S_1	4	1	2	6 x	9x	**100**	½=0.5
S_2	6	4	3	5 x	7(90) x	**30**	¾=0.75
S_3	5	2	6	4(90) x	8 x	**120**	2/4=0.5
Demand	**40**	**50**	**70**	**0**	**0**		
Column Penalty	**4/5=0.8**	**½=0.5**	**2/3=0.7**	**4/5=0.8**	**-**		

Row $S2$ represents the maximum penalty, which is 0.8.

This row's minimum C_{ij} is C_{25}=7.

This cell's maximum allocation is min (90,120) = **90**.

It satisfies S3's supply and reduces D4's demand from 120 to 30 (120 - 90=30).

Table-3

	D_1	D_2	D_3	D_4	D_5	**Supply**	**Row Penalty**
S_1	4(40) x	1	2	6 x	9x	**100**	½=0.5
S_2	6 x	4	3	5 x	7(90) x	**30**	¾=0.75
S_3	5 x	2	6	4(90) x	8 x	**30**	2/5=0.4
Demand	**0**	**50**	**70**	**0**	**0**		
Column Penalty	**4/5=0.8**	**½=0.5**	**2/3=0.7**	**-**	**-**		

Column D1 represents the maximum penalty, which is 0.8.

This row's minimum C_{ij} is C_{11}=4.

This cell's maximum allocation is min (40,100) = **40**.

It satisfies D1's demand and reduces S1's supply from 100 to 60 (100 - 40=60).

Table-4

	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	4(40) x	1	2	6 x	9x	60	½=0.5
S_2	6 x	4 x	3 (30) x	5 x	7(90) x	0	¾=0.75
S_3	5 x	2	6	4(90) x	8 x	30	2/6=0.3
Demand	0	50	40	0	0		
Column Penalty	-	½=0.5	2/3=0.7	-	-		

Row S2 represents the maximum penalty, which is 0.75.

This row's minimum C_{ij} is C_{23}=3.

This cell's maximum allocation is min (70,30) = **30**.

It satisfies D3's demand and reduces S2's supply from 70 to 40 (70 - 30=40).

Table-5

	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	4(40) x	1 (50) x	2	6 x	9x	10	½=0.5
S_2	6 x	4 x	3 (30) x	5 x	7(90) x	0	-
S_3	5 x	2 x	6	4(90) x	8 x	30	2/6=0.3
Demand	0	0	40	0	0		
Column Penalty	-	½=0.5	2/6=0.3	-	-		

Row S1 and Column D2 represents the maximum penalty, which is 0.75.

This row's minimum C_{ij} is C_{12}=1.

This cell's maximum allocation is min (50,60) = **50**.

It satisfies D2's demand and reduces S1's supply from 60 to 10 (60 - 50=10).

Table-6

	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	4(40) x	1 (50) x	2	6 x	9x	10	2
S_2	6 x	4 x	3 (30) x	5 x	7(90) x	0	-
S_3	5 x	2 x	6 (30) x	4(90) x	8 x	0	6
Demand	0	0	10	0	0		
Column Penalty	-	-	2/6=0.3	-	-		

Row S3 represents the maximum penalty, which is 6.

This row's minimum C_{ij} is C_{33}=6.

This cell's maximum allocation is min(40,30) = **30**.

It satisfies D3's demand and reduces S3's supply from 40 to 10 (40 - 30=10).

Table-7

	D_1	D_2	D_3	D_4	D_5	Supply	Row Penalty
S_1	4(40) x	1 (50) x	2 (10) x	6 x	9x	0	2
S_2	6 x	4 x	3 (30) x	5 x	7(90) x	0	-
S_3	5 x	2 x	6 (30) x	4(90) x	8 x	0	0
Demand	0	0	0	0	0		
Column Penalty	-	-	2	-	-		

Row S1 and Column D3 represents the maximum penalty, which is 6.

This row's minimum C_{ij} is C_{23}=2.

This cell's maximum allocation is min(10,10) = **10**.

It satisfies D3's demand and reduces S3's supply from 10 to 0 (10 - 10=0).

Total Transportation Cost:

$(4 \times 40) + (1 \times 50) + (2 \times 10) + (3 \times 30) + (7 \times 90) + (6 \times 30) + (4 \times 90) = 1490.$

4.1.2 Solution of Example-1 Using VAM:

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	4 30	1 70	2	6	9	**100**
S_2	6	4	3 30	5 90	7	**120**
S_3	5 10	2 50	6	4 60	8	**120**
Demand	**40**	**50**	**70**	**90**	**90**	

Total Transportation Cost:

$(4 \times 30) + (2 \times 70) + (5 \times 90) + (7 \times 30) + (5 \times 10) + (2 \times 50) + (8 \times 60) = 1550$

4.2. Example-2

A Company has three warehouse at location S1, S2, S3, which supply to distribution centre placed at D1, D2, D3, D4. Weekly warehouse capacities are 50, 60, 25 units respectively. Weekly distribution centre requirements are 60, 40, 20, 15 units respectively. Unit transportation cost (in rupees) given below:

Source	Destination				Supply
	D1	D2	D3	D4	
S1	3	2	7	6	**50**
S2	7	5	2	3	**60**
S3	2	5	4	5	**25**
Demand	**60**	**40**	**20**	**15**	

