M-Ordering on Generalized Regular Neutrosophic Fuzzy Matrices

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Abstract

The goal of this article is to announce a distinct kind of ordering termed as m-ordering on m- regular Neutrosophic Fuzzy Matrix (NSFM) as an oversimplification of the minus partial ordering for regular fuzzy matrices. A set of comparable conditions for a pair of m – regular NSVM to be under this ordering are attained. We show that this ordering is conserved under resemblance relation. Some properties under this ordering also were discussed. Further, we prove that if any two NSFMs are under m-ordering then its enrolment work (T), the non-participation (F) work and the indeterminacy work (I) fuzzy matrices are also under m- order.

Keywords: Fuzzy Matrix, Neutrosophic Fuzzy Matrix(NSFM), m-ordering, Regular NSFM, m-regular.

1.Introduction

Scientists in endless abundant fields choose with the equivocal, vague and remarkably lacking proof of showing wrong information. As a result, fuzzy set hypothesis was acquainted by L. A. Zadeh [12]. Then, the intuitionistic fuzzy sets were laid out by K. A. Atanassov [1, 2]. Evaluation of non-enrolment values are additionally not tediously workable for the vague explanation as in the event of participation esteems thus, there exists an indeterministic segment whereupon reluctance perseveres. As an impact, The neutrosophic set (NS) was declared by F. Smarandache [5,10,11] in 1999 where every component had three associated characterizing capacities, explicitly the enrolment work (T), the non-participation (F) work and the indeterminacy work (I) obvious on the universe of talk U, the three capacities are completely autonomous.

The issues with respect to a few sorts of hesitations can't tackled by the old-style grid hypothesis. That sort of issues is broken by utilizing fuzzy framework [4, 6]. In [4], Kim and Roush have perceived a model for fuzzy matrices comparable to that for Boolean Matrices by spreading the maximin procedure on fuzzy variable based math Ғ =[0,1]. In [7], Meenakshi have determined different sorts of requesting on standard fuzzy matrices. Fuzzy matrices concurred with just participation values. These matrices can't contract nonparticipation values. Intuitionistic fuzzy matrices (IFMs) declared first time by Khan, Shyamal and Pal [8]. The less incomplete requesting on neutrosophic fuzzy matrices have concentrated by Sriram and et.al[3]. In [9], Poongodi et.al have given the new portrayal on Neutrosophic fuzzy matrices.

Here, we announce a distinct kind of ordering termed as m- ordering on m - regular Neutrosophic Fuzzy Matrix (NSFM) as an oversimplification of the minus partial ordering for regular fuzzy matrices. A set of comparable conditions for a pair of m – regular NSVM to be under this ordering are attained. We show that this ordering is conserved under resemblance relation. Some properties under this ordering also were discussed. Further, we prove that if any two NSFMs are under m-ordering then its enrollment work (T), the non-participation (F) work and the indeterminacy work (I) fuzzy matrices are also under m- order.

2. Preliminaries:

In this section, some elementary definitions and consequences desired are given.

Definition 2.1

A matrix C ∈ F_n is known as to be regular that there exist a matrix U ∈ F_n, to such an extent that CUC=C. Then, at that point, U is called g-reverse of C. Let C{1} = { U / CUC = C}. F_n signifies the arrangement of all fuzzy matrices of order nxn.

Definition 2.2

A matrix C∈ F_n is known as right m – regular, assuming there exist a matrix U ∈ F_n , to such an extent that C^m U C = C^m, for some sure whole number m. U is named as right m - g inverse of C. Let $C_r\{1^m\} = \{ U / C^m U C = C^m \}$.

Definition 2.3

A matrix C∈ F_n is known as left m – regular, assuming there exist a matrix V ∈ F_n , to such an extent that C V C^m = C^m, for some sure whole number m. V is named as left m - g inverse of C.

Let C = { V / C V C^m = C^m }.

