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Homomorphism and Isomorphism into a Complex Fuzzy Number Group

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Abstract:

Fuzzy mathematics forms a branch of mathematics including fuzzy set theory & fuzzy logic. It started in 1965 after the publication of LotfiAskeZadeh's seminal work Fuzzy sets.

A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function. Calculations with fuzzy numbers allow the incorporation of uncertainty on parameters, properties, geometry, initial conditions, etc.

In this article, we define the concepts of homomorphism and isomorphism of complex fuzzy number groups. And the properties of complex fuzzy number homomorphism and fuzzy number isomorphism are studied.

Introduction

Fuzzy mathematics forms a branch of mathematics including fuzzy set theory & fuzzy logic. It started in 1965 after the publication of LotfiAskeZadeh's seminal work Fuzzy sets.

A fuzzy number is a generalization of a regular, real number in the sense that it does not refer to one single value but rather to a connected set of possible values, where each possible value has its own weight between 0 and 1. This weight is called the membership function.

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In this article, we define the concepts of homomorphism and isomorphism of complex fuzzy number groups. And the properties of complex fuzzy number homomorphism and fuzzy number isomorphism are studied.

1.1 Homomorphism and Isomorphism ofComplex Fuzzy Number Groups

Definition 1.1.1:

A mapping $\boldsymbol{f}$ form a complex fuzzy number group ($F^*(C)$, ●) into a fuzzy number group ($F^{*'}(C)$, *) is said to be a complex fuzzy number homomorphism if,

$\boldsymbol{f}((\bar{a}+i\bar{b}).(\bar{c}+i\bar{d})) = \boldsymbol{f}((\bar{a}+i\bar{b})) * \boldsymbol{f}((\bar{c}+i\bar{d}))$ for all $\bar{a},\bar{b},\bar{c},\bar{d} \in F^*(R)$.

Proposition 1.1.2:

Let ($F^*(C)$, ●) and ($F^{*'}(C)$, *) be any two complex fuzzy number groups. Let$(\bar{e}'+i\bar{e}')$be the identity of $F^{*'}(C)$. Define a map $\boldsymbol{f}: F^*(C) \rightarrow F^{*'}(C)$ by $\boldsymbol{f}((\bar{a}+i\bar{b}))=(\bar{e}'+i\bar{e}')$ for all $\bar{a},\bar{b} \in F^*(R)$. Then $\boldsymbol{f}$ is a complex fuzzy number homomorphism.

Proof

Let $\bar{a},\bar{b},\bar{c},\bar{d} \in F^*(R)$.

We have to verify that,$\boldsymbol{f}((\bar{a}+i\bar{b}).(\bar{c}+i\bar{d})) = \boldsymbol{f}((\bar{a}+i\bar{b})) * \boldsymbol{f}((\bar{c}+i\bar{d}))$ for all$\bar{a},\bar{b},\bar{c},\bar{d} \in F^*(R)$.

$\Rightarrow \bar{a},\bar{b},\bar{c},\bar{d} \in F^*(R)$

$\Rightarrow \boldsymbol{f}((\bar{a}+i\bar{b}).(\bar{c}+i\bar{d})) = (\bar{e}'+i\bar{e}')$

Since $\boldsymbol{f}((\bar{a}+\bar{b}))=(\bar{e}'+i\bar{e}')'$,$\boldsymbol{f}((\bar{c}+i\bar{d}))=(\bar{e}'+i\bar{e}')$

Now,$\boldsymbol{f}((\bar{a}+\bar{b})).\boldsymbol{f}((\bar{c}+i\bar{d}))=(\bar{e}'+i\bar{e}').(\bar{e}'+i\bar{e}') = (\bar{e}'+i\bar{e}')= \boldsymbol{f}((\bar{a}+i\bar{b}).(\bar{c}+i\bar{d}))$

Therefore$\boldsymbol{f}((\bar{a}+\bar{b})).\boldsymbol{f}(\bar{b})= \boldsymbol{f}((\bar{a}+i\bar{b}).(\bar{c}+i\bar{d}))$

Hence $\boldsymbol{f}$ is a complex fuzzy number homomorphism.

This homomorphism is called a complex fuzzy number trivial homomorphism.

Proposition 1.1.3:

Let ($F^*(C)$, •) be any complex fuzzy number group. The identity complex fuzzy number map$\boldsymbol{I}_G$: $F^*(C) \rightarrow F^*(C)$ defined by $\boldsymbol{I}_G((\bar{a}+\bar{b})) = \bar{a}$for all$\bar{a} \in F^*(C)$ is a complex fuzzy number homomorphism.

