The Study Analogue of Harnack's Theorem and Some Properties of $A(z)$ Harmonic Functions

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Abstract: In this paper we provide a definition of $A(z)$- tasks of harmonic and Devoted some properties of $A(z)$-harmonic task, and analogue of Harnack's Theorem.

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Introduction

This work is concerned with A(z)-harmonic tasks. The answer to the Beltrami equation

$$\frac{\partial f(z)}{\partial \bar{z}} - A(z)\frac{\partial f(z)}{\partial z} = 0 \qquad (1)$$

Known as the analytical task of A(z). It is widely knowledge that the link between equation (1) and Quasiconformal mappings is direct. There is a common misconception that A(z) is a measurable task and that | A (z)|C1 virtually anyin which in the area DC. Actual part of the Equation for the Solution (1)

$$u(z) = Ref(z)$$

The composition comprises of an opening and three body paragraphs. In the first paragraph, we provide a basic overview of the A(z)- analytic tasks, which will be covered in greater detail in subsequent sections on the A(z)- harm-onic task. In the next paragraph, we define A (z) harm-onic tasks, introduce the comparable Laplace operator Δ A u, and describe the taskal features, Poisson integral formula, and mean value theorem for A(z)-harmonic tasks. The third paragraph discusses Harnack's inequality and theorem on monoto-nically sequences of A(z)-harmonic tasks

$$u_j \in h_A(D)$$

1. Preliminary information

Both the solution to equation (1) and the quasiconformal homeomorphisms of Flat areas have been thoroughly investigated. We limit ourselves here to work citations ([1, 6, 8, and 11]) and the formulation of the three theorems given below:

First theorem: For each complex-measurable ℂ task, There is a one of homeom-orphic X(z) solution to the first equation that fixes the coordinates 0,1 as:

Observe that in the case of the last task is exclusively in the Area D⊂C definied, it may extend to the entire by putting it outside A=0, hence the first formulation of the 1st theorem applies for every area

$$A(z) : \|A\|_\infty < 1 \qquad D \subset C$$

$$f(z) = \Phi[X(z)],$$

$$X(z)$$

Second Theorem formulation [3]: in which is homeomorphic task, exhausts the collection of all generalized equation solutions (1). Solution according to the first Theorem, and Φ(ξ) is a homeomorphic task in area X.

Furthermore, in the case of the f(z) contains isolated singular points, (D). So, a holomorphic task possess the same types for isolated singularities.

Nota bene:

$$\Phi = fox^{-1}$$

According to Theorem 2, the A-analytic task f performs internal mapping.

That is, it transforms one open set to another.

Therefore, the maximum principle holds true for these tasks; given each confined area D⊂C, the modulus of f = constant reaches its maximum value only on that area Boundaries,

for example $|f(z)| < \max_{z\in\partial D}|f(z)|,\ z \in D$

Whenever the task is not 0, the minimal principle also holds.

For example $|f(z)| > \min_{z\in\partial z}|f(z)|,\ z\in D$

Third Theorem [6]. In the case of a task A(z) is based on a group of m-smooth class tasks A(z)∈C^m (D), so, each f solution for equation number1 and as belongs to the same class, here, let only consider the case in which A(z) stands for an anti-analytic task ∂A=0 in an area D⊂C also $|A(z)| \leq C < 1, (0 < C < 1), \forall z \in D$

$D_A = \frac{\partial}{\partial z} - \bar{A}(z)\frac{\partial}{\partial \bar{z}}$, $\bar{D}_A = \frac{\partial}{\partial \bar{z}} - A(z)\frac{\partial}{\partial z}$.

$$f \in C^m(D).$$

so we can get :

In the case of (1) is correct, so, the class of is A(z) - analytic function $f \in O_A(D)$ is defined by the fact that $\bar{D}_A f = 0$. It follows from Theorem 3 that the anti-analytic function $O_A(D) \subset C^\infty(D)$ is endlessly smooth (D).

Fourth Theorem. [**11**]. (*Analogue of Cauchy theorem*). In the case of in which $D \subset \mathbb{C}$ is an area contain piecewise smoothly boundaries ∂D, and in the case of the area D is connected as a fixed point $\xi \in D$ simply, so, :

$$\int_{\partial D} f(z)(dz + A(z)d\bar{z}) = 0$$

$$\psi(z, \xi) = z - \xi + \overline{\int_{\gamma(\xi,z)} \bar{A}(\tau)}$$

$$I(z) = \int_{\gamma(\xi,z)} \bar{A}(\tau)d(\tau)$$

is accurately specified in an area D, in which γ(ξ , z) has been a smooth curve involving the points, ξ , z∈D. An integral of

is a, because the area D is merely connected, and A (z) stands for a holomorphic function.

