Analysing And Applying Novel Integral Transforms and Differential Equations

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Abstract: Researchers are always investigating new methodologies and applications in the area of integral transforms and differential equations, leading to its ongoing evolution. This study explores new integral transformations, offering a thorough examination of recent progress and their practical consequences in solving intricate differential equations. The study involves theoretical advancements, algorithmic breakthroughs, and practical implementations across several scientific fields.
The main topics covered are the investigation of novel integral transform methods, their underlying mathematical principles, and their effectiveness in solving differential equations of varied levels of difficulty. The emphasis is placed on the interaction between theoretical breakthroughs and practical applications, highlighting the contribution of these transformational tools to progress in the fields of physics, engineering, and applied mathematics.
The state-of-the-art techniques and their influence on varied problem areas are emphasised by contributions from prominent specialists in the field. This book delves into several integral transforms, ranging from fractional calculus to wavelet transforms. It serves as a vital resource for researchers, academics, and practitioners that want to utilise these methods in order to solve advanced challenges in science and engineering.

Keywords: Differential, Algorithmic, Techniques, Advanced, Transformation

1. INTRODUCTION

Special functions theory is well known in mathematics for its apparent multiplicity of tricks and devices needed to obtain information about the subject matter. A study of the transformation theory and properties of the generalized hyper geometric function enable one to derive the properties of a large class of Special functions in a unified manner.

There are three important aspects of the study of the generalized hyper geometric function which have attracted the attention of a majority of researches in this branch of analysis.

The first aspect deals with the investigations leading to certain generalizations of the question of summing up, in terms of product of gamma function only, particular types of generalized hyper geometric function and their extensions. The second aspect comprises of the study of the interrelations that might exist between such function. The third aspect deals with the generalizations of these functions which have recently led to many remarkable applications in physics, Statistics, Number theory and Combinatory Analysis.

Integral transformations have long been useful in solving a wide range of differential and integral problems. The use of a suitable integral transform facilitates the conversion of differential and integral operators, operating within a certain domain, into multiplication operators within a different domain. To reverse the manipulated answer and get the desired solution in its original domain, one must first solve the inferred problem in the new domain and then apply the inverse transform. The Laplace transform and the Fourier integral transform are the classical integral transforms often used in solving differential equations, integral equations, and in analysis and the theory of functions. In addition, the mathematical literature contains numerous Laplace-type integral transforms, including the Laplace-Carson transform utilised in railway engineering, the z-transform applied in signal processing, the Sumudu transform used in engineering and various real-life problems, and the Hankel's and Weierstrass transform employed in heat and diffusion equations. Furthermore, we use the natural transform and Yang transform extensively in several domains of physical research and engineering.

1.1 The New Integral Transform

This study presents a novel integral transform that serves as a comprehensive and unified approach to Laplace and other established transforms for functions with exponential order. Let's start with the following: Let A denote the set of individual functions that may be transformed:

$$A = \left\{ f(t) | \exists M, k_1, k_2 > 0, |f(t)| \leq M\exp\left(\frac{|t|}{k_i}\right) \text{if } t \in (-1)^i x[0, \infty[\right\}$$

The function f(t) is strictly positive for t > 0 and equal to zero for t < 0. It is continuously differentiable in section nwise, has exponential order, and is defined in the set A. The Natural transform of the function f(t) > 0 and f(t) = 0 for t < 0 is defined as follows:

$$N\{f(t)\} = R(s,u) = \int_0^{+\infty} e^{-st} f(ut)dt \quad , s > 0, u > 0$$

The transform variables are represented by the letters s and u. Another integral transform is the Aboodh transform, which is obtained from the equation:

The equation obtained for the identical circumstances as mentioned previously is as follows:

$$A(s) = N\{f(t)\} = \frac{1}{s}\int_0^{+\infty} e^{-st} f(t)dt$$

Definition 1.1

A function f(t) is said to have an exponential order of 1/k if there are positive constants T and M such that, $| f(t) | \leq Me^{\frac{t}{k}} ( )$, for any value of t that is greater than or equal to T. For each function f (t), we assume the existence of an integral equation. The new integral transform is a composite of the integral transforms.

