Path Decomposition of Restricted Super Line Graph of Cycle Graph

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Abstract:

A decomposition of a graph *G* is a collection ψ of graphs $H_1, H_2, ...H_r$ of *G* such that every edge of *G* belongs to exactly one of H_i. If each H_i is a path, then ψ is called the path decomposition of *G*. In this paper, we discussed the path decomposition of the restricted super line graph of index 2 of *G* when *G* is isomorphic to the Cycle graph

Keywords: Path decomposition, restricted super line graph.

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1. Introduction:

The fundamental concept of path decomposition in graphs as introduced by Harary [7] continues to be of interest to researchers due to its wide range of applications in real life. The study on decomposition in paths helps us to understand, analyse and design networks effectively. Research in this area helps us analyse problems in transportation, distribution, designing, communication, team formation and event management. Extensive research has been dedicated to the study of various types of decompositions and related parameters in [1, 2, 3, 4, 6] in the context of paths, cycles and common vertices between the paths.

Graph decomposition problems rank among the most prominent areas of research in graph theory and combinatorics and further, it has numerous applications in various fields such as networking, block designs, and bioinformatics.A path decomposition of a graph *G* is a partition of edges into subgraphs H_i each of which is a path or a union of paths (linear forests). Various types of decompositions and corresponding parameters have been studied by several authors by imposing different conditions on H_i. Some of such decompositions are path decomposition, cyclic decomposition,acyclic decomposition etc.

Let G = (V, E) be a simple graph without loops or multiple edges. A path is a walk where $v_i \neq v_{j,} i \neq j$. In other words, a path is a walk that visits each vertex at most once. A decomposition of a graph G is a collection of edge-disjoint subgraphs $G_{1,} G_2, \dots . G_n$ of G such that every edge of G belongs to exactly one $G_i, 1 \leq i \leq m$. $E(G) = E(G_1) \cup E(G_2) \dots E(G_m)$. If every graph $G_i$is a path then the decomposition is called a path decomposition.All graphs considered in this paper are simple graphs. Restricted Super line graph of index r of a graph G, denoted by $RL_r(G)$, is introduced by Manjula and Sooryanarayana[8,9].It is a modificationof the concept of the super line graph $L(G)$ introduced by Bagga [5]. The vertices of $RL_r(G)$ are the r-element subsets of $E(G)$ and two vertices S and T are adjacent if there exists exactly one pair of edges, one from each of the sets S and T, which are adjacent in G.

We need a few observations to obtain the result. First consider an $n \times m$ array $R_{n,m}$ of points where a point in i^{th} row and j^{th} column is identified with the edge (x_i, y_j) of a graph G on which the vertex sets $\{x_1, x_2, \dots . x_n, y_1, y_2, \dots y_m\}$are defined. Any path on the points $R_{n,m}$ with properties (i) travels only along rows .(ii)uses at most two points from any row or column and (iii) whose end points does not lie in the same row or column defines a unique path in G.If a path with (i),(ii)&(iii) in $R_{n,m}$ uses N points then the corresponding path in G uses exactly N-1 edges and has no repeated vertices.

Now the problem of decomposing $RL_2(G)$ into paths $P_i, i \leq 2n-10$ reduces to covering of $RL_2(G)$ with paths using different points and each satisfying conditions (i),(ii)&(iii).

2. Main results:

Theorem 2.1:

$$\psi(RL_2(C_n) = \psi(RL_2(P_n) \cup X \quad for\ even\ n$$
$$= \psi(RL_2(P_n) \cup Y \quad for\ odd\ n,\ n \geq 7$$
$$where\ X = \bigcup_{i=7}^{n} P_{2i-6} \cup \frac{3(n-6)}{2}(P_9 \cup P_7 \cup P_6) \cup \frac{(n-7)(n-6)}{2} P_3 \cup P_{2n-7} \cup P_{2n-8} \bigcup_{i=8}^{n} P_{2i-12} \cup \frac{3n-12}{2} P_3$$

$$and\ Y = \bigcup_{i=7}^{n} P_{2i-6} \cup P_{2n-7} \cup P_{2n-8} \bigcup_{i=7}^{n} 2P_{2i-12} \cup P_9 \cup \frac{3n-19}{2} P_7 \cup P_6 \cup \frac{3n-13}{2} P_3 \cup P_2$$