Definition 2.4

For C∈ $Ғ_{mn}$ and D∈ $Ғ_{mn}$; the minus ordering denoted as $\overline{<}$ is demarcated as

$$C \overline{<} D \Leftrightarrow C^- C = C^- D$$

$$\text{and} \quad C C^- = D C^- \quad \text{for some } C^- \in C\{1\}.$$

Definition 2.5

An neutrosophic fuzzy matrix (NSFM) C of order m × n is defined as C= $[X_{ij,} <c_{(ij)T,}\ c_{(ij)F,}\ c_{(ij)I}>]_{mxn}$, where $c_{(ij)T,}\ c_{(ij)FF}\ c_{(ij)I}$ are called enrolment work (T), the non-participation (F) work and the indeterminacy work (I) of X_{ij} in C, which sustaining the condition $0 \le (c_{(ij)T,} + c_{(ij)F,} + c_{(ij)I}) \le 3$. For simplicity, we write C = $[c_{ij}]_{mxn}$ where $c_{ij} = < c_{(ij)T,}\ c_{(ij)F,}\ c_{(ij)I} >$. Let N_n symbolizes the arrangement of all nxn NSFM.

Let C and D be any two NSFMs. The accompanying activities are characterized for any two-component c_{ij} ∈ C and d_{ij} ∈ D, where $a_{ij} = [c_{(ij)T,}\ c_{(ij)F,}\ c_{(ij)I}]$ and $d_{ij} = [d_{(ij)T,}\ d_{(ij)F,}\ d_{(ij)I}]$ are in [0,1] with the end goal that $0 \le (c_{(ij)T} + c_{(ij)F,} + c_{(ij)I}) \le 3$ and $0 \le (d_{(ij)T,} + d_{(ij)F,} + d_{(ij)I}) \ge 3$, then

$$c_{ij} + d_{ij} = [\max\{c_{(ij)T,}, d_{(ij)T}\}, \max\{c_{(ij)F}, d_{(ij)F}\}, \min\{c_{(ij)I,} d_{(ij)I}\}]$$

$$c_{ij} . d_{ij} = [\min\{c_{(ij)T,}, d_{(ij)T}\}, \min\{c_{(ij)F}, d_{(ij)F}\}, \max\{c_{(ij)I,} d_{(ij)I}\}]$$

Here we shall track the elementary operations on NSFM.

For C = (c_{ij}) =[$c_{(ij)T,}$ $c_{(ij)F,}$ $c_{(ij)I}$] and D = (d_{ij}) =[$d_{(ij)T,}$ $d_{(ij)FF}$ $d_{(ij)I}$] of order mxn, their total indicated as C + D is characterized as,

C + D = (c_{ij} + d_{ij}) = [($c_{(ij)T,}$+ $d_{(ij)T}$), ($c_{(ij)F}$ + $d_{(ij)F}$), ($c_{(ij)I}$ + $d_{(ij)I}$])] ….(2.1)

For C = $(c_{ij})_{mxn}$ and D = $(d_{ij})_{nxp}$ their product indicated as CD is characterized as,

CD = (e_{ij}) = $\sum_{k=1}^{n} c_{jk} . d_{kj}$

$$= \sum_{k=1}^{n}\left(c_{(ik)T,} . d_{(kj)T}\right), \ \sum_{k=1}^{n}\left(c_{(ik)F} . d_{(kj)F}\right), \ \sum_{k=1}^{n} c_{(ik)I} . d_{(kj)I} \ ) \quad ....(2.2)$$

Lemma 2.6

For C, D ∈ N_{mn}

(i) If the row space of D contained in the row space of C then which is equivalent to D = UC for some U ∈ N_m

i.e. $\mathcal{R}$(D) ⊆ $\mathcal{R}$(C) ⇔ D = UC for some U ∈ N_m

(ii) If the column space of D contained in the column space of C then which is equivalent to D = CV for some V ∈ N_n

i.e. $\mathcal{C}$(D) ⊆ $\mathcal{C}$(D) ⇔ D = CV for some V ∈ N_n

Lemma 2.7

For C ∈ N_{mn} and D ∈ N_{nm}, the following hold.