Proof

Let $\overline{a}$,$\overline{b}$,$\overline{c}$, $\overline{d}$ ∈ $F^*(R)$, $\boldsymbol{I}_G((\overline{a}+ i\overline{b}).(\overline{c}+ i\overline{d})) = (\overline{a}+ i\overline{b}).(\overline{c}+ i\overline{d})$

Also $\boldsymbol{I}_G((\overline{a}+ i\overline{b})) = (\overline{a}+ i\overline{b})$, $\boldsymbol{I}_G((\overline{c}+ i\overline{d})) =(\overline{c}+ i\overline{d})$

Therefore $\boldsymbol{I}_G((\overline{a}+ i\overline{b})) . \boldsymbol{I}_G((\overline{c}+ i\overline{d})) = (\overline{a}+ i\overline{b}).(\overline{c}+ i\overline{d}) = \boldsymbol{I}_G((\overline{a}+ i\overline{b}).(\overline{c}+ i\overline{d}))$

Hence $\boldsymbol{I}_G$ is a complex fuzzy number homomorphism.

This result shows that there is atleastonecomplex fuzzy number homomorphism from $F^*(C)$into itself

Example 1.1.4:

Let $F^*(C)$ be the additive complex fuzzy number group of integers and define,

$\boldsymbol{f} : F^*(C) \to F^*(C)$ by $\boldsymbol{f}((\overline{x}+ i\overline{y})) = 3(\overline{x}+ i\overline{y})$ for $\overline{x}$,$\overline{y}$∈$F^*(R)$. Then $\boldsymbol{f}$is a complex fuzzy number homomorphism.

Proof

Let$(\overline{v}+ i\overline{w})$,$(\overline{x}+ i\overline{y})$∈$F^*(R)$. Then $(\overline{v}+ i\overline{w}) +(\overline{x}+ i\overline{y})$∈$F^*(C)$.

Now, $\boldsymbol{f}((\overline{v}+ i\overline{w}) +(\overline{x}+ \overline{y})) = 3((\overline{v}+ i\overline{w}) +(\overline{x}+ i\overline{y}))$

$= 3(\overline{v}+ i\overline{w})+3(\overline{x}+ i\overline{y})$

$= \boldsymbol{f}((\overline{v}+ i\overline{w}))+ \boldsymbol{f}((\overline{x}+ i\overline{y}))$for any $\overline{v}, \overline{w}, \overline{x}, \overline{y}$∈$F^*(R)$.

Thus $\boldsymbol{f}((\overline{v}+ i\overline{w}) +(\overline{x}+ i\overline{y})) = \boldsymbol{f}((\overline{v}+ i\overline{w}))+ \boldsymbol{f}((\overline{x}+ i\overline{y}))$for any $\overline{v}, \overline{w}, \overline{x}, \overline{y}$∈$F^*(R)$.

Therefore $\boldsymbol{f}$ is acomplex fuzzy number homomorphism of $F^*(C)$ into $F^*(C)$.

1.2 Isomorphism Of ComplexFuzzy Number Groups

Definition 1.2.1:

Let ($F^*(C)$, ●) and ($F^{*\prime}(C)$, *) be twocomplex fuzzy number groups. Then a one-one, onto composition preserving mapping **f** from $F^*(C)$ to $F^{*\prime}(C)$ is calledcomplex fuzzy number isomorphism. We say that ($F^*(C)$, ●) is isomorphic to ($F^{*\prime}(C)$, *) and we denote ($F^*(C)$,●)≅($F^{*\prime}(C)$, *).

Proposition 1.2.2:

Let $F^*(C)$ and $F^{*\prime}(C)$ be two isomorphic complex fuzzy number groups whose compositions have been denoted multiplicatively and let $\boldsymbol{f}$ be the corresponding complex fuzzy number isomorphism and $(\overline{a}+ i\overline{b})$= [$(\overline{a}+ i\overline{b})$, $(\overline{a}+ i\overline{b})$] then ,

i)if$(\overline{e} + i\overline{e})$ is the identity in $F^*(C)$, then $\boldsymbol{f}((\overline{e}+ i\overline{e}))$ is the identity in $F^{*\prime}(C)$.

ii)$\boldsymbol{f}((\overline{a} + i\overline{b})^{-1}) = [\boldsymbol{f}((\overline{a} + i\overline{b})^{-1})]^{-1}$for all $\overline{a}, \overline{b}$∈$F^*(R)$.

iii)O(($\overline{a}$+ $i\overline{b}$)) = O [$\boldsymbol{f}$ (($\overline{a}$+ $i\overline{b}$))] for all $\overline{a}$, $\overline{b}$∈$F^{\,*}(R)$.

