# An Eoq Model for Time Dependent Deterorating Items with Octagonal Fuzzy Quadratic Demand

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***Article Info***
***Page Number: 2271 - 2278***
***Publication Issue:***
***Vol 72 No. 1 (2023)***

**Abstract:** This paper represents an inventory model of time dependent deteriorating items under octagonal fuzzy without allowing shortages in inventory. Demand is time dependent and is quadratic in nature. The model is compared between crisp, fuzzy and octagonal fuzzy demand. The result is validated with the help of numerical example under fuzzy and octagonal fuzzy quadratic demand. The main purpose of this work is to find optimal cost under these constraints.

***Article History:***
***Article Received: 15 November 2022***
***Revised: 24 December 2022***
***Accepted: 18 January 2023***

**Keywords:** Inventory, Fuzzy and Octagonal Fuzzy time dependent quadratic Demand

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**Introduction:**

In the production domain, there is a lot of complex scenario for productions. In large companies, the factors influencing any decision is always under complex situation needs to be made favoring the company. The factors like, raw material, customer demand, capacity of equipment etc. are the main reason for complex situation. It is possible to make suitable decision to get rid of these uncertainties with the help of mathematical model and optimize the objectives and results. For these kinds of various critical situations, the fuzzy theory helps us to get optimal results. Fuzzy set theory has been applied to several fields like optimization and other areas. In the field of research, L. A. Zadeh [1] developed the fuzzy set theory which is used in several research in different fields like mathematics, statistics. Garai, Chakraborty and Roy [2] developed a fuzzy inventory model in which demand is price dependent and holding cost is fuzzy. Sen and Chakrabarti [3] introduced an EOQ model for healthcare industries with exponential demand pattern and time dependent delayed deterioration under fuzzy and neutrosophic fuzzy environment. Saha and Chakrabarti [4] schemed A fuzzy inventory model for deteriorating items with linear price dependent demand in a supply chain. Sen and Chakrabarti [5] also developed an industrial production inventory model with deterioration under neutrosophic fuzzy optimization. Rajput et. al. [6] expanded FEOQ model with octagonal fuzzy demand rate and optimize with signed distance method. Khanra, Sudhansu and Chaudhuri [7] modified A note on an order level inventory model for deteriorating items with time dependent quadratic demands. Kar, Mondal, Roy [8] developed An Inventory Model under Space Constraint in Neutrosophic Environment: An Neutrosophic Geometric Programming Approach. Rajeswari et. al. [9] modeled Octagonal fuzzy neutrosophic number and its application to reusable container inventory model with container shrinkage. Zita [10] proposed a Fuzzy inventory model with allowable shortages and backorder. Kundu and Chakrabarti [11] also developed an EOQ model for deteriorating items with fuzzy demand and fuzzy partial backlogging. Chakrabarti and Chaudhuri [12] insisted an EOQ model deterioratng items with a inear trend in demand and shortages in all cycles. Goyal [13] developed an EPQ model with stock dependent demand and time varying deterioration with shortages under inflationary environment. Jaggi and Tiwari designed [14] Effect of deterioration on two warehouse inventory model with imperfect quality. Soni and Shah [15] modeled Optimal ordering policy for stock dependent demand under progressive payment. Manna [16] proposed An EOQ Model for deteriorating item with non-linear demand under inflation, time discounting and a trade credit policy.

Here we discuss the basics of Fuzzy, mainly triangular fuzzy and octagonal fuzzy with defuzzification under signed distance method.

**Basic Preliminaries:**

**Fuzzy Set:**

A Fuzzy Set A is defined by a membership function $\mu_A(x)$ which maps each and every element of X to [0, 1]. i.e. $\mu_A(x) \rightarrow [0,1]$, where X is the underlying set. In simple, a fuzzy set is a set whose boundary is not clear. On the other hand, a fuzzy set is a set whose element are characterized by a membership function as above.

A triangular fuzzy number is a fuzzy set. It is denoted by A = <a, b, c> and is defined by the following membership function:

$$\mu_A(x) = \begin{cases} 0, & a \le x, \\ \dfrac{x-a}{b-a}, & a \le x \le b, \\ \dfrac{c-x}{c-b}, & b \le x \le c, \\ 0, & x \ge c, \end{cases}$$