(i) The row space of CD, which is contained in the row space of C, i.e. $\mathcal{R}$(CD) ⊆ $\mathcal{R}$(C)

(ii) The column space of CD, which is contained in the column space of D , i.e. $\mathcal{C}$(CD) ⊆ $\mathcal{C}$(D)

Lemma: 2.8

For C=[C_T, C_F, C_I] ∈ N_{mn} and D=[D_T, D_F, D_I] ∈ N_{nm}, the following hold.

(i) C^T = [$C_T{}^T$, $C_F{}^T$, $C_I{}^T$]

(ii) CD = [C_TD_T, C_FD_F, C_ID_I]

3. m -Ordering on m – Regular Neutrosophic Fuzzy Matrices.

In this part, we announce a distinct kind of ordering termed as m- ordering on m - regular Neutrosophic Fuzzy Matrix (NSFM) as an oversimplification of the minus partial ordering for regular fuzzy matrices. A set of comparable conditions for a pair of m – regular NSVM to be under this ordering are attained. We show that this ordering is conserved under resemblance relation. Some properties under this ordering also were discussed. Further, we prove that if any two NSFMs are under m-ordering then its enrolment work (T), the non-participation work(F) and the indeterminacy work (I) fuzzy matrices are also under m- order.

3.1. Right m -Regular Neutrosophic Fuzzy Matrices

A matrix C=[C_T, C_F, C_I] ∈ N_n is known to be right m – regular if the matrix satisfies the equation C^m U C = C^m where U∈ N_n and m be any sure whole number. Let C{1_r^m}={ U / C^m U C = C^m }.

3.2. Left m -Regular Neutrosophic Fuzzy Matrices

A matrix C=[C_T, C_F, C_I] ∈ N_n is known to be left m – regular if the matrix satisfies the equation CVC^m = C^m, where V∈ N_n and m be any positive integer. Let C{1_ℓ^m}= { V / C V C^m = C^m }.

In the following example, we illustrated that right m – g inverse and left m – g inverse both are distinct.

Example 3.3

Let us consider $C = \begin{pmatrix} [0, 0,1] & [0.2,0.5, 0] & [0, 0, 0] \\ [0, 0, 1] & [0.5, 1, 0] & [0.3,0.5, 0] \\ [0.1,0.5, 1] & [0, 0, 0] & [0, 0, 0] \end{pmatrix} \in (N)_{3U3}$.

For this C, $C^2 = \begin{pmatrix} [0, 0, 1] & [0.2,0.5, 0] & [0.2, 0.5, 0] \\ [0.1, 0.5, 1] & [0.5, 1, 0] & [0.3, 0.5, 0] \\ [0, 0, 1] & [0.1, 0.5, 0] & [0, 0, 0] \end{pmatrix}$

$$C^3 = \begin{pmatrix} [0.1, 0.5, 1] & [0.2,0.5, 0] & [0.2, 0.5,0] \\ [0.1, 0.5, 1] & [0.5, 1, 0] & [0.3, 0.5, 0] \\ [0, 0, 1] & [0.1, 0.5, 0] & [0.1, 0.5, 0] \end{pmatrix}$$

For $U = \begin{pmatrix} [0.4, 0.5, 1] & [0, 0, 0] & [0.3, 0.5, 0] \\ [0.2, 0.5, 1] & [0.5, 1, 0] & [0, 0, 0] \\ [0, 0, 1] & [0.4, 0.5, 0] & [0, 0, 0] \end{pmatrix}$,

$C^3 U C = C^3$. Hence C is 3 – regular NSFM. For m = 3, $C^3 U C = C^3$ but $C U C^3 \neq C^3$.

3.4. m -Ordering

For $C \in (N)_n^{(m)}$,the set of all m-regular fuzzy matrices and $D \in N_n$; the m - ordering symbolized as $C \overset{m}{\leq} D$ is defined as

$$C \overset{m}{\leq} D \Leftrightarrow \quad C^m U = D^m U \quad \text{for some } U \in C\{1_r^k\}$$
$$\text{and} \quad V C^m = V D^m \quad \text{for some } V \in C\{1_\ell^k\}.$$

In all-purpose, if $U \in C\{1_r^m\}$, then U need not be a g – inverse of C^m. This is showed in the succeeding illustration.

