Reliability Analysis and Statistical Fitting for the Transmuted Weibull Model in R

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Abstract

Accelerated life testing is a fundamental practice in reliability engineering, allowing the evaluation of component or device performance over extended lifetimes impractical to encounter during design. This study delves into the application of the transmuted Weibull distribution to model lifetime data, showcasing its versatility in real-world scenarios. The evaluation includes critical metrics such as Akaike's information criterion (AIC), Bayesian information criterion (BIC), coefficient of determination, and standard error for distribution comparison. Utilizing Maximum Likelihood Estimation (MLE) for parameter estimation, a simulation study is conducted with varying sample sizes, and the R programming language is employed for in-depth analysis. Real data analysis involves comparing the Transmuted Weibull (twd) model with other models using goodness-of-fit criteria. Maximum Likelihood Estimates (MLEs) are obtained, and the likelihood ratio test demonstrates the (twd) model's superior alignment with the data. The study concludes with the simplicity of producing Quick Fit plots for analysis using R software. The presented approach provides a comprehensive understanding of reliability characteristics, combining theoretical insights with practical applications and numerical analyses

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1. Introduction

A key component of reliability engineering is accelerated life testing. In order to evaluate the performance of a component or device over lifetimes that would be impractical to encounter under design conditions at the time of product introduction, it is a means to shorten the time to failure. Identification of stress factors that can be changed in a controlled manner during testing to hasten the degradation of component materials is the key to this testing. The study of dependability benefits from the modelling of failure times. Therefore, probability distributions that link a given value of the examined variable with the chance of occurrence must be used in order to statistically model the objects under study [1].

Exponential, Gamma, Lognormal, and Weibull distributions are those that are most frequently employed to represent failure times, according to [2]. Choosing the distribution that most closely matches the failure times constitutes the analysis [3].function, and average time to failure after specifying the distribution that describes the data [4] Software that identifies the distributions that fit the failure times the best is typically used for this modelling. In contrast to software that solely supports analytical computations, R [5] permits the application of numerical and analytical approaches. Additionally, a few techniques can be employed to establish or suggest which model describes the failure time data more accurately. Graphs and numeric techniques can be separated out of this group. One graphical method is the paper of probability, which linearizes the accumulated density function [6]. Additionally, the authors in [15] proposed an exploration of integral equations, with a particular focus on Volterra integral equations. The authors in[16] proposed an algorithmic application for transforming positive original responses in an academic setting. Finally, the authors in [17, 18] used some statistical analysis methods to discuss the investigation of the relationship between petroleum prices and the real exchange rate in Iraq. Also, they described the studies of Iraq's economy, respectively. In this study, we emphasise showing how the transmuted Weibull distribution can be used to describe lifetime, using examples from actual data. The transmuted Weibull distribution, also known as the three-parameter Weibull distribution, is used to model data sets. The Akaike's information criterion(AIC), the Bayesian information criterion(BIC), the coefficient of determination, and the standard error are used to compare the distributions. The maximum likelihood method is used to estimate the parameters of the probability distributions.

2. Methodology

The approach to working with the Weibull distribution is presented in this section. When a random variable X has the following probability density function (pdf), it is said to have a Weibull distribution with parameters $\eta > 0$ and $\sigma = 1$

$$g\,(x) = \frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)^{\eta}}, \mathrm{x} > 0. \qquad (1)$$

The probability density function (PDF) of the Weibull distribution is depicted in Figure 1 for different values of the shape parameter (η) while keeping the scale parameter (σ) fixed at 1.