Now, use NVAM and VAM, respectively, to solve the following problem:

4.2.1 Solution of Example-2 Using NVAM:

Source	Destination				Supply
	D1	**D2**	**D3**	**D4**	
S1	3 10	2 40	7	6	**50**
S2	7 25	5	2 20	3 15	**60**
S3	2 25	5	4	5	**25**
Demand	**60**	**40**	**20**	**15**	

Total Transportation Cost:

$$(3 \times 10) + (2 \times 40) + (7 \times 25) + (2 \times 20) + (3 \times 15) + (2 \times 25) = 420$$

4.2.2 Solution of Example-2 Using VAM:

Source	Destination				Supply
	D1	**D2**	**D3**	**D4**	
S1	3 10	2 40	7	6	**50**
S2	7 25	5	2 20	3 15	**60**
S3	2 25	5	4	5	**25**
Demand	**60**	**40**	**20**	**15**	

Total Transportation Cost:

$$(3 \times 10) + (2 \times 40) + (7 \times 25) + (2 \times 20) + (3 \times 15) + (2 \times 25) = 420$$

4.3. Example-3

A Company has three plants at location S1, S2, S3, which supply to warehouse located at D1, D2, D3. Weekly plant capacities are 7,12,11 units respectively. Weekly warehouse requirements are 10, 10, 10 units respectively. Unit transportation cost (in rupees) given below:

Source	Destination			Supply
	D1	D2	D3	
S1	1	2	6	**7**
S2	0	4	2	**12**
S3	3	1	5	**11**
Demand	**10**	**10**	**10**	

Now, use NVAM and VAM, respectively, to solve the following problem:

4.3.1 Solution of Example-3 Using NVAM:

Source	Destination			Supply
	D1	D2	D3	
S1	1 7	2	6	**7**
S2	0 2	4	2 10	**12**
S3	3 1	1 10	5	**11**
Demand	**10**	**10**	**10**	

Total Transportation Cost:

$$(1 \times 7) + (0 \times 2) + (2 \times 10) + (3 \times 1) + (1 \times 10) = 40$$

4.3.2 Solution of Example-3 Using VAM:

Source	Destination			Supply
	D1	D2	D3	
S1	1 7	2	6	**7**
S2	0 2	4	2 10	**12**
S3	3 1	1 10	5	**11**
Demand	**10**	**10**	**10**	

Total Transportation Cost:
$$(1 \times 7) + (0 \times 2) + (2 \times 10) + (3 \times 1) + (1 \times 10) = 40$$

5. Observation and findings

The New Vogel's Approximation Method (NVAM) provides an equivalent or less viable answer to VAM, as shown in the examples above. Results obtained using NVAM are very near to or the same as the ideal result. The following is a comparison table of the NVAM and VAM results:

Example	Transportation Problems		
	Optimal Solution	NVAM	VAM
1	1400	1490	1550
2	395	420	420
3	40	40	40

6. Conclusion

This work proposes a new method called "New Vogel's Approximation Method (NVAM)" to extend the feasible solution in VAM that is widely used transportation needs in supply chain. It is evident from the above instances that NVAM provides the same realistic alternative as VAM and that this approach may also be used to identify an optimal situation or an answer that comes close to being optimal.

References

[1] Hillier, F. S. and G. J. Lieberman. (1995). Introduction to Operations Research, 6th ed. New York: McGraw-Hill, Inc.

[2] Islam, M. A., Khan, A. R., Uddin, M. S., & Malek, M. A. (2012). Determination of basic feasible solution of transportation problem: a new approach. Jahangirnagar University Journal of Science, 35(1), 101-108.

[3] Khan, A. R. (2011). A Re-solution of the Transportation Problem: An Algorithmic Approach, Jahangirnagar University Journal of Science, Vol. 34, No. 2, pp. 49-62 ISSN 1022-8594.

[4] Babu, M. A., Helal, M. A., Hasan, M. S., & Das, U. K. (2013). Lowest Allocation Method (LAM): a new approach to obtain feasible solution of transportation model. International Journal of Scientific and Engineering Research, 4(11), 1344-1348.

[5] Das, U. K., Babu, M. A., Khan, A. R., & Uddin, M. S. (2014). Advanced Vogel's Approximation Method (AVAM): a new approach to determine penalty cost for better feasible solution of transportation problem. International Journal of Engineering Research & Technology (IJERT), 3(1), 182-187.

[6] Shore, H. H. (1970). The Transportation Problem and the Vogel Approximation Method, Decision Sciences 1(3-4), 441–457.

[7] Taha, H. A. (1987). Operations Research: An Introduction, New York: Macmillan Publishing Company.

[8] Reeb, J. E. & Leavengood, S. (2002). Transportation problem: A special case for linear programming, Oregon: Oregon State University Extension Service Publications EM 8779.

[9]   Wagner, H. (1969). Principles of Operations Research, New Jersey: Prentice-Hall, Englewood Cliffs.

[10] Mathirajan, M. & Meenakshi, B. (2004). Experimental Analysis of Some Variants of Vogel's Approximation Method, Asia-Pacific Journal of Operational Research 21(4), 447-462.

[11] Korukoğlu, S., & Ballı, S. (2011). An improved Vogel's approximation method for the transportation problem. Mathematical and Computational Applications, 16(2), 370-381.