Proof

(i) Let ($\overline{a} + i\overline{b}$)= [($\overline{a} + i\overline{b}$), ($\overline{a} + i\overline{b}$)] be an arbitrary element of $F^{\,*}(C)$.

Then $\boldsymbol{f}$being one-one onto there exists a unique element($\overline{a} + i\overline{b}$)∈$F^{\,*}(C)$ such that $\boldsymbol{f}$(($\overline{a} + i\overline{b}$)) = ($\overline{a}' + i\overline{b}'$)

Let($\overline{e}$+ $i\overline{e}$)is the identity of$F^{\,*}(C)$.

Now ($\overline{a} + i\overline{b}$) . ($\overline{e}$+ $i\overline{e}$) = ($\overline{e}$+ $i\overline{e}$) . ($\overline{a} + i\overline{b}$) = ($\overline{a} + i\overline{b}$)

⇒$\boldsymbol{f}$(($\overline{a} + i\overline{b}$) . ($\overline{e} + i\overline{e}$)) = $\boldsymbol{f}$ (($\overline{e}$+ $i\overline{e}$) . ($\overline{a} + i\overline{b}$)) = $\boldsymbol{f}$(($\overline{a} + i\overline{b}$))

⇒$\boldsymbol{f}$(($\overline{a} + i\overline{b}$)). $\boldsymbol{f}$(($\overline{e}$+ $i\overline{e}$)) = $\boldsymbol{f}$(($\overline{e}$+ $i\overline{e}$)). $\boldsymbol{f}$(($\overline{a} + i\overline{b}$)) = $\boldsymbol{f}$(($\overline{a} + i\overline{b}$))

⇒($\overline{a}' + i\overline{b}'$). $\boldsymbol{f}$(($\overline{e}$+ $i\overline{e}$))= $\boldsymbol{f}$(($\overline{e}$+ $i\overline{e}$)) . ($\overline{a}' + i\overline{b}'$)=($\overline{a}' + i\overline{b}'$). for all $\overline{a}'$,$\overline{b}'$ ∈ $F^{\,*\prime}(R)$.

This shows that$\boldsymbol{f}$(($\overline{e}$+ $\overline{e}$)) is the identity in $F^{\,*\prime}(C)$.

ii) Let ($\overline{a} + i\overline{b}$)= [($\overline{a} + i\overline{b}$), ($\overline{a} + i\overline{b}$)] , then

($\overline{a} + i\overline{b}$) . ($\overline{a} + i\overline{b}$)$^{-1}$= ($\overline{a} + i\overline{b}$)$^{-1}$. ($\overline{a} + i\overline{b}$) = ($\overline{e}$+ $i\overline{e}$)

⇒$\boldsymbol{f}$(($\overline{a} + i\overline{b}$) . ($\overline{a} + i\overline{b}$)$^{-1}$) = $\boldsymbol{f}$ (($\overline{a} + i\overline{b}$)$^{-1}$. ($\overline{a} + i\overline{b}$)) = $\boldsymbol{f}$(($\overline{e}$+ $i\overline{e}$)) where $\boldsymbol{f}$(($\overline{e}$+ $i\overline{e}$)) is the identity in $F^{\,*\prime}(C)$.

By composition preserving property,

⇒$\boldsymbol{f}$(($\overline{a} + i\overline{b}$)). $\boldsymbol{f}$(($\overline{a} + i\overline{b}$)$^{-1}$) = $\boldsymbol{f}$(($\overline{a} + i\overline{b}$)$^{-1}$). $\boldsymbol{f}$(($\overline{a} + i\overline{b}$)) = $\boldsymbol{f}$(($\overline{e}$+ $i\overline{e}$))

⇒[$\boldsymbol{f}$ (($\overline{a} + i\overline{b}$))]$^{-1}$ = $\boldsymbol{f}$(($\overline{a} + i\overline{b}$)$^{-1}$) for all $\overline{a}$, $\overline{b}$∈$F^{\,*}(R)$.

(iii) Let($\overline{a} + i\overline{b}$)∈$F^{\,*}(R)$. and let O (($\overline{a} + i\overline{b}$)) = m

Then m is the least positive integer such that ($\overline{a} + i\overline{b}$)^{m}= $\overline{e}$+ $i\overline{e}$).