3.5. Example

Let $C = \begin{pmatrix} [0.3, 0.5, 0.1] & [0.7, 1, 0] \\ [0.5, 1, 0.7] & [0, 0, 0] \end{pmatrix}$

For this C, $C^2 = \begin{pmatrix} [0.5, 0.7, 0.1] & [0.3, 0.5, 0] \\ [0.3, 0.5, 0.7] & [0.5, 1, 0] \end{pmatrix}$

For $U = \begin{pmatrix} [0.3, 1, 0.1] & [0.7, 0.9, 0] \\ [0.5, 0.8, 0.7] & [0, 0, 0] \end{pmatrix}$

$C^2 U C = C^2$. Hence $U \in C\{1_r^2\}$ and C is 2 – regular, but

$C^2 U C^2 = \begin{pmatrix} [0.3, 0.3, 0.1] & [0.5, 0.5, 0] \\ [0.5, 0.5, 0.7] & [0.3, 0.5, 0] \end{pmatrix} \neq C^2$, Therefore, U is not in $C^2\{1\}$. Thus, U is a 2-g- inverse of C but U is not a g – inverse for C^2.

Lemma.3.6. For $C=[C_T, C_F, C_I] \in (N)_n^{(m)}$ and $D= [D_T, D_F, D_I] \in (N)_n$; the following are equivalent.

i. $C_T \overset{m}{\leq} D_T \Leftrightarrow C_T^m = D_T^m U_T C_T = C_T V_T D_T^m$ for some $U_T, V_T \in C_T\{1^m\}$

ii. $C_F \overset{m}{\leq} D_F \Leftrightarrow C_F^m = D_F^m U_F C_F = C_F V_F D_F^m$ for some $U_F, V_F \in C_F\{1^m\}$

iii. $C_I \overset{m}{\leq} D_I \Leftrightarrow C_I^m = D_I^m U_I C_I = C_I V_I D_I^m$ for some $U_I, V_I \in C_F\{1^m\}$

Proof: (i)

$C_T \overset{m}{<} D_T \Rightarrow C_T^m U_T = D_T^m U_T$ where $U_T \in C_T\{1^m_r\}$

and $V_T C_T^m = V_T D_T^m$ where $V_T \in C_T\{1^m_\ell\}$

Now,

$C_T^m = (C_T^m U_T) C_T = D_T^m U_T C_T$ where $U_T \in C_T\{1^m_r\}$

$C_T^m = C_T (V_T C_T^m) = C_T V_T D_T^m$ where $V_T \in C_T\{1^m_\ell\}$

$C_T^m = D_T^m U_T C_T = C_T V_T D_T^m$ where $U_T, V_T \in C_T\{1^m\}$

Thus $C_T^m = D_T^m U_T C_T = C_T V_T D_T^m$ where $U_T, V_T \in C_T\{1^m\}$ holds.

Let $Z_T = U_T C_T U_T$ for $U_T \in C_T\{1^k_r\}$

$C_T^m Z_T C_T = C_T^m (U_T C_T U_T) C_T = (C_T^m U_T C_T) U_T C_T = C_T^m U_T C_T = C_T^m$

$\Rightarrow Z_T \in C_T\{1^m_r\}$

Equally, $C_T Z_T C_T^m = C_T^m$ for $Z_T = V_T C_T V_T$ for $V_T \in C_T\{1^m_\ell\}$

$\Rightarrow Z_T \in C_T\{1^m_\ell\}$

Thus, for $U \in C_T\{1^m\}$, $Z = U_T C_T U_T \in C_T\{1^m_r\}$ when $U_T \in C_T\{1^m_r\}$ and $Z = U_T C_T U_T \in C_T\{1^m_\ell\}$ when $U_T \in C_T\{1^m_\ell\}$.