It has been integration path, and corresponds with an anti-derivative,

Theorem 5. [**10**]. In the event that D is merely a connected and convex region, the kernel-style task

$$I'(z) = \bar{A}(z) \qquad (2)$$

$$k(z, \xi) = \frac{1}{2\pi i} \cdot \frac{1}{z - \xi + \overline{\int_{\gamma(\xi,z)} \bar{A}(\tau)}\, d\tau}$$

Is there A(z)-analytic task out of a point in which z=ξ In the case of so, so, k∈ O_A (D\{ξ}) is an answer; also, z=ξ the task k(z,ξ) is easy task at z=ξ.

Proof. A simple check shows that the task

$$\psi(z,\xi) = z-\xi+\overline{I(z)} = z-\xi+\overline{\int_{\gamma(\xi\ ,z)}\bar{A}(\tau)d\tau},$$

is $A(z)$ – analytic in D:

$$\frac{\partial}{\bar{z}}\left[\, z-\ \xi\ \ +\overline{I(z)}\,\right]\frac{\partial}{\bar{z}}\overline{I(z)}=\frac{\overline{\partial I(Z)}}{\partial z}\,\bar{A}(z)\frac{\partial}{z}\left[\, z-\ \xi\ \ +\overline{I(z)}\,\right]$$

i.e. $\psi(z,\xi) \in O_A(D)$.

The task $\psi(\xi,z) = z-\xi+\overline{\int_{\gamma(\xi\ ,z)}\bar{A}(\tau)d\tau}$ has a unique simple zero at the point $z=\xi$. In fact, $|\xi,z|$ is a segment which connects the points $\xi,z\in D$, so,

$$z-\xi+\overline{\int_{\gamma(\xi\ ,z)}\bar{A}(\tau)d\tau} = z-\xi+\overline{\int_{|\xi,z\,|}\bar{A}(\tau)d\tau}$$

and since $|A(z)|\le c<1$, we have

$$\left|z-\ \xi+\overline{\int_{\gamma(\xi\ ,z)}\bar{A}(\tau)d\tau}\right| \ge |\ z-\xi| - \left|\ \int_{|\xi,z\,|}\bar{A}(\tau)d\tau\right| \ge$$

$$\ge |\ z-\xi| - \int_{|\xi,z\,|}|\ A(\tau)|\,|\ d\tau| \ge |\ z-\xi| - c\cdot\int_{|\xi,z\,|}\ |\ d\tau| = (1-c)|\ z-\xi| > 0,$$

$$z\neq\ \xi.$$

the task $\psi(z,\xi)$ has only one zero and it is simple at the point $z=\xi$, therefore, k (z, ξ) is holomorphic in $D\setminus\{\xi\}$. $z=\xi$ is its simple pole.

Remark 1. Notably, area D has been convex; K (z, ξ) possesses a simple single-pole point z = ξ. In the case of region D C has not been Convex and it is merely simple-Linked, regardless of the tasks:

$$\psi(\xi,z) = \xi - \mathrm{z} + \overline{\int_{\gamma(z\ ,\xi)}\bar{A}(\tau)d\tau}$$

Theorem 6: Let $D\subset\mathbb{C}$ be any arbitrary convex area, and let $G\subset D$ be any arbitrary subarea with a smooth or piecewise smooth border ∂G.

Therefore, the formula (3) applies to any task f(z) ∈ $O_A(G)\cap C(\bar{G})$

(3) $$f(z) = \int_{\partial G} K\ (\xi\ ,z\ )\, f(\ \xi\ )\ (d\xi + A(\xi)d\bar{\xi}\ )\quad ,\mathrm{z}\in G\ \ .$$