The function in set A requires the constant M to be a positive value. The integral equation defines a new integral transform indicated by the operator E(.).

$$E(s,u) = N\{f(t)\} = \frac{1}{su}\int_0^{+\infty} e^{-\frac{st}{u}} f(t)dt \quad (1.1) \text{ or}$$

$$E(s,u) = N\{f(t)\} = \frac{1}{s}\int_0^{+\infty} e^{-st} f(ut)dt$$

2. LITERATURE REVIEW

Hasan, Sameer & Abubakar, Umar & Kaurangini, Muhammad (2023) This paper presents the introduction of a novel integral transform known as the SUM transform, along with an investigation of its characteristics. Furthermore, the use of the SUM integral transform in resolving ordinary and partial differential equations is shown via several instances. By correctly and precisely substituting the parameters of the SUM integral transform, it may be simplified to certain integral transforms found in the literature as specific instances.

Gwila, Nbila & Farhat, Ali (2022) Integral transforms are a valuable technique for addressing initial value issues and certain integral equations. They allow for the conversion of these problems into algebraic equations, which may then be solved by using inverse transforms. Recently, a number of essential transforms have been introduced and used to solve initial value issues and certain integral equations. These include the Sumudu, Natural, Al-Tememe, Elzaki, Aboodh, T Kashuri, Fundo, Mahgoub, Kamal, Polynomial, Al-Zughair, Rangaig, Mohand, and Al-Zughair Transforms. This study presents and examines an integral transform that belongs to the class of Laplace integral transform and is associated with the Jafari general integral transform. In addition, theorems and several applications for addressing initial value issues have been presented.

Ajel Mansour, Iman & A. Mehdi, Sadiq & Kuffi, Emad (2021) This work introduces a novel complex integral transform called the complicated SEE transform. An analysis is conducted on the characteristics of this transformation. The use of this intricate integral transform is also employed to convert the main issue into a straightforward algebraic equation. To acquire the solution to this fundamental issue, one must solve the given algebraic equation and use the inverse of the complex integral transform. Ultimately, the intricate integral transform is used to resolve ordinary differential equations of higher order. Additionally, we present some significant engineering and physics applications.

Ajel Mansour, Iman & Kuffi, Emad & A. Mehdi, Sadiq (2021) This paper introduces a significant modification to the standard method of addressing differential equations with constant coefficients. It focuses on the application of a modified version of the method, known as SEE change, to handle cases where the derivative is incomplete. The suitability of this modified method is demonstrated through the analysis of three specific equations: the wave equation, the heat equation, and the Laplace equation. The paper concludes by presenting the particular solutions obtained for these equations.

Kilicman, Adem & Gadain, Hassan (2010) The integral transform technique is extensively used for solving differential equations that are accompanied with starting values or boundary conditions, which may be expressed as integral equations. Watugala presented the Sumudu transform as a novel integral transform to address certain ordinary differential equations in control engineering. Subsequently, it was shown that the Sumudu transform had very distinctive and advantageous characteristics. This research examines the efficacy of a notable integral transform in solving linear ordinary differential equations with both constant and nonconstant coefficients, as well as systems of differential equations.

3. BASIC CONCEPTS

To address problems without using a novel frequency domain, one may use a novel integral transform that has both scale and unit-preserving properties. In this section, we provide some concise definitions and distinctive features of a unique general integral transform.

Definition 1. Consider a function $u(t)$ that is integrable and defined for $t \geq 0$. Let $\phi(s)$ be a non-zero function and $\sigma(s)$ be a positive real function. The formula provides the generic integral transform of $u(t)$.

$$T\{u(t)\} = \phi(s) \int_0^\infty \exp(-\sigma(s)t)u(t)dt.$$

Assuming that the integral is present for a certain $\sigma(s)$.