**Defuzzification of Triangular fuzzy number:**

Defuzzification, i.e. Signed distance for A = <a, b, c>, a triangular fuzzy number, the signed distance of A measured from $O_1$ is given by

$$d(A, O_1) = \tfrac{1}{4}(a + 2b + c)$$

**Octagonal Fuzzy Number:**

A fuzzy number $\tilde{Z} = (z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8)$ is a octagonal fuzzy number, if its membership function $\mu_{\tilde{Z}(x)}$ is

$$\mu_{\tilde{Z}(x)} = \begin{cases} 0, & if\ x < z_1\ \&\ z_8 \le x \\ k\left(\frac{x-z_1}{z_2-z_1}\right), & if\ z_1 \le x \le z_2 \\ k, & if\ z_2 \le x \le z_3 \\ k+(1-k)\left(\frac{x-z_3}{z_4-z_3}\right), & if\ z_3 \le x \le z_4 \\ 1, & if\ z_4 \le x \le z_5 \\ k+(1-k)\left(\frac{z_6-x}{z_6-z_5}\right), & if\ z_5 \le x \le z_6 \\ k, & if\ z_6 \le x \le z_7 \\ k\left(\frac{z_8-x}{z_8-z_7}\right), & if\ z_7 \le x \le z_8 \end{cases} \quad for\ \ 0 \le k \le 1$$

**Defuzzification of Octagonal fuzzy number:**

Let $A_o$ be a OFN. Then $m(A_o)$, the measure of $A_o$ by signed distance method is given as $m(A_o) = \frac{1}{4}[(z_1 + z_2 + z_7 + z_8)k + (z_3 + z_4 + z_5 + z_6)(1 - k)]$ where $0 \le k \le 1$

**Assumptions and Notations:**

i. Demand is dependent on time and is quadratic i.e. $D = at^2 + bt + c$, where a, b and c are constants.

a ,b, c are first considered as crisp then fuzzy and then octagonal fuzzy.

ii. θ is rate of deterioration and is dependent of time.

iii. Replenishment is instantaneous and lead time is zero.

iv. The cycle time is uncertain.

v. Shortages are not allowed.

vi. q is the initial stock level at the beginning of the inventory.

vii. The total cost of deterioration items is D.

viii. T is the length of a cycle.

ix. I(t) is the inventory level at any time t.

x. h is the holding cost per unit time.

xi. A is the setup cost per cycle.

xii. C is the deterioration cost per unit.

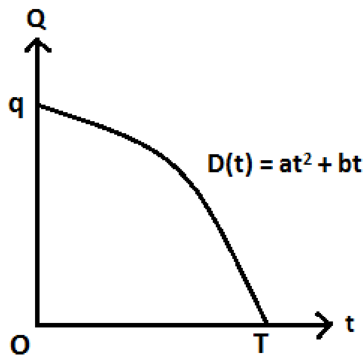
xiii. TAC is the total inventory cost.

xiv. $TAC^+$ is the total fuzzy inventory cost.

xv. $TAC^{++}$ is the total octagonal fuzzy inventory cost.

**Mathematical Model:**

The inventory level is q at time t = 0. Then the inventory level decreases due to demand and deterioration and reaches to zero at t = T.




The change in the inventory level can be described by the following differential equation:

**Case 1 (Crisp Model):**

$I'(t) + t\theta I(t) = - D(t) \qquad 0 \le t \le T$

$I'(t) + t\theta I(t) = - (at^2 + bt + c)$

With boundary condition $I(0) = q$ and $I(T) = 0$

The solution of the above differential equation is given by

$I(t) = - [ct + \frac{b}{2}t^2 + \frac{a}{3}t^3 - \frac{c\theta}{3}t^3 - \frac{\theta b}{8}t^4 - \frac{\theta a}{15}t^5 ] + q ( 1 - \frac{\theta}{2}t^2 )$

The inventory in a cycle is given by:

$I_T \qquad = \int_0^T I(T)\, dt$

$\qquad = qT - [ \frac{c}{2}T^2 + (\frac{b+q\theta}{6}) T^3 + (\frac{a-c\theta}{12}) T^4 - \frac{\theta b}{40} T^5 - \frac{\theta a}{90}T^6 ]$

Total deterioration is a cycle is given by

$D \qquad = q - \text{total demand}$

$\qquad = q - \int_0^T (at^2 + bt + c )\, dt$

$\qquad = q - \frac{a}{3}T^3 - \frac{b}{2}T^2 - cT$

Average cost of the system is given by

$TAC \qquad = \frac{1}{T} [A + C.D + hI_T ]$

$\qquad = \frac{1}{T} [A + C(q - \frac{a}{3}T^3 - \frac{b}{2}T^2 - cT) + h(qT - [ \frac{c}{2}T^2 + (\frac{b+q\theta}{6}) T^3 + (\frac{a-c\theta}{12}) T^4 - \frac{\theta b}{40} T^5 - \frac{\theta a}{90}T^6$
$])]$