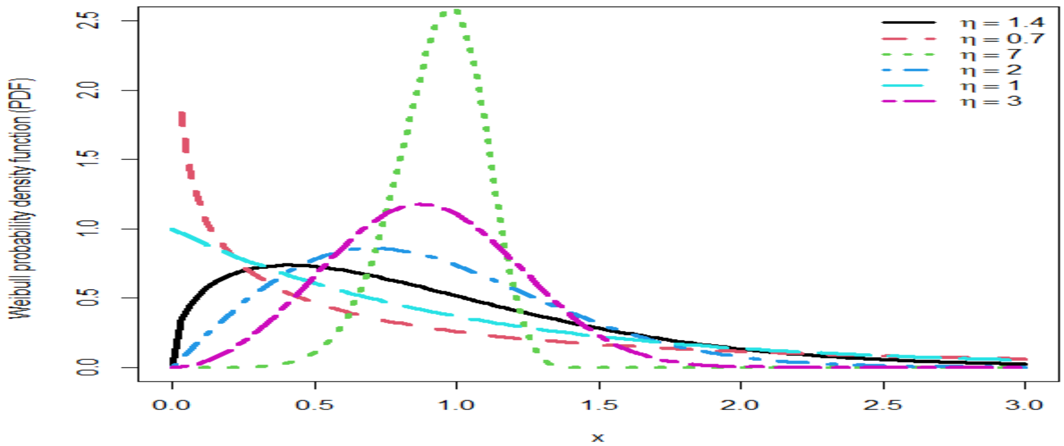
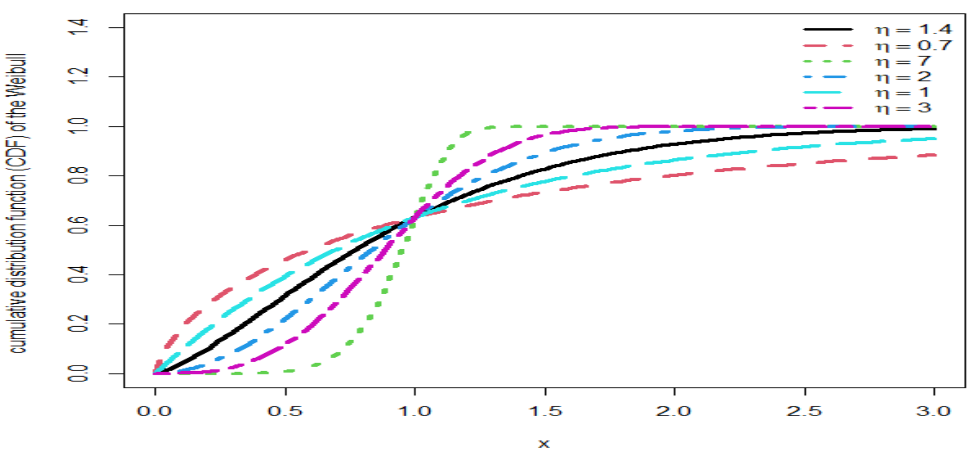


Figure 1: probability density function (PDF) of the Weibull distribution for different values of (η) and (σ) =1.

The cumulative distribution function (cdf) of the Weibull Distribution can be expressed as follows.

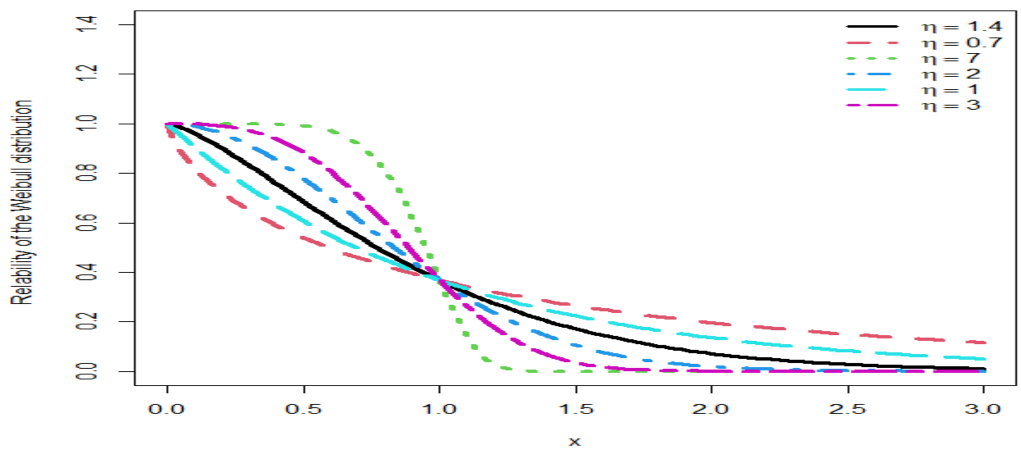
$$G(x) = 1 - e^{-\left(\frac{x}{\sigma}\right)^{\eta}}. \quad (2)$$

The cumulative distribution function (CDF) of the Weibull distribution is illustrated in Figure 2 for varying values of the shape parameter (η), with the scale parameter (σ) held constant at 1.



Figur 2: cumulative distribution function (CDF) of the Weibull distribution for different values of η and σ =1.

The reliability is the complement of the Cumulative Distribution Function (CDF), representing the probability that failure will not happen until time (t), as given by [7,14]. Reliability of the Weibull distribution is illustrated in Figure 3 for varying values of the shape parameter (η), with the scale parameter (σ) held constant at 1.



Figur 3: Relability of the Weibull distribution for different values of η and σ =1.

$$R(t) = e^{-\left(\frac{t}{\sigma}\right)^{\eta}}, \tag{3}$$

where σ corresponds to the mean time to failure $(mttf)$ specifically when the slope, η is set to one. The relationship between η and mttf is established through a gamma function of η, as demonstrated in the subsequent equation:

$$mttf = \sigma\Gamma\left[1 + \frac{1}{\eta}\right] \tag{4}$$

When η = 1.0, mttf = σ , the Exponential distribution.