Proof. Fixing a point z ∈ G and small circle U (z , ε) $\subset\ G$, $\varepsilon>0$, the following theorem holds: (4)

$$\int_{\partial G} K(\xi, z) f(\xi)(d\xi + A(\xi)d\bar{\xi})$$

$$= \int_{|\xi - z| = c} K(\xi, z) f(\xi)(d\xi + A(\xi)d\bar{\xi}),$$

but according to the Stokes formula we have:

$$\int_{|\xi - z| = \varepsilon} K(\xi, z) f(\xi)(d\xi + A(\xi)d\bar{\xi}) = \int_{|\xi - z| = \varepsilon} f(\xi) w(\xi, z) =$$
$$\int_{|\xi - z| \le \varepsilon} d[f(\xi) w(\xi, z)] = \int_{|\xi - z| \le \varepsilon} df(\xi) w(\xi, z) +$$
$$\int_{|\xi - z| \le \varepsilon} f(\xi) dw(\xi, z)$$
$$\longrightarrow 0 + f(z) = f(z), for\ \varepsilon \longrightarrow 0$$ ■

2. *A*(z)−harmonic task

As stated earlier, the A(z)-harmonic task is the real component of A(z)-analytical tasks. The imaginary component of the analytical task is harmonic. A(z)-harmonic tasks exist when A(z) represents anti-analytic tasks.

Theorem 7: The real component of the analytic task f(z) ∈ $O_A(G)$ satisfies the following equations.

$$\Delta_A u = 0 \qquad (4)$$

in which

$$\Delta_A = \frac{\partial}{\partial z}\left[\frac{1}{1-|A|^2}\left[(1 + |A^2|)\frac{\partial u}{\partial z} - 2A\frac{\partial u}{\partial z}\right]\right] + \frac{\partial}{\partial z}\left[\frac{1}{1-|A|^2}\left[(1 + |A^2|)\frac{\partial u}{\partial z} - 2\bar{A}\frac{\partial u}{\partial z}\right]\right].$$

Note Theorem 7 gives the following determinations for the A(z)-harmonic task.

Definition 1.

In area G, a task of twice differentiable function $u \in C^2(G), u : G \to R$ is A(z)-harmonic if, called *A*(z)−harmonic, it is a solution to the differential equation (4).

$h_A(G)$is the symbol for a class for A(z)-harmonic tasks in area (G), and both the real and imaginary components of the A(z)-analytic task $f(z) \in O_A(G)$ are A(z)-harmonic tasks. Likewise, the opposite is true for Areas with a simple link.

Theorem 8. *f(z)*∈ $O_A(G$, such that u =Re f, exists in the case of the task $u(z) \in h_A(G)$, (G), in which G is a simply connected area.

For A(z)-harmonic tasks, theoretically, operator *A*(z)−has the similar role as u operator concerning harmonic and subharmonic tasks (Namely, we must provide the integral principle. Assume G⊂C to be the convex area, and let

$$\psi(z, \xi) = z - \xi + \overline{\int_{\gamma(\xi, z)} \bar{A}(\tau) d\tau}$$

correspond to the appropriately defined task for G.

Theorem 9. Poisson's formula (by Poisson's Theorem) holds in the case of a task u(z) has been A(z)-harmonic in the lemniscate L(a,R)⊂D, continuous in its closure, specifically u(z)∈h_A (L(a,R))∩C(L ̅(a,R))

(5) $$u(z) = \frac{1}{2\pi r}\oint_{|\psi(\xi,a)|=R} \mathrm{u}(\xi)\frac{r^2-|\psi(\xi,a)|^2}{|\psi(\xi,z)|^2}\left| d\xi + A(\xi)d\bar{\xi}\right|.$$

Other side in the case of the tasks φ(ξ) continuous at the boundaries of the lemniscate $L(a,R) \subset D$, So, the Task:

$$u(z) = \frac{1}{2\pi r}\oint_{|\psi(\xi,a)|=R} \varphi(\xi)\frac{r^2-|\psi(\xi,a)|^2}{|\psi(\xi,z)|^2}\left| d\xi + A(\xi)d\bar{\xi}\right| \qquad (6)$$

Is the Lemniscate a Solution to the Dirichlet Problem:

$L(a,r)$: $\Delta_A u = 0 \ \forall\ z \in L(a,R)$, $u|\partial L(a,R) = \varphi$.