Now, $C_T^m Z_T = C_T^m (U_T C_T U_T) = (C_T^m U_T C_T) U_T$

$= C_T^m U_T = (D_T^m U_T C_T) U_T = D_T^m (U_T C_T U_T) = D_T^m Z$

Hence $C_T^m Z_T = D_T^m Z_T$ for some $Z_T \in C_T\{1^m_r\}$.

Correspondingly, $Z_T C_T^m = Z_T D_T^m$ for some $Z_T \in C_T\{1^m_\ell\}$.

Henceforth $C_T \overset{m}{\leq} D_T$. Thus (i) holds.

In the same way, $C_F \overset{m}{\leq} D_F \Leftrightarrow C_F^m = D_F^m U_F C_F = C_F V_F D_F^m$ for some $U_F, V_F \in C_F$ *$\{1^m\}$*

$C_I \overset{m}{\leq} D_I \Leftrightarrow C_I^m = D_I^m U_I C_I = C_I V_I D_I^m$ for some $U_I, V_I \in C_I$ *$\{1^m\}$*

Proof of (ii) and (iii) are like that of proof of (i).

Lemma 3.7. For $C, D \in (N)_n^{(m)}$

(i) If D is right m-regular and $R(C^m) \subseteq R(D^m)$ then $C^m = C^m D^- D$ for every right m-g inverse D^- of D.

(ii) If D is left m-regular and $C(C^m) \subseteq C(D^m)$ then $C^m = D D^- C^m$ for every left m-g inverse D^- of D.

Proof:

(i) $R(C^m) \subseteq R(D^m) \Rightarrow C^m = U D^m$ (By Lemma (2.6))

$= U D^m D^- D$ for every $D^- \in D\{1_r^m\}$.

$= C^m D^- D$

Thus (i) holds.

(ii) $C(C^m) \subseteq C(D^m) \Rightarrow C^m = D^m V$ (BV Lemma (2.6))

$= D D^- D^m V$ for every $D^- \in D\{1_\ell^m\}$.

$= D D^- C^m$

Thus (ii) holds.

Theorem 3.8. For $C=[C_T, C_F, C_I]$, $D=[D_T, D_F, D_I] \in (N)_n^{(m)}$,

(i) if $C_T \overset{m}{<} D_T$ then $R(C_T^m) \subseteq R(D_T^m)$, $C(C_T^m) \subseteq C(D_T^m)$ and $C_T^m U D_T = C_T^m = D_T V C_T^m$ for each $U \in D_T\{1_r^m\}$ and for each $V \in D_T\{1_\ell^m\}$.

(ii) if $C_F \overset{m}{<} D_F$ then $R(C_F^m) \subseteq R(D_F^m)$, $C(C_F^m) \subseteq C(D_F^m)$ and $C_F^m U D_F = C_F^m = D_F V C_F^m$ for each $U \in D_F\{1_r^m\}$ and for each $V \in D_F\{1_\ell^m\}$.

(iii) if $C_I \overset{m}{<} D_I$ then $R(C_I^m) \subseteq R(D_I^m)$, $C(C_I^m) \subseteq C(D_I^m)$ and $C_I^m U D_I = C_I^m = D_I V C_I^m$ for each $U \in D_I\{1_r^m\}$ and for each $V \in D_I\{1_\ell^m\}$.

Proof:

(i) If $C_T \overset{m}{<} D_T \Rightarrow C_T^m = C_T V D_T^m = D_T^m U C_T$ (By Lemma (3.6))

$\Rightarrow C_T^m = V D_T^m = D_T^m U$, where $V = C_T V$ and $U = U C_T$

$\Rightarrow R(C_T^m) \subseteq R(D_T^m)$ and $C(C_T^m) \subseteq C(D_T^m)$ (By Lemma (2.6))

$\Rightarrow C_T^m U D_T = C_T^m = D_T V C_T^m$ for each $U \in D_T\{1_r^m\}$ and for each $V \in D_T\{1_\ell^m\}$

(By Lemma 3.6)

Proof of (ii) and (iii) are like that of proof of (i).

Theorem 3.9. For $C, D \in (N)_n^{(m)}$, the following hold.