When $\eta >$ 1.0, mttf is less than σ.

When $\eta <$ 1.0, mttf is greater than σ.

When $\eta = \frac{1}{2}$, mttf= 2σ .

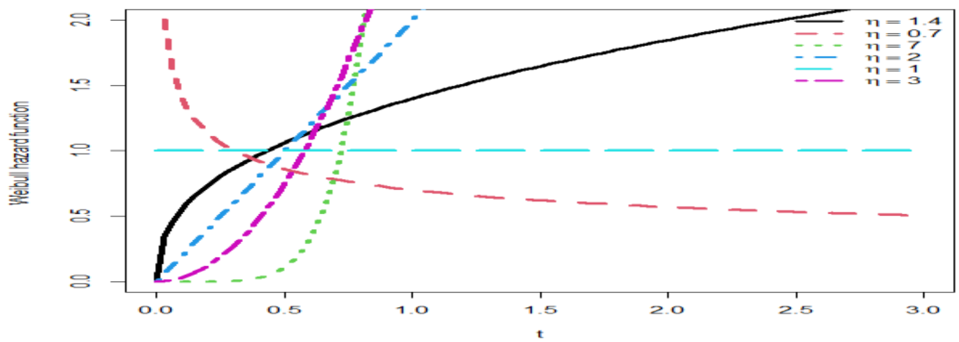
It is essential to differentiate between $mtbf$ (Mean Time Between Failures) and $mttf$ (Mean Time To Failure)[8], as they represent distinct concepts. $mtbf$ denotes the average time interval between occurrences of failures and is computed by dividing the cumulative operational time of all units by the total count of observed failures. These two parameters possess dissimilar characteristics, although they equate when instances of system suspensions are absent. However, under scenarios involving suspensions, substantial discrepancies may emerge. mtbf finds relevance primarily in systems that are capable of being repaired. Moreover, the Weibull hazard function, denoted as h(t), plays a critical role in depicting the instantaneous rate of failures and is mathematically expressed as follows:

$$h(t) = \frac{f(t)}{R(t)} = \frac{\frac{\eta}{\sigma}(\frac{x}{\sigma})^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)^{\eta}}}{e^{-\left(\frac{x}{\sigma}\right)^{\eta}}},$$

undergoes compensation and simplification, resulting in

$$h(x) = \frac{\eta}{\sigma}(\frac{x}{\sigma})^{\eta-1} \tag{5}$$

The Weibull hazard function, of the Weibull distribution is illustrated in Figure 4 for varying values of the shape parameter (η), with the scale parameter (σ) held constant at 1



Figur 4: Weibull hazard function for different values of η and σ =1.

2.2 Weibull Distribution transformation(twd)

If a random variable X's cumulative distribution function (cdf) is as follows, then it is said to have a transmuted distribution.

$$F(x) = (1 + \lambda)G(x) - \lambda G(x)^2, |\lambda| \leq 1 \quad (6)$$

we now have the CDF of a transmuted Weibull distribution using (2) and (6).

$$F(x) = \left(1 - e^{-\left(\frac{x}{\sigma}\right)^{\eta}}\right)\left(1 + \lambda e^{-\left(\frac{x}{\sigma}\right)^{\eta}}\right). \quad (7)$$

Therefore, the probability density function (pdf) of the transmuted Weibull distribution with parameters, η, σ, and λ is

$$f(x) = \frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)^{\eta}} \left[(1 - \lambda + 2\lambda e^{-\left(\frac{x}{\sigma}\right)^{\eta}}\right]. \quad (8)$$

A transmuted Weibull distribution's possible pdf shapes are shown in Figure 5 for a range of parameter values (λ, η, and σ = 1). Additionally, Figure 6 depicts the cumulative distribution function (CDF).

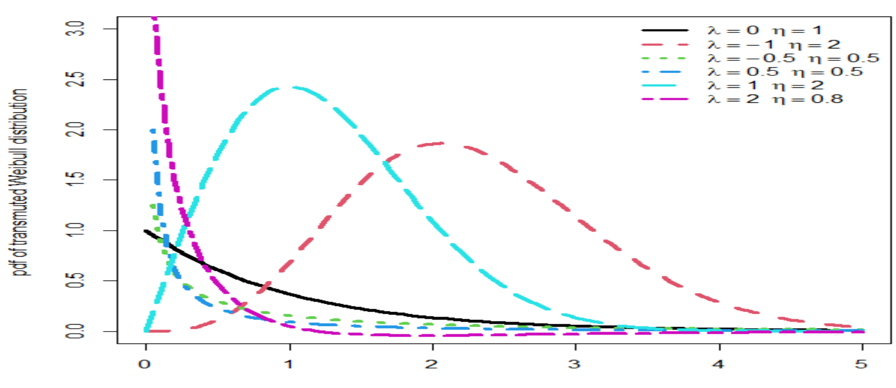


Figure 5: probability density function (PDF) of the transmuted Weibull distribution for different values of (η, λ) and (σ) =1.