Theorem 10. in the case of task u is an $A(z)$ –harm-onic in a Lemn-iscate

$$L(z,R) = \{\xi \in G: |\psi(z,\xi)| < R\} \subset G$$

so, the following equality holds for any r $< R$:

$$u(z) = \frac{1}{2\pi r}\oint_{|\psi(\xi,z)|=r} \mathrm{u}(\xi)\left| d\xi + A(\xi)d\bar{\xi}\right|.$$

Proof. Since $u \in h_A(L(z,R))$ so, there is a task $f(z) \in O_A(L(z,R))$ for which $u(z) = Rf(z)$.

we expand the task $f(z)$ in the area $L(z,R)$ in a

Taylor series

$$f(\mathrm{z}) = \sum_{n=0}^{\infty} c_n\,\psi^n(\xi, z).$$

If $r < R$, the Series converges uniformly in a lemniscate $|\psi(\xi,z)| \le r$.

$$u(z) = \frac{1}{2}\left(f(\mathrm{z}) + \overline{f(\mathrm{z})}\right) = \frac{1}{2}\sum_{n=0}^{\infty}\left[c_n\psi^n(\xi,z) + \overline{c_n\psi^n(\xi,z)}\right] \qquad (7)$$

using $d\psi(\xi,z) = d\xi + A(\xi)d\bar{\xi} = rie^{it}dt,\ 0 \le t \le 2\pi$

and $\left|d\xi + A(\xi)d\bar{\xi}\right| = rdt$,

Compute the following Integrals:

$$\oint_{|\psi(\xi,z)|=r} \psi^n(\xi,z)\left| d\xi + A(\xi)d\bar{\xi}\right| = r^{n+1}\int_0^{2\pi} e^{tin}\, dt = \begin{cases} 0, & n \geq 1 \\ 2\pi r, & n = 0 \end{cases}$$

$$\oint_{|\psi(\xi,z)|=r} \overline{\psi^n(\xi,z)}\left| d\xi + A(\xi)d\bar{\xi}\right| = r^{n+1}\int_0^{2\pi} e^{-tin}\, dt = \begin{cases} 0, & n \geq 1 \\ 2\pi r, & n = 0 \end{cases}$$

Integrating the part-equality (15) in a perimeter for lemniscate yields a subsequent equivalence:

$$\oint_{|\psi(\xi,z)|=r} \mathrm{u}(\xi)\left| d\xi + A(\xi)d\bar{\xi}\right| = \pi r(c_0 + \bar{c}_0) = 2\pi r u(z) \qquad \blacksquare$$

Theorem 11. (Fubini’s Theorem). In the case of $f(x,y)$ is continuous throughout the rectangular region, R: $a \leq x \leq b$, $c \leq y \leq d$, $(a,b,c,d \in R)$,so,

$$\iint\limits_R f(x,y)dA = \int\limits_c^d \int\limits_a^b f(x,y)dxdy = \int\limits_a^b \int\limits_c^d f(x,y)dydx.$$

Theorem 12. Task $u \in C(G)$, the subsequent Statements have been Comparable:

1) $u \in h_A(D)$;
2) for any $z \in G$ and $L(z,r) \subset\subset$ G the following equality holds

   $u(z) = \frac{1}{2\pi r}\oint_{|\psi(\xi,z)|=r} \mathrm{u}(\xi)\left| d\xi + A(\xi)d\bar{\xi}\right|;$
3) for any $z \in G$ and $L(z,r) \Subset$G the following equality holds

   $u(z) = \frac{1}{\pi r^2}\iint_{|\psi(\xi,z)|\leq r} \mathrm{u}(\xi)\mathrm{d}\mu$ (8)

   $Where$ $\mathrm{d}\mu(1 - |A(\xi)|^2)\frac{d\xi \wedge d\bar{\xi}}{2i}$.

Proof. 1 ⟹ 2 based on a mean magnitude, A theory (10). 2 3 stems based on Fubini's familiar formula. Hypothesis (11):

$$\frac{1}{\pi r^2}\iint_{|\psi(\xi,z)|\leq r} \mathrm{u}(\xi)\mathrm{d}\mu = \frac{1}{\pi r^2}\int_0^r dt \int_{|\psi(\xi,z)|=t} \mathrm{u}(\xi)\left| d\xi + A(\xi)d\bar{\xi}\right| =$$

$$= \frac{1}{\pi r^2}\int_0^r 2\pi t u(z)dt = u(z).$$

here we are using the following obvious equality,

$$\mathrm{d}\mu\,(1-|A(\xi)|^2)\frac{d\xi\wedge d\bar{\xi}}{-2i}=\frac{i}{2}\left(d\xi+A(\xi)d\bar{\xi}\right)\wedge\left(d\bar{\xi}+\bar{A}(\xi)d\bar{\xi}\right)=$$

$$=\frac{i}{2}d\psi(\xi,z)\wedge d\bar{\psi}(\xi,z)=dt\otimes|d\psi(\xi,z)|=dt\otimes\left|d\xi+A(\xi)d\bar{\xi}\right|.$$

Fix a lemniscate $L(a,R)\subset G$ to prove that 3⟹1is true. Apply the poisson formula (5) to the task

$$v\in h_A(L(a,R))\cap C(\bar{L}(a,R)) : \mathrm{v}|_{\partial L(a,R)}=u|_{\partial L(a,R)}$$

Using the auxiliary task $u_1=v-u$, in which $u_{1|_{\partial L(a,R)}}=0$.