**Case 2 (Fuzzy Model, when a, b and c of demand is Fuzzy):**

$I'(t) + t\theta I(t) = - \tilde{D}(t) \qquad 0 \le t \le T$

$I'(t) + t\theta I(t) = - (\tilde{a}t^2 + \tilde{b}t + \tilde{c})$

With boundary condition $I(0) = q$ and $I(T) = 0$

The solution of the above differential equation is given by

$I(t) = - [\tilde{c}t + \frac{\tilde{b}}{2}t^2 + \frac{\tilde{a}}{3}t^3 - \frac{\tilde{c}\theta}{3}t^3 - \frac{\theta\tilde{b}}{8}t^4 - \frac{\theta\tilde{a}}{15}t^5 ] + q ( 1 - \frac{\theta}{2}t^2 )$

The inventory in a cycle is given by:

$I_T \qquad = \int_0^T I(T)\, dt$

$\qquad = qT - [ \frac{\tilde{c}}{2}T^2 + (\frac{\tilde{b}+q\theta}{6}) T^3 + (\frac{\tilde{a}-\tilde{c}\theta}{12}) T^4 - \frac{\theta\tilde{b}}{40} T^5 - \frac{\theta\tilde{a}}{90}T^6 ]$

Total deterioration is a cycle is given by

D $= q - \text{total demand}$

$= q - \int_0^T (\tilde{a}t^2 + \tilde{b}t + \tilde{c})\, dt$

$= q - \frac{\tilde{a}}{3}T^3 - \frac{\tilde{b}}{2}T^2 - \tilde{c}T$

Average cost of the system is given by

$TAC^{+} = \frac{1}{T}[A + C.D + hI_T]$

$= \frac{1}{T}[A + C(q - \frac{\tilde{a}}{3}T^3 - \frac{\tilde{b}}{2}T^2 - \tilde{c}T) + h(qT - [\frac{\tilde{c}}{2}T^2 + (\frac{\tilde{b}+q\theta}{6})T^3 + (\frac{\tilde{a}-\tilde{c}\theta}{12})T^4 - \frac{\theta\tilde{b}}{40}T^5 - \frac{\theta\tilde{a}}{90}T^6])]$

$= \frac{1}{T}[A + C(q - \frac{(a_1+2a_2+a_3)}{12}T^3 - \frac{(b_1+2b_2+b_3)}{8}T^2 - \frac{(c_1+2c_2+c_3)}{4}T) + h(qT - [\frac{(c_1+2c_2+c_3)}{8}T^2 + (\frac{(b_1+2b_2+b_3)+4q\theta}{24})T^3 + (\frac{(a_1+2a_2+a_3)-(c_1+2c_2+c_3)\theta}{48})T^4 - \frac{\theta(b_1+2b_2+b_3)}{160}T^5 - \frac{\theta(a_1+2a_2+a_3)}{360}T^6])]$

**Case 3 (Octagonal Fuzzy Model, when a, b and c of demand is Octagonal Fuzzy):**

$I'(t) + t\theta I(t) = -\tilde{D}(t) \qquad 0 \le t \le T$

$I'(t) + t\theta I(t) = -(\tilde{a}t^2 + \tilde{b}t + \tilde{c})$

With boundary condition $I(0) = q$ and $I(T) = 0$

The solution of the above differential equation is given by

$I(t) = -[\tilde{c}t + \frac{\tilde{b}}{2}t^2 + \frac{\tilde{a}}{3}t^3 - \frac{\tilde{c}\theta}{3}t^3 - \frac{\theta\tilde{b}}{8}t^4 - \frac{\theta\tilde{a}}{15}t^5] + q(1 - \frac{\theta}{2}t^2)$

The inventory in a cycle is given by:

$I_T = \int_0^T I(T)\, dt$

$= qT - [\frac{\tilde{c}}{2}T^2 + (\frac{\tilde{b}+q\theta}{6})T^3 + (\frac{\tilde{a}-\tilde{c}\theta}{12})T^4 - \frac{\theta\tilde{b}}{40}T^5 - \frac{\theta\tilde{a}}{90}T^6]$

Total deterioration is a cycle is given by

D $= q - \text{total demand}$

$= q - \int_0^T (\tilde{a}t^2 + \tilde{b}t + \tilde{c})\, dt$

$= q - \frac{\tilde{a}}{3}T^3 - \frac{\tilde{b}}{2}T^2 - \tilde{c}T$

Average cost of the system is given by

$TAC^{++} = \frac{1}{T}[A + C.D + hI_T]$

$= \frac{1}{T}[A + C(q - \frac{\tilde{a}}{3}T^3 - \frac{\tilde{b}}{2}T^2 - \tilde{c}T) + h(qT - [\frac{\tilde{c}}{2}T^2 + (\frac{\tilde{b}+q\theta}{6})T^3 + (\frac{\tilde{a}-\tilde{c}\theta}{12})T^4 - \frac{\theta\tilde{b}}{40}T^5 - \frac{\theta\tilde{a}}{90}T^6])]$