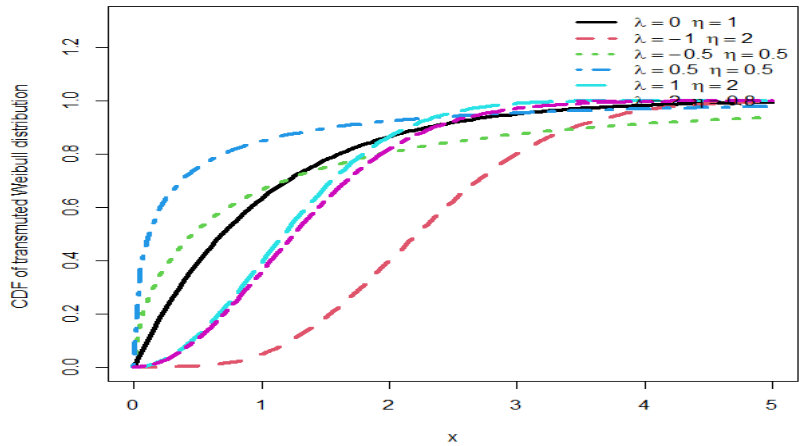


Figure 6: CDF of the transmuted Weibull distribution for different values of (η, λ) and (σ) =1.

The transmuted Weibull distribution, owing to its analytical structure, proves to be a valuable tool for modeling the failure time of a system. The reliability function R(t) = 1 − F(t) signifies the probability that an item will not fail before time t. An illustration of the reliability function for the transmuted Weibull distribution can be found in Aryal [9]. This distribution serves as an extended model capable of analyzing complex data across a wide range of scenarios, offering a generalization of several commonly used distributions. Notably, setting $\eta = 1$ results in the transmuted exponential distribution, as discussed by Shaw et al. [10]. The standard Weibull distribution is a distinctive case with $\lambda = 0$. When both η and λ are set to 1, the distribution transforms into an exponential distribution characterized by the parameter σ^2. Figure 7 visually illustrates various potential shapes of the Reliability Function of transmuted Weibull distribution. These shapes correspond to selected values of the parameters λ and η , with $\sigma = 1$ held constant.

$$R(t) = e^{-\left(\frac{t}{\sigma}\right)^{\eta}}\left[1-\lambda+\lambda e^{-\left(\frac{t}{\sigma}\right)^{\eta}}\right]. \qquad (9)$$

The other characteristic of interest of a random variable is the hazard rate function defined by

$$h(x) = \frac{f(x)}{1-F(x)}, \qquad (10)$$

which is a significant amount that describes phenomena in life. Given that it has endured till time t, it might be informally understood as the conditional likelihood of failure. Given below (11) is the hazard rate function for a Weibull random variable that has been transformed in figure 8.

$$h(x) = \frac{\eta}{\sigma}\left\{\frac{1-\lambda+2\lambda e^{-(\frac{x}{\sigma})^{\eta}}}{1-\lambda+\lambda e^{-(\frac{x}{\sigma})^{\eta}}}\right\}\left(\frac{x}{\sigma}\right)^{n-1} \qquad (11)$$

The cumulative hazard rate function $(H(x))$ of a transmuted Weibull random variable is provided in Aryal's work [9]. In Figure 7, the reliability characteristics of a transmuted

Weibull distribution are visualized while varying the parameter λ across the range from -1 to 1. It's important to observe that the left part of the figure displays this behavior. Furthermore, Figure 8 depicts the hazard rate function's behavior for a transmuted Weibull distribution.