(i) $C \overset{m}{\leq} C$

(ii) $C \overset{m}{\leq} D$ and $D \overset{m}{\leq} C$ then $C^m = D^m$

(iii) $C \overset{m}{\leq} D$ and $D \overset{m}{\leq} E$ then $C \overset{m}{\leq} E$

Proof :

(i) $C \overset{m}{\leq} C$ is trivial.

(ii) $C \overset{m}{\leq} D \Rightarrow C^m = D^m U C$ for $U \in C\{1_r^m\}$ (By Lemma 3.6)

$D \overset{m}{<} C \Rightarrow D^m = D V C^m$ for $V \in C\{1_\ell^m\}$ (By Lemma 3.6)

Now, $C^m = D^m U C = (D V C^m) U C = D V (C^m U C) = D V C^m = D^m$

Hence, $C \overset{m}{\leq} D$ and $D \overset{m}{\leq} C \Rightarrow C^m = D^m$

(iii) $C \overset{m}{<} D \Rightarrow C^m = C^m D^- D = D D^- C^m$ (By Theorem (3.8) and Lemma (3.6))

$D \overset{m}{\leq} E \Rightarrow D^m = E^m D^- D = D D^- E^m$ (By Theorem (3.5) and Lemma (3.6))

Let $Z = D^- D U$ for $D^- \in D\{1_r^m\}$ and $U \in C\{1_r^m\}$

Then, $C^m Z C = (C^m D^- D) U C = C^m U C = C^m$

Therefore, $Z \in C\{1_r^m\}$

If $Z = V D D^-$ for $D^- \in D\{1_\ell^m\}$ and $V \in C\{1_\ell^m\}$ then it follows that $C Z C^m = C^m$

Therefore, $Z \in C\{1_\ell^m\}$.

Since $C \overset{m}{\leq} D$ and $D \overset{m}{\leq} E$, then we have

$$\begin{aligned}
C^m Z &= C^m (D^- D U) \\
&= (C^m D^- D) U \\
&= C^m U && \text{(BV Theorem (3.8))} \\
&= D^m U \\
&= (E^m D^- D) U && \text{(BV Lemma (3.6))} \\
&= (DD^- E^m) U \\
&= E^m (D^- D U) \\
&= E^m Z && \text{for some } Z \in C\{1_r^m\}.
\end{aligned}$$

and $Z C^m = Z C^m$ for some $Z \in C\{1_\ell^m\}$ can be proved in a similar manner.

Hence, $Z \in C\{1^m\}$ with $C^m Z = E^m Z$ and $Z C^m = Z E^m$. Therefore $C \overset{m}{\leq} E$.

4. Properties of m -Ordering on m – Regular Neutrosophic Fuzzy Matrices.

Proposition 4.1. For $C, D \in (N)_n^{(m)}$, $C \overset{m}{\leq} D \Leftrightarrow C^T \overset{m}{\leq} D^T$

Proof.

$C \overset{m}{\leq} D \Leftrightarrow \quad C^m C^- = D^m C^-$ for some $C^- \in C\{1_r^m\}$

and $C^- C^m = C^- D^m$ for some $C^- \in C\{1_\ell^m\}$

$C^- \in C\{1_r^m\} \Leftrightarrow (C^-)^T \in C\{1_\ell^m\}$.

$$\begin{aligned}
& C^m C^- = D^m C^- \\
\Leftrightarrow\ & (C^m C^-)^T = (D^m C^-)^T \\
\Leftrightarrow\ & (C^-)^T (C^m)^T = (C^-)^T (D^m)^T \\
\Leftrightarrow\ & (C^T)^- (C^m)^T = (C^T)^- (D^m)^T
\end{aligned}$$

Thus, $C^m C^- = D^m C^- \Leftrightarrow (C^T)^- (C^m)^T = (C^T)^- (D^m)^T$

Similarly, $C^- C^m = C^- D^m \Leftrightarrow (C^m)^T (C^T)^- = (D^m)^T (C^T)^-$

Hence $C \overset{m}{\leq} D \Leftrightarrow C^T \overset{m}{\leq} D^T$

Proposition 4.2. $C, D \in (N)_n^{(m)}$, $C \overset{m}{\leq} D \Leftrightarrow RCR^T \overset{m}{\leq} RDR^T$ for few permutation matrix R.