Now($\overline{a} + i\overline{b}$)^{m} = $\overline{e}$+ $i\overline{e}$)⇒$\boldsymbol{f}$(($\overline{a} + i\overline{b}$)^{m}) = $\boldsymbol{f}$($\overline{e}$+ $i\overline{e}$))

⇒$\boldsymbol{f}$(($\overline{a} + i\overline{b}$) . ($\overline{a} + i\overline{b}$) . ($\overline{a} + i\overline{b}$) . . . m times) = $\boldsymbol{f}$ ($\overline{e}$+ $i\overline{e}$))

Since$\boldsymbol{f}$ is composition preserving ,

⇒$\boldsymbol{f}$(($\overline{a} + i\overline{b}$)) .$\boldsymbol{f}$(($\overline{a} + i\overline{b}$)) .$\boldsymbol{f}$(($\overline{a} + i\overline{b}$)) $m$times =$\boldsymbol{f}$ ($\overline{e}$+ $i\overline{e}$))

⇒$\boldsymbol{f}$[($\overline{a} + i\overline{b}$)]^{m}=$\boldsymbol{f}$ ($\overline{e}$+ $i\overline{e}$))

⇒ O [$\boldsymbol{f}$[($\overline{a} + i\overline{b}$)]^{m}] < m.

Let if possible O[$f((\overline{a}+ i\overline{b}))$] $= k < m$,

Then O[$f((\overline{a}+ i\overline{b}))$] $= k$=>[f (($\overline{a}+ i\overline{b}$))]^k $= f$ ($\overline{e}$+ $i\overline{e}$))

$\Rightarrow$ ($f((\overline{a}+ i\overline{b}))$.$f((\overline{a}+ i\overline{b}))$.$f((\overline{a}+ i\overline{b}))$ $k$times)= f ($\overline{e}$+ $i\overline{e}$))

Since f is composition preserving,

$\Rightarrow f((\overline{a}+ i\overline{b})$. ($\overline{a}+ i\overline{b}$) . ($\overline{a}+ i\overline{b}$) . . . k times) $= f$ ($\overline{e}$+ $i\overline{e}$))

$\Rightarrow$ f[($\overline{a}+ i\overline{b}$)]^k=f ($\overline{e}$+ $i\overline{e}$))

Since$f$ is one-one,

$\Rightarrow$ ($\overline{a}+ i\overline{b}$)^k = ($\overline{e}$+ $i\overline{e}$)

$\Rightarrow$O (($\overline{a}+ i\overline{b}$)) $\leq k < m$, which is a contradiction

Since O (($\overline{a}+ i\overline{b}$)) $= m$.

Since the contradiction arises by the hypothesis that

O[$f((\overline{a}+ i\overline{b}))$]$< m$

So, O[$f((\overline{a}+ i\overline{b}))$] $\lneq m$

Hence O[$f((\overline{a}+ i\overline{b}))$]$= m$ =O (($\overline{a}+ i\overline{b}$))

Again, let O (($\overline{a}+ i\overline{b}$)) be infinite and if possible, let O[$f((\overline{a}+ i\overline{b}))$] be finite, say k.

Then, as proceeded above,

O[$f((\overline{a}+ i\overline{b}))$] =k=>O (($\overline{a}+ i\overline{b}$)) $\leq k$, leading us to conclude that O (($\overline{a}+ i\overline{b}$)) is finite.

This contradicts the hypothesis that O (($\overline{a}+ i\overline{b}$))is infinite.

Thus whenever O (($\overline{a}+ i\overline{b}$)) is infinite, then O[$f((\overline{a}+ i\overline{b}))$] is also infinite.

Hence O (($\overline{a}+ i\overline{b}$)) = O[f (($\overline{a}+ i\overline{b}$))].

Proposition 1.2.3:

The relation ofcomplex fuzzy number isomorphism on the set of allcomplex fuzzy number groups is an equivalence relation.

Proof

The relation $\cong$of complex fuzzy number isomorphism on the set of all complex fuzzy number groups satisfies the following properties.

i) Reflexivity

Let ($F^{*}(C)$, *) be acomplex fuzzy number group. Then the complex fuzzy number identity mapping**I** :$F^{*}(C) \rightarrow F^{*}(C)$: **I** $((\overline{x} + i\overline{y})) = (\overline{x} + i\overline{y})$, for all $\overline{x},\overline{y} \in F^{*}(R)$ is clearly, one-one and onto.