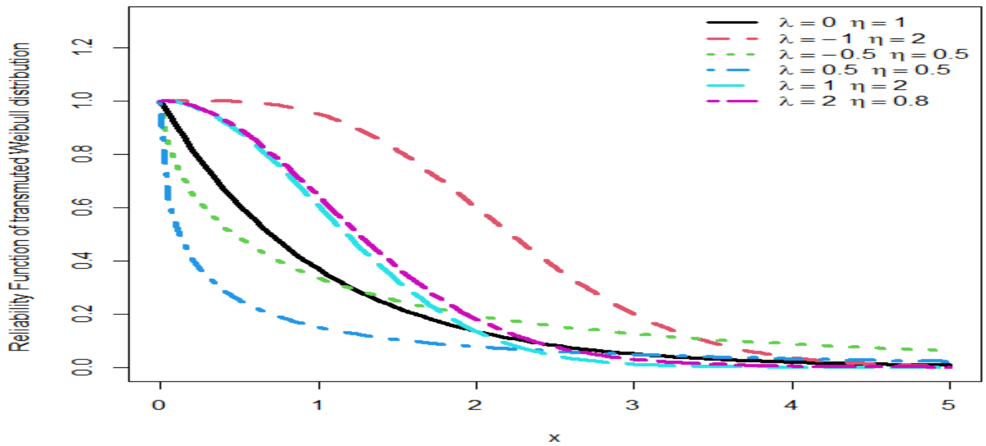


Figure 7: Reliability Function of transmuted Weibull distribution different values of (η, λ) and (σ) =1.

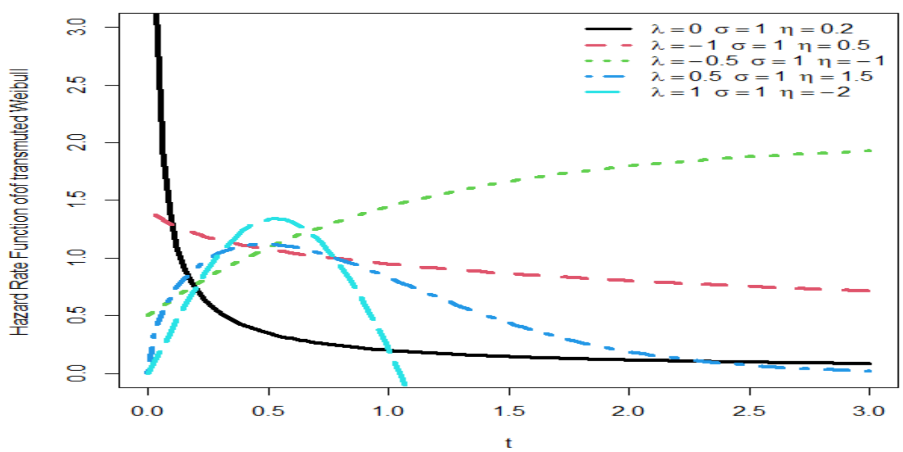


Figure 8: hazard rate Function of transmuted Weibull distribution different values of (η, λ) and (σ) =1.

$$H(x) = \int_0^x h(x)dx = \left(\frac{x}{\sigma}\right)^n - \ln\left[1 - \lambda + \lambda \exp\left(-\left(\frac{x}{\sigma}\right)^\eta\right)\right]. \quad (12)$$

The mean residual life MRL at a given time x measures the expected remaining life time of an individual of age x. It is given by[10]

$$m(x) = E(X - x|X \geq x)$$
$$= \frac{1}{R(x)}\int_0^\infty R(u)du. \quad (13)$$

It's noteworthy that $m(0)$ represents the mean time to failure. The Mean Residual Life

(MRL) can be expressed in terms of the cumulative hazard rate function as demonstrated by the integral equation:

$$m(x) = \int_0^{\infty} exp[H(X) - H(x+t)]\, dt.$$

Additionally, the mean residual life can be connected to the failure rate hazard function of the random variable through the relationship $\acute{m}(x) = m(x)h(x) - 1$.

For a transmuted Weibull random variable, the MRL function $m(x)$ can be represented using the incomplete Gamma function, as outlined in Equation (14) below:

$$m(x) = \frac{\sigma}{\eta} \frac{\exp\left((\frac{x}{\sigma})^{\eta}\right)}{\left[1-\lambda+\lambda\exp\left(-(\frac{x}{\sigma})^{\eta}\right)\right]} (1-\lambda)\Gamma\left(\frac{1}{\eta}, \left[\frac{x}{\sigma}\right]^{\eta}\right) + \lambda 2^{\frac{1}{\eta}}\Gamma\left((1-\lambda)\Gamma\left(\frac{1}{\eta}, \left[\frac{x}{\sigma}\right]^{\eta}\right)\right). \quad (14)$$

Here $\Gamma(a,x) = \int_x^{\infty} e^{-1} z^{a-1}\, dz$ represents the upper incomplete Gamma function.