$= \frac{1}{T}[A + C(q - \frac{[(a_1+a_2+a_7+a_8)k + (a_3+a_4+a_5+a_6)(1-k)]}{12}T^3 - \frac{[(b_1+b_2+b_7+b_8)k + (b_3+b_4+b_5+b_6)(1-k)]}{8}T^2 - \frac{[(c_1+c_2+c_7+c_8)k + (c_3+c_4+c_5+c_6)(1-k)]}{4}T) + h(qT - [\frac{[(c_1+c_2+c_7+c_8)k + (c_3+c_4+c_5+c_6)(1-k)]}{8}T^2 + (\frac{[(b_1+b_2+b_7+b_8)k + (b_3+b_4+b_5+b_6)(1-k)]+4q\theta}{24})T^3 + (\frac{[(a_1+a_2+a_7+a_8)k + (a_3+a_4+a_5+a_6)(1-k)]-[(c_1+c_2+c_7+c_8)k + (c_3+c_4+c_5+c_6)(1-k)]\theta}{48})T^4 - \frac{\theta[(b_1+b_2+b_7+b_8)k + (b_3+b_4+b_5+b_6)(1-k)]}{160}T^5 - \frac{\theta[(a_1+a_2+a_7+a_8)k + (a_3+a_4+a_5+a_6)(1-k)]}{90}T^6])]$

**Problem and Solution Procedure:**

The problem is to minimize TAC, $TAC^{+}$, $TAC^{++}$

Here we use Lingo Software for optimization.

**Illustrative example:**

For Crisp Model: A = 100, C = 12, a = 3, b = 7, c = 2, h = 6, q = 500, θ = 0.25

For Fuzzy Model: A = 100, C = 12, a = <2.5, 3, 3.4>, b = <6.6, 7, 7.5>, c = <1.9, 2, 2.1>, h = 6, q = 500, θ = 0.25

For Octagonal Fuzzy Model: A = 100, C = 12, a = <2.1, 2.3, 2.6, 3, 3.1, 3.2, 3.3, 3.5>, b = <6.4, 6.6, 6.8, 7, 7.2, 7.3, 7.4, 7.5>, c = <1, 1.3, 1.6, 1.9, 2, 2.1, 2.5, 2.9>, h = 6, q = 500, θ = 0.25

**Comparison of Models:**

TAC = 1745.06 at T = 3.652

$TAC^{+}$ = 1736.78 at T = 3.649

$TAC^{++}$ = 1699.56 at T = 3.645

**Sensitivity Analysis:**

| Parameters | % Change | TAC | $TAC^{+}$ | $TAC^{++}$ |
|---|---|---|---|---|
| A | - 50 % | 1731.65 | 1724.72 | 1679.33 |
| | - 25 % | 1739.52 | 1728.55 | 1684.94 |
| | + 25 % | 1752.64 | 1742.46 | 1705.63 |
| | + 50 % | 1764.43 | 1751.06 | 1720.19 |
| C | - 50 % | 1729.61 | 1723.27 | 1672.74 |
| | - 25 % | 1737.92 | 1725.56 | 1688.04 |
| | + 25 % | 1754.04 | 1745.96 | 1709.77 |
| | + 50 % | 1765.74 | 1752.21 | 1722.03 |
| h | - 50 % | 1730.05 | 1723.72 | 1679.33 |
| | - 25 % | 1730.89 | 1727.45 | 1684.94 |
| | + 25 % | 1753.01 | 1743.96 | 1705.63 |
| | + 50 % | 1765.03 | 1752.95 | 1720.19 |
| q | - 50 % | 1695.26 | 1689.03 | 1647.93 |
| | - 25 % | 1702.43 | 1697.23 | 1679.43 |
| | + 25 % | 1781.77 | 1778.07 | 1767.79 |
| | + 50 % | 1796.33 | 1791.66 | 1784.96 |
| θ | - 50 % | 1731.06 | 1725.01 | 1678.98 |
| | - 25 % | 1739.40 | 1728.42 | 1684.28 |
| | + 25 % | 1753.02 | 1742.59 | 1706.14 |
| | + 50 % | 1764.65 | 1752.00 | 1721.01 |

It is clear that the change in inventory cost due to cost parameters holding cost (h), setup cost (A), deterioration cost (C) and deterioration (θ) and initial inventory (q) are highly sensitive with respect to all the three models.

## Conclusion:

We have developed an inventory model for time dependent deteriorating items with price quadratic demand under crisp, fuzzy and octagonal fuzzy environment. This model doesn't have any shortages. Here the comparison of the model has been portrayed with its crisp, fuzzy and octagonal demands and hence its cost. For the fuzzy and octagonal fuzzy model, we have used signed distance method for defuzzification techniques and thus we obtain the minimum cost for each case. We also observe that the result is quite impressive with octagonal fuzzy parameters and the cost is much less compare to its crisp and fuzzy model. In the present situation, the fuzziness occurs in the different parameters which affects to the whole inventory management.

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