To every $L(z,r)\subset\subset L(a,R)$, equality (8) holds since $v(z)\in h_A(L(a,R))$ and $u(z)$ fulfil the Theorem condition. From the following can be inferred the required statement.

Lemma 1. in the case of the mean value condition 3 for task $u\in C(G)$, u≢ const, has been True, i.e. for every z∈G and $L(a,R)\subset\subset$ G , the equivalence (8) holds, so, u(z) cannot attain its maximum or minimum magnitude within G.

Proof. Indeed, presume:

$$\exists z^0\in G : u(z^0)=\sup_G u(z),$$

fix $L=L(z^0,r)\subset G,$

and write the equality (8)

$$u(z^0)=\frac{1}{\pi r^2}\iint_L \mathrm{u}(\xi)\mathrm{d}\mu=\frac{1}{\pi r^2}\iint_{L\cap\{u(\xi)=u(z^0)\}}\mathrm{u}(\xi)\mathrm{d}\mu+\frac{1}{\pi r^2}\iint_{L\cap\{u(\xi)<u(z^0)\}}\mathrm{u}(\xi)\mathrm{d}\mu$$

$$=\frac{1}{\pi r^2}\iint_{L\cap\{u(\xi)=u(z^0)\}}\mathrm{u}(z^0)\mathrm{d}\mu+\frac{1}{\pi r^2}\iint_{L\cap\{u(\xi)<u(z^0)\}}\mathrm{u}(\xi)\mathrm{d}\mu$$

$$=\frac{1}{\pi r^2}\iint_L \mathrm{u}(z^0)\mathrm{d}\mu-\frac{1}{\pi r^2}\iint_{L\cap\{u(\xi)<u(z^0)\}}\mathrm{u}(z^0)\mathrm{d}\mu+\frac{1}{\pi r^2}\iint_{L\cap\{u(\xi)<u(z^0)\}}\mathrm{u}(\xi)\mathrm{d}\mu$$

$$=u(z^0)-\frac{1}{\pi r^2}\iint_{L\cap\{u(\xi)<u(z^0)\}}[u(z^0)-u(\xi)]\mathrm{d}\mu\,. \qquad (9)$$

Since $u(z^0)-u(\xi)\geq 0\ \forall\xi\in L(z^0,r)$ then from (9) it follows that

$L(z^0,r)\cap\{u(\xi)<u(z^0)\}=\emptyset$, **i.e.** $u(\xi)\equiv u(z^0)$ in (z^0,r) .

Changing u(z) to -u(z) reveals that the Minimum principle holds for u(z) under the conditions of Lemma 1, i.e., in the case of u(z) is less than u. (z)

$\exists z^0 \in G : u(z^0) = \inf_G u(z)$.

Then $u(\xi) \equiv u(z^0) \;\; \forall z \in G$. ■

It suffices to observe, to conclude the proof of Theorem (12), that the auxiliary Task $u_1 = v - u$, for which $u_1 \in C(\bar{L}(a,R))$ and $u_{1|_{\partial L(a,R)}} = 0$, the condition (3). Based on Lemma (1), $u_1 = v - u \equiv 0$ i.e.

$u(z) \equiv v(z) \in h_A(L(a,R))$. ■

Corollary 1. (Extremum principle). In the case of the task $u \in h_A(D)$ touches its extreme in G, so, $u \equiv$ constant.

Corollary 2. The Dirichlet problem $\triangle_A u(z) = 0 \; z \in G, u \in h_A(G) \cap C(\bar{G})$, $u|_{\partial G} = \varphi, \varphi \in C(\partial G)$ takes the distinctive solution.

Proof. Assume two solutions u_1 and u_2 are existing. So, their difference $v = u_1 - u_2 \in h_A(D)$ has been continuous in $\bar{D}$ and $v|_{\partial D} \equiv 0$. Therefore, by extremum standard $v|_D \equiv 0$,

For example, $u_1 \equiv u_2$.

■

3. Equivalence for Harnack's theorem

Remark 2. Here, the equivalence for familiar Harnack's Inequality has presented, that is vital in proving Harnack's eorem.