3. Maximum Likelihood Estimators of (twd)

Take into account a set of random samples, denoted as $x_1, x_2, \ldots, x_n$, which comprises n observations originating from the transmuted Weibull distribution (twd), (η, σ, λ), characterized by its probability density function as cited in Ahmad's work [11,13]. The likelihood function corresponding to Equation (15) is expressed as follows:

$$L = \left(\frac{\eta}{\sigma}\right)^n e^{\left[-\sum_{i=1}^{n}\left(\frac{x_i}{\sigma}\right)^{\eta}\right]} \prod_{i=1}^{n} \left\{\left(\frac{x_i}{\sigma}\right)^{n-1} \times \left[1-\lambda+2\lambda\exp^{(-\frac{x_i}{\sigma})^{\eta}}\right]\right\}. \quad (15)$$

Hence, the log-likelihood function $l = lnL$ become

$$l = n\ln\frac{\eta}{\sigma} - \sum_{i=1}^{n}\left(\frac{x_i}{\sigma}\right)^{\eta} + \sum_{i=1}^{n} \ln\left(\frac{x_i}{\sigma}\right)^{\eta-1} + \sum_{i=1}^{n} \ln\left[1-\lambda+2\lambda\exp^{(-\frac{x_i}{\sigma})^{\eta}}\right]$$

$$\text{l} = n\ln\eta - n\eta\ln\sigma + (\eta-1)\sum_{i=1}^{n}\ln(x_i) - \sum_{i=1}^{n}\ln\left(\frac{x_i}{\sigma}\right)^{\eta-1} + \sum_{i=1}^{n}\ln\left[1-\lambda+2\lambda\exp^{(-\frac{x_i}{\sigma})^{\eta}}\right]. \quad (16)$$

Hence, the Maximum Likelihood Estimates (MLEs) for η, σ, and λ, which aim to optimize Equation (16), must adhere to the subsequent set of normal equations.

$$\frac{\partial l}{\partial \eta} = \frac{n}{\eta} + \sum_{i=1}^{n}\left[1-\left(\frac{x_i}{\sigma}\right)^{\eta}\right]\ln\left(\frac{x_i}{\sigma}\right) - 2\lambda\sum_{i=1}^{n}\frac{\ln(\frac{x_i}{\sigma})(\frac{x_i}{\sigma})^{\eta}e^{-(\frac{x_i}{\sigma})^{\eta}}}{1-\lambda+2\lambda e^{-(\frac{x_i}{\sigma})^{\eta}}} = 0, \quad (17)$$

$$\frac{\partial l}{\partial \sigma} = \frac{n}{\eta} + \sum_{i=1}^{n}\left[1-\left(\frac{x_i}{\sigma}\right)^{\eta}\right] - \frac{2\lambda\eta}{\sigma}\sum_{i=1}^{n}\frac{(\frac{x_i}{\sigma})^{\eta}e^{-(\frac{x_i}{\sigma})^{\eta}}}{1-\lambda+2\lambda e^{-(\frac{x_i}{\sigma})^{\eta}}} = 0, \quad (18)$$

$$\frac{\partial l}{\partial \lambda} = \sum_{i=1}^{n}\frac{2e^{-(\frac{x_i}{\sigma})^{\eta}-1}}{1-\lambda+2\lambda e^{-(\frac{x_i}{\sigma})^{\eta}}} = 0. \quad (19)$$

The (MLE) estimate, denoted as $\hat{\theta} = (\hat{\eta}, \hat{\sigma}, \hat{\lambda})$, for the parameter set $\theta = (\eta, \sigma, \lambda)$, is acquired through the resolution of this non-linear system of equations, as detailed in Zaindin's work [11].

4. Application

4.1. Simulation study

as a method, involves the representation or emulation of real-world phenomena using specific models. Given the intricacies of complex operations encountered in reality, which may be challenging to comprehend and analyze directly, models resembling real-world scenarios become invaluable. Simulation serves as a tool to enhance understanding and analysis by providing insights into the underlying processes or real-world situations.

In this section, we present a simulation study wherein data is generated using the inverse transformation method of the cumulative distribution function. The primary objective is to assess the performance of estimators, specifically Maximum Likelihood Estimators (MLEs). The evaluation is based on the comparison of their estimates and Mean Squared Errors (MSEs). The simulation tests are carried out with varying sample sizes (n = 25, 50,100, 150) for both the Weibull distribution and its transformation. The implementation utilizes the R programming language, adjusting values for the two parameters (η, σ) and (η, σ , λ). The experiment is iterated 1000 times for each combination of sample size and shape parameter values. Tables (1), (2), and (3) present the estimated parameters and MSEs for the estimations of (η, σ) in three distinct cases. Case (1) is outlined in Table (1), Case (2) in Table (2), and Case (3) in Table (3).