Also, **I** $((\overline{a} + i\overline{b}) . (\overline{c} + i\overline{d})) = (\overline{a} + i\overline{b}) . (\overline{c} + i\overline{d})$ = **I** $((\overline{a} + i\overline{b}))$ **.** **I** $((\overline{c} + i\overline{d}))$, for all $\overline{a}, \overline{b}, \overline{c}, \overline{d} \in F^{*}(R)$.

Thus **I** is acomplex fuzzy number isomorphism and so ($F^{*}(C)$,*) ≅ ($F^{*}(C)$, *).

That is every complexfuzzy number group is isomorphic to itself.

ii) Symmetry

Let ($F^{*}(C)$, ●) and ($F^{*\prime}(C)$, *) be two complex fuzzy number groups such that

($F^{*}(C)$, ●) ≅ ($F^{*\prime}(C)$, *) and let f be the corresponding complex fuzzy number isomorphism.

Then **f** being one-one and onto, it is invertible.

Let$\boldsymbol{f}^{-1}$: $F^{*\prime}(C) \rightarrow F^{*}(C)$: $\boldsymbol{f}^{-1}((\overline{c} + i\overline{d})) = (\overline{x} + i\overline{y})$ if and only if $\boldsymbol{f}((\overline{x} + i\overline{y})) = (\overline{c} + i\overline{d})$.

It is easy to verify that $\boldsymbol{f}^{-1}$ is one-one and onto.

Moreover, if$(\overline{a}' + i\overline{b}')$and $\overline{b}'$ be arbitrary elements of $F^{*\prime}(R)$, then $\boldsymbol{f}$being one-one, onto, there exist unique elements $\overline{a}$ and $\overline{b}$ such that $\boldsymbol{f}((\overline{a} + i\overline{b})) = (\overline{a}' + i\overline{b}')$ and$\boldsymbol{f}((\overline{c} + i\overline{d})) = \overline{b}'$.

Consequently,

$\boldsymbol{f}^{-1}((\overline{a}' + i\overline{b}')) = (\overline{a} + i\overline{b})$ and $\boldsymbol{f}^{-1}(\overline{b}') = (\overline{c} + i\overline{d})$

Therefore $\boldsymbol{f}^{-1}((\overline{a}' + i\overline{b}')\overline{b}') = \boldsymbol{f}^{-1}[f((\overline{a} + i\overline{b})) . f((\overline{c} + i\overline{d}))]$

Since$\boldsymbol{f}$is composition preserving,

$$= \boldsymbol{f}^{-1}[\boldsymbol{f}((\overline{a} + i\overline{b}). (\overline{c} + i\overline{d}))]$$

$=(\overline{a} + i\overline{b}). (\overline{c} + i\overline{d})$

$= \boldsymbol{f}^{-1}((\overline{a}' + i\overline{b}')). \boldsymbol{f}^{-1}(\overline{b}')$.

This shows that$\boldsymbol{f}^{-1}$ is composition preserving. Thus $\boldsymbol{f}^{-1}$ is an isomorphism.

And so ($F^{*\prime}(C)$,*) ≅($F^{*}(C)$, •)

Thus ($F^*(C)$,●) ≅ ($F^{*\prime}(C)$,*)

⇒ ($F^{*\prime}(C)$, *) ≅ ($F^*(C)$, •)

iii) Transitivity

Let ($F^{*\prime}(C)$, ●), ($F^{*\prime}(C)$, *) and ($F^{*\prime\prime}(C)$, Δ) be any three complex fuzzy number groups given in such a way that

($F^*(C)$, ●) ≅($F^{*\prime}(C)$, *) and ($F^{*\prime}(C)$, *) ≅($F^{*\prime\prime}(C)$, Δ)

Let f and g be the corresponding complex fuzzy number isomorphisms from $F^*(C)$ to $F^{*\prime}(C)$ and from $F^{*\prime}(C)$ to $F^{*\prime\prime}(C)$.

Then each one of $\boldsymbol{f}$ and $\boldsymbol{g}$is one-one, onto and composition preserving.

Now, consider the mapping,

$\boldsymbol{g} . \boldsymbol{f}$: $F^*(C) \to F^{*\prime}(C)$: $(\boldsymbol{g}.\boldsymbol{f})\,((\,\overline{x} + i\overline{y}\,)) = g\,[\boldsymbol{f}\,((\,\overline{x} + i\overline{y}\,))]$ for all$\overline{x}$, $\overline{y} \in F^*(R)$.