Definition 2. The Gamma function is precisely defined as:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \, \Re(z) > 0$$

Definition 3. Consider the function (t) defined on the interval $[0,\infty)$. The Laplace transform of the function $f(t)$ is denoted as $F(s)$ and is defined as:

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$$

The inverse Laplace Transform of $F(s)$ is defined as:

$$f(t) = \mathcal{L}^{-1}(F(s))$$

Definition 4. The Caputo fractional derivative is precisely defined as:

$${}_0^C D_u^\gamma m(u) = \frac{1}{\Gamma(n-\gamma)} \int_0^u (u-x)^{n-\gamma-1} m^{(n)}(x)dx.$$

Lemma 1. The general integral transform of the t α function has been modified and is now expressed as:

$$T\{t^\alpha\} = \frac{\Gamma(\alpha)\phi(s)}{\sigma(s)^{\alpha+1}}, \qquad \alpha > 0$$

Proof. By using the novel universal integral transform on the t α function, we get

$$T\left\{t^{\alpha}\right\} = \phi(s)\int_{0}^{\infty} e^{-\sigma(s)t} t^{\alpha} dt$$

Setting $u = \sigma(s)t$ implies $du = \sigma(s)dt$, then

$$T\left\{t^{\alpha}\right\} = \phi(s)\int_{0}^{\infty} e^{-u} \frac{u^{\alpha}}{\sigma(s)^{\alpha}} \frac{du}{\sigma(s)}$$

$$T\left\{t^{\alpha}\right\} = \frac{\phi(s)}{\sigma(s)^{\alpha+1}}\int_{0}^{\infty} e^{-u} u^{\alpha} du$$

Let $\Gamma(\alpha)$ be the Gamma function, which is defined as follows:

$$\Gamma(\alpha) = \int_{0}^{\infty} e^{-u} u^{\alpha} du$$

Thus, we get

$$T\left\{t^{\alpha}\right\} = \frac{\phi(s)\Gamma(\alpha+1)}{\sigma(s)^{\alpha+1}}.$$

4. GENERAL INTEGRAL TRANSFORM METHOD ON LINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

Now let's look at the Caputo fractional derivative. Applying the universal integral transform, we can solve fractional ordinary differential equations. Take into consideration the following ordinary differential equation of fractional order:

$$\frac{d^{\gamma} m(u)}{dt^{\gamma}} = \frac{d^{2} m(u)}{dt^{2}} + \frac{dm(u)}{dt} + m(u) + c,$$

where the starting condition is $m(0) = f(0)$. From Eq. we derive the generic integral transform as:

$$T\left\{\frac{d^{\gamma} m(u)}{dt^{\gamma}}\right\} = T\left\{\frac{d^{2} m(u)}{dt^{2}}\right\} + T\left\{\frac{dm(u)}{dt}\right\} + T\left\{m(u)\right\} + T\left\{c\right\},$$

$$T\left\{m(u)\right\}\sigma(s)^{\gamma} - \sigma(s)^{\gamma-1}\phi(s)m(0)$$

$$= T\left\{m(u)\right\}\sigma(s)^2 - \sigma(s)\phi(s)m(0) - \phi(s)m'(0) + \sigma(s)T\left\{m(u)\right\} - \phi(s)m(0)$$

$$+ T\left\{m(u)\right\} + c\frac{\phi(s)}{\sigma(s)}$$

$$T\left\{m(u)\right\}\left(\sigma(s)^\gamma - \sigma(s)^2 - \sigma(s) - 1\right)$$

$$= \sigma(s)^{\gamma-1}\phi(s)m(0) - \sigma(s)\phi(s)m(0) - \phi(s)m'(0) - \phi(s)m(0) + c\frac{\phi(s)}{\sigma(s)}$$

$$T\left\{m(u)\right\}\left(\sigma(s)^\gamma - \sigma(s)^2 - \sigma(s) - 1\right)$$

$$= \sigma(s)^{\gamma-1}\phi(s)f(0) - \sigma(s)\phi(s)f(0) - \phi(s)m'(0) - \phi(s)f(0) + c\frac{\phi(s)}{\sigma(s)}$$

$$T\left\{m(u)\right\} = \frac{\sigma(s)^{\gamma-1}\phi(s)f(0) - \sigma(s)\phi(s)f(0) - \phi(s)m'(0) - \phi(s)f(0) + c\frac{\phi(s)}{\sigma(s)}}{\left(\sigma(s)^\gamma - \sigma(s)^2 - \sigma(s) - 1\right)}$$

$$T\left\{m(u)\right\} = \left\{\frac{(\sigma(s)^{\gamma-1}\phi(s) - \sigma(s)\phi(s) - \phi(s))f(0) - \phi(s)m'(0) + c\frac{\phi(s)}{\sigma(s)}}{\left(\sigma(s)^\gamma - \sigma(s)^2 - \sigma(s) - 1\right)}\right\}$$

$$T\left\{m(u)\right\} = f(0)\frac{\phi(s)}{\sigma(s)}\frac{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1} + \frac{-\phi(s)m'(0)}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1}$$
$$+c\frac{\phi(s)}{\sigma(s)}\frac{1}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1}$$