Table 1: MSE of the parameter estimations and a comparison of the two methods of estimation at the sample sizes (25,50,100,150) For the initial value set (η=1.4, σ =1,λ= 1).

methods	S. Size	p	Estimate				MSE
		p	value	MSE	AIC	BIC	
wd	25	η	1.17624	0.050067	37.1835	39.6213	0.8243463
		σ	0.77094	0.05246599			
twd		η	1.216697	0.033600034	-27.1590	-23.5024	0.8079190
		σ	0.8438874	0.02437114			
		λ	0.1837507	0.1000136			
wd	50	η	1.392868	5.085831e-05	80.79324	84.61728	0.7721594
		σ	0.9338179	0.004380075			
twd		η	1.345351	0.002986467	-70.7308	-64.9947	0.7668581
		σ	0.8755625	0.01548470			
		λ	-0.154844	0.4288210			
wd	100	η	1.428079	0.0007884332	-155.737	-147.922	0.770635
		σ	0.9994493	3.033195e-07			
twd		η	1.493404	0.008724310	166.5986	171.809	0.76131055
		σ	1.4929016	0.24295198			
		λ	0.8856499	0.1487259			
wd	150	η	1.417022	0.0002897649	244.5604	250.5817	0.773764
		σ	0.9813339	0.0003484228			
twd		η	1.349451	0.002555234	-234.443	-225.411	0.7550213

		σ	0.8987777	0.01024596			
		λ	-0.208523	0.5020060			

Table 2: MSE of the parameter estimations and a comparison of the two methods of estimation at the sample sizes (25,50,100,150) For the initial value set (η=0.7, σ =1,λ= 1).

methods	S. Size	p	Estimate				MSE
		p	value	MSE	AIC	BIC	
wd	25	η	0.7417809	0.00174564	56.86037	59.29812	9.343934
		σ	0.9683636	0.001000863			
twd		η	0.6083443	0.0084007664	-20.3684	-24.7117	4.232673
		σ	0.7120392	0.0829214265			
		λ	0.1836635	0.6664052			
wd	50	η	0.6964223	1.279985e-05	100.5557	104.3798	6.985404
		σ	0.872006	0.01638245			
twd		η	0.6726759	0.0007466078	-90.4933	-84.7572	6.674713
		σ	0.7666121	0.0544698910			
		λ	-0.1548461	1.3336697			
wd	100	η	0.7140396	0.0001971093	225.752	230.9624	6.380177
		σ	0.9988995	1.211052e-06			
twd		η	0.7168592	0.0002842329	-215.754	-207.939	6.397812
		σ	1.0122492	0.0001500432			
		λ	0.0146814	0.9708526			
wd	150	η	0.7020947	4.387814e-06	352.3819	358.4032	6.27817
		σ	1.057965	0.003359905			
twd		η	0.6747204	0.0006390567	-314.963	-305.931	5.331955
		σ	0.8077863	0.0369460885			
		λ	-0.2085476	1.4605874			

Table 3: MSE of the parameter estimations and a comparison of the two methods of estimation at the sample sizes (25,50,100,150) For the initial value set (η=7, σ =1,λ= 1).

methods	S. Size	p	Estimate				MSE
		p	value	MSE	AIC	BIC	
wd	25	η	5.881204	1.251704	-13.3299	-10.8921	0.07438244
		σ	0.9493018	0.002570306			
twd		η	6.083648	0.8397011	23.35441	27.01104	0.07300441
		σ	0.9666174	0.001114399			

		λ	0.1837536	6.662582e-01			
wd	50	η	7.363925	0.1324413	-48.0245	-44.2004	0.0662862
		σ	1.013052	0.0001703603			
twd		η	6.085781	0.8357973	56.58280	62.31887	0.0571142
		σ	1.2965697	0.0879536			
		λ	1.1410028	0.019881			
wd	100	η	7.837957	0.70192	-129.096	-123.885	0.0580619
		σ	0.9835058	0.000272057			
twd		η	6.644950	0.12606	134.90048	132.7159	0.0481874
		σ	1.4501735	0.202656188			
		λ	1.1123366	0.01261			
wd	150	η	7.044783	0.002005491	-126.376	-120.355	0.0542712
		σ	1.001298	1.683608e-06			
twd		η	6.884547	0.01334	684.60432	693.6362	0.0460602
		σ	1.4397453	0.19337			
		λ	1. 244403	0.05973			

The presented tables provide a thorough comparison between the Weibull and Transmuted Weibull distributions across different sample sizes. This comparison relies on the Mean Squared Error (MSE) of parameter estimates, as well as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values. The results highlight the significant impact of both distribution choice and sample size on the accuracy of parameter estimates and the overall goodness of fit for the models. Consistently, the Transmuted Weibull distribution outperforms the Weibull distribution in terms of parameter estimation accuracy and goodness of fit across the examined scenarios.