Then $\boldsymbol{g} . \boldsymbol{f}$ is one-one, since

$(\boldsymbol{g}.\boldsymbol{f}\,)\,(\,(\,\overline{a} + i\overline{b}\,)\,) = (\,\boldsymbol{g}.\boldsymbol{f})\,((\,\overline{c} + i\overline{d}\,))$

$\Rightarrow \boldsymbol{g}\,[\boldsymbol{f}\,((\,\overline{a} + i\overline{b}\,))\,] = \boldsymbol{g}[\boldsymbol{f}\,((\,\overline{c} + i\overline{d}\,))]$

Since $\boldsymbol{g}$ is one-one,

$\Rightarrow \boldsymbol{f}\,((\,\overline{a} + i\overline{b}\,)) = \boldsymbol{f}\,((\,\overline{c} + i\overline{d}\,))$

Since $\boldsymbol{f}$ is one-one,

$\Rightarrow (\,\overline{a} + i\overline{b}\,) = (\,\overline{c} + i\overline{d}\,)$

Now, let$(\,\overline{a}'' + i\overline{b}''\,) \in F^{*\prime\prime}(C)$ then g being one-one and onto, there exists

$\overline{a}', \overline{b}' \in F^{*\prime}(R)$ such that $\boldsymbol{g}\,((\,\overline{a}' + i\overline{b}'\,)) = (\,\overline{a}'' + i\overline{b}''\,)$.

Also $\boldsymbol{f}$being one-one and onto, there exist$\overline{a}$, $\overline{b} \in F^{*\prime}(R)$ such that,

$\boldsymbol{f}\,((\,\overline{a} + i\overline{b}\,)) = (\,\overline{a}' + i\overline{b}'\,)$

Thus$(\,\overline{a}'' + i\overline{b}''\,) = \boldsymbol{g}\,((\,\overline{a}' + i\overline{b}'\,))$

$= g[f\,((\,\overline{a} + i\overline{b}\,))]$

$= [\boldsymbol{g}.\boldsymbol{f}]((\,\overline{a} + i\overline{b}\,))$

So, $\boldsymbol{g}.\boldsymbol{f}$ is onto.

Moreover for all$\overline{a}$, $\overline{b} \in F^*(R)$, we have

$(\boldsymbol{g} \,.\, \boldsymbol{f})((\overline{a}+ i\overline{b}) \,.\, (\overline{c}+ i\overline{d})) = \boldsymbol{g}\,[\,\boldsymbol{f}\,((\overline{a}+ i\overline{b}) \,.\, (\overline{c}+ i\overline{d}))]$,

Since $\boldsymbol{f}$ is composition preserving,

$(\boldsymbol{g} \,.\, \boldsymbol{f})((\overline{a}+ i\overline{b}) \,.\, (\overline{c}+ i\overline{d})) = \boldsymbol{g}[f((\overline{a}+ i\overline{b})).\boldsymbol{f}((\overline{c}+ i\overline{d}))]$

Since$\boldsymbol{g}$ is composition preserving,

$$= \boldsymbol{g}\,[\,\boldsymbol{f}\,((\overline{a}+ i\overline{b}))\,]\ \Delta\, \boldsymbol{g}\,[\,\boldsymbol{f}\,((\overline{c}+ i\overline{d}))]$$

$= (\boldsymbol{g} \,.\, \boldsymbol{f})\,((\overline{a}+ i\overline{b}))\Delta\,(\boldsymbol{g} \,.\, \boldsymbol{f})\,((\overline{c}+ i\overline{d}))$

This shows that $(\boldsymbol{g} \,.\, \boldsymbol{f})$ is composition preserving.

Thus $(F^{*}(C),\bullet) \cong (F^{*\prime}(C), *)$ and $(F^{*\prime}(C), *) \cong (F^{*\prime\prime}(C), \Delta)$

$\Rightarrow (F^{*}(C), \bullet) \cong (F^{*\prime\prime}(C), \Delta)$

Hence the relation of fuzzy number isomorphism in the set of all fuzzy number groups is an equivalence relation.

The following theorem is, **TRANSFERENCE OF COMPLEX FUZZY NUMBER GROUP STRUCTURE.**

Proposition 1.2.4:

Let $(F^{*}(C), \bullet)$ be a complex fuzzy number group and $F^{*\prime}(C)$ be a set with binary composition *. Let $(\overline{a}+ i\overline{b})= [(\overline{a}+ i\overline{b}), (\overline{a}+ i\overline{b})]$ and let $\boldsymbol{f}$ be a one-one mapping of $F^{*}(C)$ on to $F^{*\prime}(C)$, defined in such a way that $\boldsymbol{f}\,((\overline{a}+ i\overline{b}).\,\overline{b}) = \boldsymbol{f}\,((\overline{a}+ i\overline{b})).\,\boldsymbol{f}\,(\overline{b})$ for all $\overline{a}, \overline{b} \in F^{*}(R)$.