Theorem 13: Assume u (z) has the A(z)-harmonic task in a lemniscate. L (a, R) $\subset D$, continuous in its closure, specifically u(z)$\in h_A$ (L a, R))$\cap C(\bar{L}$(a, R)) , in which $D \subset \mathbb{C}$ has been convex area. In the case of u(z)≥ 0 in a lemniscate L (a, R), so, it will be right Harnack's inequality.

$$\frac{r-\rho}{r+\rho}u_j(a) \le u_j(z) \le \frac{r+\rho}{r-\rho}u_j(a), \quad z \in \partial L(a,\rho). \qquad (10)$$

Proof. In L(a, r) The Poisson's formula was written as (cf. (12))

$$u_j(z) = \frac{1}{2\pi r}\oint_{|\psi(\xi,a)|=r} u_j(\xi)\frac{r^2-|\psi(z,a)|^2}{|\psi(\xi,z)|^2}\left| d\xi + A(\xi)d(\bar{\xi}\right|, \ z \in \ L(a, r),\ j= 1,2\cdots$$

This formula implies the following inequality**:**

$$\frac{r^2-\rho^2}{(r+\rho)^2}u_j(a) \le u_j(z) \le \frac{r^2-\rho^2}{(r-\rho)^2}u_j(a), \quad z \in \partial L(a,\rho) = \{|\psi(z,a)| = \rho\}$$

Which is equivalent to

$$\frac{r-\rho}{r+\rho}u_j(a) \le u_j(z) \le \frac{r+\rho}{r-\rho}u_j(a), \quad z \in \partial L(a,\rho). \qquad \blacksquare$$

Theorem 14. A monotonically increasing series for A(z)-harmonic tasks u_j∈h_A(D). If it converges uniformly (in D) to, or meets uniformly to definite A(z)-harmonic tasks $u \in h_A(D)$.

Proof. It suffices to demonstrate the theorem given a monotonically growing sequence such as $u_j \longrightarrow u(z)\,, u(z) \in (-\infty\,, +\infty\,]$. We correct a random convex area GD In which a lemniscate can be defined as $(a,r) = \{\xi \in G : |\psi(\xi,a)| < r\} \subset G$, a∈G ,r>0 it can be assuming that $u_j \ge u_1(z) \ge 0\ \forall\, z \in G$ given that u_j≥u_1 (z) and, should it be necessary, adding positive constant. Using the formula for the mean value (10) so, :

$$u_j(z) = \frac{1}{\pi r^2}\iint_{|\psi(\xi,a)|\le r} u_j(\xi)\mathrm{d}\mu$$

According to Levy's theorem, this equivalence applies to you as well (the priory u is feasibly not bounded)

$$u(z) = \frac{1}{\pi r^2}\iint_{|\psi(\xi,a)|\le r} \mathrm{u}(\xi)\mathrm{d}\mu$$

(11)

***Case* I**. U represents not bounded task, and the equation reads ∃ a∈G∶u(a)=+∞. In the case of this is the case, the left side of equation (10) suggests that the value of u_j (z) as j→ ∞, as uniformly evaluated in ∂L(a,ρ),∀ ρ<r, meets to +∞. Here, demonstrating that u(z)≡+∞ in G besides u_j (z) as j→ ∞uniformly converges to + in arbitrary L(a,ρ)⊂G is not a difficult task at

all.

***Case* II** . $u(z) < \infty \; \forall \; z \in G$. So, the right-hand of (10) indicates that

$$u_{j+m}(z) - u_j(z) \leq \frac{r+\rho}{r-\rho}\left(u_{j+m}(a) - u_j(z)\right) \;, z \in \partial L(a, \rho).$$

Moreover, the sequence $u_j(z)\; as\; j \to \infty$ uniformly converges in $L(a, \rho), \rho < r$.
As a consequence of this, in any compact KG, u j (z) uniformly meets u(z), with continuous u(z) in G. (11). The both theorems support this proposition (12). As a result, u(z)h A. (G). Because GD stands for a fixed arbitrary convex area, u(z) has been A(z) in D.■

Conclusions

1- Study some properties of $A(z)$ – harmonic tasks.
2- Proving an analog of the Schwarz inequality for analytic tasks A(z).
3-Proving the integral formula of Poisson for A (z)-analytic tasks.
4-Demonstrate an analog of Harnack's theorem for $A(z)$ - harmonic tasks.

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