Proof: As $P_n \subset C_n$,in [1] it is proved that if $H \subset G$, then $RL_2(H) \subset RL_2(G)$.So $RL_2(P_n) \subset RL_2(C_n)$. Let $V(C_n) = \{v_i, 1 \leq i \leq n\}$ & $E(C_n) = E(P_n) \cup \{e_n e_1\}$

Thus $V(RL_2(C_n)) = V(RL_2(P_n)) \cup e_i e_n\ for\ 1 \leq i < n$ .So

$$\psi(RL_2(C_n)) = \psi(RL_2(P_n)) \cup \psi\left(E\begin{pmatrix} e_ie_n, e_je_k \\ 1 \le i < n, \\ 1 \le j,k \le n, j < k \end{pmatrix}\right) for\, 1 \le i < n$$

$$\psi(E(e_ie_n, e_je_k)) = \left\{ \begin{matrix} \psi\left(E\begin{pmatrix} e_i, e_n \ \& \ e_k, e_l \\ 1 \le i < n \\ 1 \le k < l \le 4 \end{pmatrix}\right) \cup \psi\left(E\begin{pmatrix} e_i, e_n \ \& \ e_k, e_5 \\ 1 \le i < n \\ 1 \le k < 5 \end{pmatrix}\right) \cup \psi\left(E\begin{pmatrix} e_i, e_n \ \& \ e_k, e_6 \\ 1 \le i < n \\ 1 \le k < 6 \end{pmatrix}\right) \cup \\ \psi\left(E\begin{pmatrix} e_i, e_j \ \& \ e_k, e_7 \\ 1 \le i < n \\ 1 \le k < 7 \end{pmatrix}\right) \cup .... \cup \psi\left(E\begin{pmatrix} e_i, e_n \ \& \ e_k, e_{n-2} \\ 1 \le i < n \\ 1 \le k < n-2 \end{pmatrix}\right) \\ \cup \psi\left(E\begin{pmatrix} e_i, e_n \ \& \ e_k, e_{n-1} \\ 1 \le i < n, 1 \le k < n-1 \end{pmatrix}\right) \cup \psi\left(E\begin{pmatrix} e_i, e_n \ \& \ e_i, e_n \\ 1 \le i < n \end{pmatrix}\right) ------(1) \end{matrix} \right\}$$

So, the path decomposition between these vertices e_ie_n, e_je_k, can be obtained by partitioning the edges between these vertices as follows.

Path decomposition between the vertices $e_i, e_n \ \& \ e_k, e_l; 1 \le i < n, 1 \le k < l < n,$ is given as in table 2.1. Thus these edges are decomposed into $P_9 \cup P_6 \cup \frac{(n-6)}{2} P_7 \cup P_4 \cup P_2$

Path decomposition between the vertices $e_ie_n \ \& \ e_ke_5$ are given as shown in table 2.2

And are decomposed into $P_8 \cup P_7 \cup \frac{(n-8)}{2} P_3 \cup P_2$

Path decomposition between the vertices $e_ie_n \ \& \ e_ke_6, 1 \le i < n, 1 \le k < 6,$ is given as in table 2.3. Further none of the vertex $e_je_n, j > 6$ is adjacent to $e_ke_6, k \ne 1$. Thus these edges are decomposed into $P_{10} \cup P_7 \cup \frac{(n-6)}{2} P_3$

Table 2.1	$e_{1,2}$	$e_{1,3}$	$e_{1,4}$	$e_{2,3}$	$e_{2,4}$	$e_{3,4}$
$e_{1,n}$		1	1	1	1	
$e_{2,n}$			1			1
$e_{3,n}$		1	1			1
$e_{4,n}$	1		1	1		(1)
$e_{5,n}$	1	1			1	1
$e_{6,n}$	1	1	1			
$e_{7,n}$	1	1	1			
$e_{8,n}$	1	1	1			
$e_{9,n}$	1	1	1			
$e_{7,j}$ . .						
$e_{n-3,n}$	1	1	1			
$e_{n-2,n}$	1		1			
$e_{n-1,n}$	1	1				