Proof:

Since C is m- regular, it can be verified that RCR^T is m- regular and RC^-R^T is a m-g inverse of RCR^T for each m-g inverse C^- of C.

Now,
$$\begin{aligned}
(RCR^T)^- (RCR^T)^m &= R C^- R^T R C^m R^T \\
&= R C^- (R^T R) C^m R^T \\
&= R (C^- C^m) R^T \\
&= R(C^- D^m)R^T \\
&= (R C^- R^T)(R D^m R^T) \\
&= (R C R^T)^- (R D R^T)^m
\end{aligned}$$

Hence $(R\,C\,R^T)^-\,(R\,C\,R^T)^m = (R\,C\,R^T)^-\,(R\,D\,R^T)^m$

Similarly $(R\,C\,R^T)^m\,(R\,C\,R^T)^- = (R\,D\,R^T)^m\,(R\,C\,R^T)^-$

Hence $(R\,C\,R^T) \overset{m}{\leq} (R\,D\,R^T)$

Conversely, if $(R\,C\,R^T) \overset{m}{<} (R\,D\,R^T)$, then by the Preceding part,

$$C = R^T(R\,C\,R^T)\,R \overset{m}{\leq} R^T(R\,D\,R^T)\,R = D$$

Thus $C \overset{m}{\leq} D$.

Proposition 4.3. For C, D$\in$(N)$_n^{(m)}$, if $C \overset{m}{\leq} B$ with B^m is idempotent, then C^m is idempotent.

Proof: Since $C \overset{m}{\leq} D$, By Lemma (3.6)

$$\begin{aligned} C^{2m} &= C^m C^m \\ &= (C \vee D^m)(D^m \cup C) \\ &= C \vee (D^{2m}) \cup C = (C \vee D^m) \cup C = C^m \cup C = C^m \end{aligned}$$

Remark 4.4

In the above Proposition (4.3), if $C \overset{m}{\leq} D$ with C idempotent then D need not be idempotent. This is demonstrated in the following.

4.5. Example

Consider $C = \begin{pmatrix} [0.5, 1, 0.4] & [0.5, 1, 0] \\ [0.5, 1, 0.9] & [0.5, 1, 0] \end{pmatrix}$ and $D = \begin{pmatrix} [0, 0, 1] & [0.5, 1, 0.5] \\ [0.5, 1, 0] & [0, 0, 1] \end{pmatrix} \in ($ N$_{mn}$

For this D, $D^2 = \begin{pmatrix} [0.5, 1, 0.5] & [0, 0, 1] \\ [0, 0, 1] & [0.5, 1, 0.5] \end{pmatrix}$

Here $C^2 = C$ and for $C^- = C$, $C \overset{2}{<} D$, but D is not idempotent. Since $D^2 \neq D$.

Proposition 4.6. . *For* C, D$\in$(N)$_n^{(m)}$, if $C \overset{m}{\leq} D$ then $D^m = 0$ implies $C^m = 0$.

Proof:

Since $C \overset{m}{\leq} D \Rightarrow C^m = C \vee D^m$ (By Lemma (3.6))

$= 0$

Proposition 4.7 . For C , D$\in$(N)$_n^{(m)}$, if $D \overset{m}{\leq} C$ then $C^m = 0$ implies $D^m = 0$.

Proof:

Since $D \overset{m}{\leq} C \Rightarrow D^m = D \vee C^m$ (By Lemma (3.6))

$= 0$

Conclusion:

Ordering moralities are essential for classifying and grading real world problems. This article affords a distinct type of ordering called m-ordering which has extensive application in neutrosophic fuzzy matrices. This paper is an extension of minus ordering in regular fuzzy matrices to m-ordering on m-regular neutrosophic fuzzy matrices.

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