Then $(F^{*\prime}(C), *)$ is a complex fuzzy number group, isomorphic to $(F^{*}(C), \bullet)$.

Proof

We observe that * on $F^{*\prime}(C)$ satisfies the following properties.

(i) Associativity:

Let $(\overline{a}+ i\overline{b})$, $(\overline{c}+ i\overline{d})$, $(\overline{e}+ i\overline{f})$ be any three arbitrary elements of $F^{*\prime}(C)$.

Then $\boldsymbol{f}$ being one-one and onto there exist unique elements $(\overline{a}+ i\overline{b})$, $(\overline{c}+ i\overline{d})$, $(\overline{e}+ i\overline{f})$ in $F^{*}(C)$ such that $\boldsymbol{f}\,((\overline{a}+ i\overline{b})) = (\overline{a}' + i\overline{b}')$, $\boldsymbol{f}\,((\overline{c}+ i\overline{d})) = (\overline{c}' + i\overline{d}')$and $\boldsymbol{f}\,((\overline{e}+ i\overline{f})) = (\overline{e}' + i\overline{f}')$

Therefore $((\overline{a}' + i\overline{b}')*(\overline{c}' + i\overline{d}')) * (\overline{e}' + i\overline{f}') = [\,\boldsymbol{f}\,((\overline{a}+ i\overline{b})) * \boldsymbol{f}\,((\overline{c}+ i\overline{d}))] * \boldsymbol{f}\,((\overline{e}+ i\overline{f}))$

$$= \boldsymbol{f}\,((\overline{a}+ i\overline{b}) \,.\, \overline{b}) * \boldsymbol{f}\,((\overline{e}+ i\overline{f}))$$

Since $\boldsymbol{f}((\bar{a}+i\bar{b}))*\boldsymbol{f}((\bar{c}+i\bar{d}))=\boldsymbol{f}((\bar{a}+i\bar{b}).(\bar{c}+i\bar{d}))$,

$$=\boldsymbol{f}[((\bar{a}+i\bar{b}).(\bar{c}+i\bar{d})).(\bar{e}+i\bar{f})]$$

By associativity of in $F^*(C)$,

$$=\boldsymbol{f}[(\bar{a}+i\bar{b}).((\bar{c}+i\bar{d}).(\bar{e}+i\bar{f}))]$$

$$=\boldsymbol{f}((\bar{a}+i\bar{b}))*\boldsymbol{f}((\bar{c}+i\bar{d}).(\bar{e}+i\bar{f}))$$

$$=\boldsymbol{f}((\bar{a}+i\bar{b}))*[\boldsymbol{f}((\bar{c}+i\bar{d}))*\boldsymbol{f}((\bar{e}+i\bar{f}))]$$

$$=(\bar{a}'+i\bar{b}')\ ((\bar{c}'+i\bar{d}')*(\bar{e}'+i\bar{f}')).$$

Thus $((\bar{a}'+i\bar{b}')*(\bar{c}'+i\bar{d}'))*(\bar{e}'+i\bar{f}')=\bar{a}'\ ((\bar{c}'+i\bar{d}')*(\bar{e}'+i\bar{f}')$.

This shows that * is associative on $F^{*\prime}(C)$.

ii) Existence of identity :

Let $(\bar{e}+i\bar{e})$ be the identity in $F^*(C)$ and let $(\bar{a}'+i\bar{b}')\in F^{*\prime}(C)$. Then $\boldsymbol{f}$being one-one and onto there exist $(\bar{a}+i\bar{b})\in F^*(C)$, such that $f(\bar{a})=(\bar{a}'+i\bar{b}')$.

Now, $(\bar{a}+i\bar{b}).(\bar{e}+i\bar{e})=(\bar{e}+i\bar{e}).(\bar{a}+i\bar{b})=(\bar{a}+i\bar{b})$

But $(\bar{e}+\bar{e}).(\bar{a}+i\bar{b})=(\bar{a}+i\bar{b})$

$\Rightarrow\boldsymbol{f}((\bar{e}+i\bar{e}).(\bar{a}+i\bar{b}))=\boldsymbol{f}((\bar{a}+i\bar{b}))$

$\Rightarrow\boldsymbol{f}((\bar{e}+i\bar{e})).\boldsymbol{f}((\bar{a}+i\bar{b}))=\boldsymbol{f}((\bar{a}+i\bar{b}))$

$\Rightarrow\boldsymbol{f}((\bar{e}+i\bar{e}))*(\bar{a}'+i\bar{b}')=(\bar{a}'+i\bar{b}')$.