Table 2.2	$e_{1,5}$	$e_{2,5}$	$e_{3,5}$	$e_{4,5}$
$e_{1,n}$	1	1		
$e_{2,n}$			1	
$e_{3,n}$	1	1		1
$e_{4,n}$		1		1
$e_{5,n}$	1			1
$e_{6,n}$		1	1	1
$e_{7,n}$	○			
$e_{8,n}$	1			
$e_{9,n}$	1			
$e_{7,j}$ . .				
$e_{n-3,n}$	1			
$e_{n-2,n}$	1			
$e_{n-1,n}$	1			

Path decomposition between the vertices $e_i e_n$ & $e_k e_7, 1 \le i < n,\ 1 \le k < 7,$ is given as in table 2.4. Further none of the vertex $e_j e_n,\ j > 7$ is adjacent to $e_k e_7, k \ne 1$. Thus these edges are decomposed into $P_{12} \cup P_7 \cup \frac{(n-4)}{2} P_3 \cup P_2$

By continuing in the similar way Path decomposition between the vertices $e_i e_n$ & $e_k e_{n-2}, 1 \le i < n,\ 1 \le k < n,$ is given as $P_{2n-6} \cup P_7 \cup \frac{(3n-22)}{2} P_3 \cup P_2$

Path decomposition between the vertices $e_i e_n$ & $e_k e_{n-1}, 1 \le i < n,\ 1 \le k < n-1$ is given as shown in table 3.5 and is $P_{2n-7} \cup P_{2n-8} \cup 2P_{2n-12} \cup 2P_{2n-16} .... \cup 2P_4$

Path decomposition between the vertices $e_i e_n$ & $e_i e_n, 1 \le i < n,$ is given as

$e_1 e_n, e_3 e_n, e_2 e_n --P_3; e_1 e_n, e_4 e_n, e_3 e_n --P_3; e_1 e_n, e_5 e_n, e_4 e_n --P_3;.....$
$e_1 e_n, e_{4n-2} e_{n,} e_{n-1} e_n --P_3$ &

$e_2 e_n, e_{n-1} e_n, e_3 e --P_3; e_4 e_n, e_{n-1} e_n, e_5 e_n --P_3; .... e_{n-4} e_n, e_{n-1} e_n, e_{n-3} e_n --P_3$

Thus there are $\frac{(3n-12)}{2} P_3$ distinct paths between these vertices.

Table 2.3	$e_{1,6}$	$e_{2,6}$	$e_{3,6}$	$e_{4,6}$	$e_{5,6}$
$e_{1,n}$	1	1			
$e_{2,n}$			1		
$e_{3,n}$	1	1		1	
$e_{4,n}$	1		1		1
$e_{5,n}$		1	1		1
$e_{6,n}$	1				1
$e_{7,n}$		1	1	1	1
$e_{8,n}$	1				
$e_{9,n}$	1				
$e_{7,j}$ . .					
$e_{n-3,n}$	1				
$e_{n-2,n}$	1				
$e_{n-1,n}$	1				

Table 2.4	$e_{1,7}$	$e_{2,7}$	$e_{3,7}$	$e_{4,7}$	$e_{5,7}$	$e_{6,7}$
$e_{1,n}$	1	1				
$e_{2,n}$			1			
$e_{3,n}$	1	1		1		
$e_{4,n}$	1		1		1	
$e_{5,n}$				1		1
$e_{6,n}$	1	1	1	1		1
$e_{7,n}$	1					1
$e_{8,n}$	1	1	1	1	1	1
$e_{9,n}$	1					
$e_{7,j}$ . .						
$e_{n-3,n}$	1					
$e_{n-2,n}$	1					
$e_{n-1,n}$	1					