We get when we use the inverse general integral transform.

$$m(u) = T^{-1}\left\{f(0)\frac{\phi(s)}{\sigma(s)}\frac{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1}\right\}$$

$$+ T^{-1}\left\{\frac{-\phi(s)m'(0)}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1}\right\}$$

$$+ T^{-1}\left\{c\frac{\phi(s)}{\sigma(s)}\frac{1}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1}\right\}$$

Assume that $A = f(0)\frac{\phi(s)}{\sigma(s)}\frac{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1}$. Then, we have

$$A = f(0)\frac{\phi(s)}{\sigma(s)}\frac{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)}{\sigma^\gamma(s)-\sigma^2(s)-\sigma(s)-1}$$

$$=f(0)\frac{\phi(s)}{\sigma(s)}\left(1+\frac{1}{\sigma^{\gamma}(s)-\sigma^{2}(s)-\sigma(s)-1}\right)$$

$$=f(0)\frac{\phi(s)}{\sigma(s)}-f(0)\frac{\phi(s)}{\sigma(s)}\frac{1}{1-\left(\sigma^{\gamma}(s)-\sigma^{2}(s)-\sigma(s)\right)}$$

$$=f(0)\frac{\phi(s)}{\sigma(s)}-f(0)\frac{\phi(s)}{\sigma(s)}\sum_{j=0}^{\infty}\left(\sigma^{\gamma}(s)-\sigma^{2}(s)-\sigma(s)\right)^{j}$$

$$=f(0)\frac{\phi(s)}{\sigma(s)}-f(0)\frac{\phi(s)}{\sigma(s)}\sum_{j=0}^{\infty}\sum_{n=0}^{j}(-1)^{n}\binom{j}{n}(\sigma^{\gamma}(s))^{j-n}\left(\sigma^{2}(s)+\sigma(s)\right)^{n}$$

$$=f(0)\frac{\phi(s)}{\sigma(s)}$$

$$-f(0)\frac{\phi(s)}{\sigma(s)}\sum_{j=0}^{\infty}\sum_{n=0}^{j}(-1)^{n}\binom{j}{n}(\sigma^{\gamma}(s))^{j-n}\sum_{k=0}^{n}\binom{n}{k}(\sigma^{2}(s))^{n-k}(\sigma(s))^{k}$$

$$=f(0)\frac{\phi(s)}{\sigma(s)}-f(0)\frac{\phi(s)}{\sigma(s)}\sum_{j=0}^{\infty}\sum_{n=0}^{j}(-1)^{n}\binom{j}{n}\sum_{k=0}^{n}\binom{n}{k}(\sigma(s))^{\gamma(j-n)+2n-k}$$

$$=f(0)\frac{\phi(s)}{\sigma(s)}-f(0)\sum_{j=0}^{\infty}\sum_{n=0}^{j}(-1)^{n}\binom{j}{n}\sum_{k=0}^{n}\binom{n}{k}\frac{\phi(s)}{\sigma(s)^{-\gamma(j-n)-2n+k+1}}$$

5. CONCLUSION

In this research, the authors have successfully developed a new integral transform called the "NE transform" and conducted an analysis of this innovative integral transform. The authors also introduced the essential characteristics (linearity, scaling, translation, convolution) of the suggested transform, along with its corresponding inverse transform. The definition and uses of the new transform for solving ordinary differential equations have been shown. In the future, the "NE integral transform" may be used to address intricate issues in the fields of science, medicine, and engineering by enhancing existing mathematical models. Additionally, we will present the complicated transform and a dual "NE integral transform." Integral transformations are useful tools for mathematicians because they simplify problems in one domain into another, making them easier to manipulate and solve. They have a wide range of applications in mathematics and physics, including the solution of integral and differential equations as well as issues involving signal and image processing. Both the nature of the issue and the intended outcomes of the changed function dictate which integral transform is most suitable. The characteristics of the original function and the needs of the situation dictate which transform is best, as each has pros and cons.

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