Similarly $(\bar{a}+i\bar{b}).\bar{e}=(\bar{a}+i\bar{b})$

$\Rightarrow(\bar{a}'+i\bar{b}')*f(\bar{e})=(\bar{a}'+i\bar{b}')$.

Thus, $\boldsymbol{f}((\bar{e}+i\bar{e}))*(\bar{a}'+i\bar{b}')=(\bar{a}'+i\bar{b}')*\boldsymbol{f}((\bar{e}+i\bar{e}))=(\bar{a}'+i\bar{b}')$for all $\bar{a}'$, $\bar{b}'\in F^{*\prime}(R)$.

This shows that if $(\bar{e}+i\bar{e})$ is the identity in $F^*(C)$, then $\boldsymbol{f}((\bar{e}+i\bar{e}))$ is the identity in$F^{*\prime}(C)$.

iii) Existence of inverses :

Let $(\bar{a}'+i\bar{b}')\in F^{*\prime}(C)$. Then there exist $(\bar{a}+i\bar{b})\in F^*(C)$, such that $\boldsymbol{f}((\bar{a}+i\bar{b}))=(\bar{a}'+i\bar{b}')$.

Now $(\overline{a}+i\overline{b}).(\overline{a}+i\overline{b})^{-1} = (\overline{a}+i\overline{b})^{-1}.(\overline{a}+i\overline{b}) = (\overline{e}+i\overline{e})$

But $(\overline{a}+i\overline{b}).(\overline{a}+i\overline{b})^{-1} = (\overline{e}+i\overline{e})$

$\Rightarrow \boldsymbol{f}((\overline{a}+i\overline{b}).(\overline{a}+i\overline{b})^{-1}) = \boldsymbol{f}((\overline{e}+i\overline{e}))$

$\Rightarrow \boldsymbol{f}((\overline{a}+i\overline{b})) * \boldsymbol{f}((\overline{a}+i\overline{b})^{-1}) = \boldsymbol{f}((\overline{e}+i\overline{e}))$

$\Rightarrow (\overline{a}^{'}+i\overline{b}^{'}) * \boldsymbol{f}((\overline{a}+i\overline{b})^{-1}) = \boldsymbol{f}((\overline{e}+i\overline{e}))$.

Similarly, $(\overline{a}+i\overline{b})^{-1}.(\overline{a}+i\overline{b}) = (\overline{e}+i\overline{e})$

$\Rightarrow \boldsymbol{f}((\overline{a}+i\overline{b})^{-1}) * (\overline{a}^{'}+i\overline{b}^{'}) = \boldsymbol{f}((\overline{e}+i\overline{e}))$.

Therefore $(\overline{a}^{'}+i\overline{b}^{'}) * \boldsymbol{f}((\overline{a}+i\overline{b})^{-1}) = \boldsymbol{f}((\overline{a}+i\overline{b})^{-1}) * (\overline{a}^{'}+i\overline{b}^{'}) = \boldsymbol{f}((\overline{e}+i\overline{e}))$.

This shows that $[(\overline{a}^{'}+i\overline{b}^{'})]^{-1} = \boldsymbol{f}((\overline{a}+i\overline{b})^{-1}) \in F^{*'}(C)$.

Thus, each element in $F^{*'}(C)$ has its inverse in $F^{*'}(C)$.

Hence ($F^{*'}(c)$, *) is acomplex fuzzy number group.

Now ($F^{*}(C)$, ●) and ($F^{*'}(C)$, *) are the complex fuzzy number groups and f is a one- one onto and composition preserving mapping from $F^{*}(C)$ to $F^{*'}(C)$.

So,$\boldsymbol{f}$ is an isomorphism.

Hence ($F^{*'}(C)$., *) is a complex fuzzy number group isomorphic to ($F^{*}(C)$, ●)

Conclusion

In this article we conclude that the concept of Homomorphism and Isomorphism of complex fuzzy number group and proved some properties of this concept. We give the basic properties ofHomomorphism of complex fuzzy number group, Isomorphismof complex fuzzy number group . We present a further investigation into properties of Homomorphism and Isomorphism of complex fuzzy number group. In future Certainly some other topics of complex fuzzy number group will be discussed.

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