Hence there are

$$\left(P_9 \cup P_6 \cup \frac{(n-6)}{2} P_7 \cup P_4 \cup P_2\right) \cup \left(P_8 \cup P_7 \cup \frac{(n-8)}{2} P_3 \cup P_2\right) \cup \left(P_{10} \cup P_7 \cup \frac{(n-6)}{2} P_3\right)........$$

$$\cup.\left(P_{2n-6} \cup P_7 \cup \frac{(3n-22)}{2} P_3\right) \cup \left(P_{2n-7} \cup P_{2n-8} \cup 2P_{2n-12} \cup 2P_{2n-16}.......... \cup 2P_4\right) \cup \left(\frac{(3n-12)}{2} P_3\right)$$

$$\Rightarrow \left(P_{2n-6} \cup P_{2n-8} \cup 2P_{2n-10} \cup 2P_{2n-16}..........P_{10} \cup P_8\right) \cup \left(\frac{(n-1)}{2} + 1 + 1....(n-6)\right) P_7 \cup P_6 \cup P_9 \cup$$

$$\left(\frac{(3n-22)}{2} + \frac{(3n-24)}{2} + ... \frac{(n-6)}{2} + \frac{(n-8)}{2}\right) P_3 \cup \left(P_{2n-7} \cup P_{2n-8} \cup 2P_{2n-12} \cup 2P_{2n-16}.......... \cup 2P_4\right)$$

$$\Rightarrow \bigcup_{i=7}^{n} P_{2n-6} \cup P_{2n-7} \cup P_{2n-8} \bigcup_{i=8}^{n} P_{2n-12} \cup P_9 \cup P_8 \cup \frac{3(n-6)}{2} P_7 \cup P_6 \bigcup \frac{(n^2-10n-7)}{2} P_3$$

$$\textit{Therefore } \psi\left(RL_2(C_n)\right) = \psi\left(RL_2(P_n)\right) \cup \bigcup_{i=7}^{n} P_{2n-6} \cup P_{2n-7} \cup P_{2n-8} \bigcup_{i=8}^{n} P_{2n-12} \cup P_9 \cup P_8 \cup \frac{3(n-6)}{2} P_7 \cup P_6 \bigcup$$

$$\frac{(n^2-10n-7)}{2} P_3 \quad \textit{for } 1 \le i < n \quad \textit{for even } n$$

Table 2.5	$e_{1,\kappa-1}$	$e_{2,\kappa-1}$	$e_{3,\kappa-1}$	$e_{4,\kappa-1}$	$e_{5,\kappa-1}$	$e_{6,\kappa-1}$	$e_{7,\kappa-1}$	$e_{8,\kappa-1}$	……	$e_{\kappa-5,\kappa}$	$e_{\kappa-6,\kappa}$	$e_{\kappa-5,\kappa}$	$e_{\kappa-4,\kappa}$	$e_{\kappa-3,\kappa-1}$	$e_{\kappa-2,\kappa-1}$
$e_{1,\kappa}$			1	1	1	1	1	1	……	1	1	1	1	1	1
$e_{2,\kappa}$		1		1	1	1	1	1	……	1	1	1	1	1	1
$e_{3,\kappa}$			1		1	1	1	1	……	1	1	1	1	1	1
$e_{4,\kappa}$		1		1		1	1	1	……	1	1	1	1	1	1
$e_{5,\kappa}$		1	1		1		1	1	……	1	1	1	1	1	1
$e_{6,\kappa}$		1	1	1		1		1	……	1	1	1	1	1	1
$e_{7,\kappa}$		1	1	1			1		……	1	1	1	1	1	1
$e_{8,\kappa}$		1	1	1	1	1		1	……		1	1	1	1	1
$e_{9,n}$		1	1	1	1	1	1		……			1	1	1	1
$e_{10,n}$ ⋮									……						
$e_{\kappa-7,\kappa}$		1	1	1	1	1	1	1	……	1		1	1	1	1
$e_{\kappa-6,\kappa}$		1	1	1	1	1	1	1	……		1		1	1	1
$e_{\kappa-5,\kappa}$		1	1	1	1		1	1	……	1		1		1	1
$e_{\kappa-4,\kappa}$		1	1	1	1	1		1	……						1
$e_{\kappa-3,\kappa}$		1	1	1	1	1	1		……	1	1		1		
$e_{\kappa-2,\kappa}$									……						
$e_{\kappa-1,\kappa}$		1	1	1	1	1	1	1	……	1	1	1	1		

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