4.2. Real data

In this section, we showcase two instances where the Transmuted Exponential Weibull (TEW) model is juxtaposed with other related models. To ensure a balanced comparison, we employ various goodness of fit criteria. The R software is used to conduct numerical analyses to determine the distribution that best fits each data set. The Maximum Likelihood Estimates (MLEs) of the parameters of the distributions are displayed in the subsequent tables. The models are selected using the Akaike Information Criterion (AIC), also known as the Bayesian Information Criterion (BIC). The data used in this context is purely for illustrative purposes. All crucial numerical computations have been executed using the R software. Our first dataset pertains to the analysis of gear data, obtained from the smithdat folder within the SuperSMITH installation, as shown in Table (4). These data points, representing subjects, have been fitted using the Weibull distribution, and the estimated parameters are outlined in the table below. It’s noteworthy that the subject data have been modeled using both the Weibull and the transmuted Weibull distributions. Table (5) provides the MLEs and the values of maximal log-likelihoods for the Weibull and transmuted Weibull distributions. The likelihood ratio test can be employed to illustrate that the transmuted Weibull distribution aligns more closely with the input data than the standard 3-parameter Weibull distribution.

Table 4: The gears data obtained from the smithdat folder on SuperSMITH installation.

4325.816	6089.124	6281.571	7329.370	7586.772
8361.412	9136.757	9794.200	10939.03	10942.62
11090.46	11635.25	12160.14	13057.69	14307.81

Table 5: The normality tests of the original and transformed datasets.

Datasets	Parameter MLE	Std. Dev.	K-Smirov		Statistics			
			Stat.	p-value	AIC	BIC	W	A
twd	η =0.900515	0.0716	0.05342	0.93779	187.61	195.43	0.03546	0.2542
	σ= 0.98870	46.23688	Min(-log(Likelihood)) =90.80609					
	λ =1.1285464	52.776732						
wd	η = 0.973725	0.138803	0.11596	0.9227	379.47	381.46	0.03914	3.134
	σ =14167.5	3467.59	Min(-log(Likelihood)) =-211.479					

Where A= Anderson-Darling statistic.
W= Cramer-von Misses statistic.

Quick Fit functions have fit characteristics included in the function name, and reasonable defaults are used, making it straightforward to get a full analysis. Figure 9 simply demonstrates the simplicity of producing a Quick Fit plot.

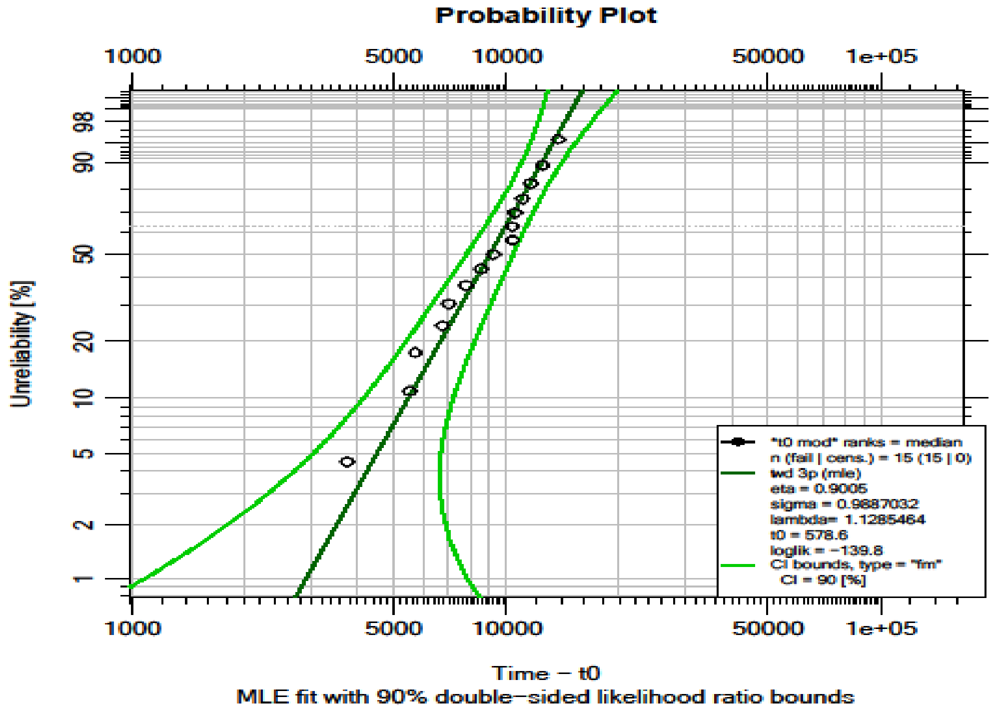


Figure 9: A Probability Plot with Quick Fit.

Figure 10 shows a multi-distribution and the best distribution is the transformed Weibull with a 3-parameter. When comparing the R^2 in the three cases (transformed Weibull with 3-parameters $= 0.9825$; transformed Weibull with 2 -parameter$= 0.9783$ and lognormal $= 0.9732$) it is concluded that the best value is transformed Weibull with 3 parameters.

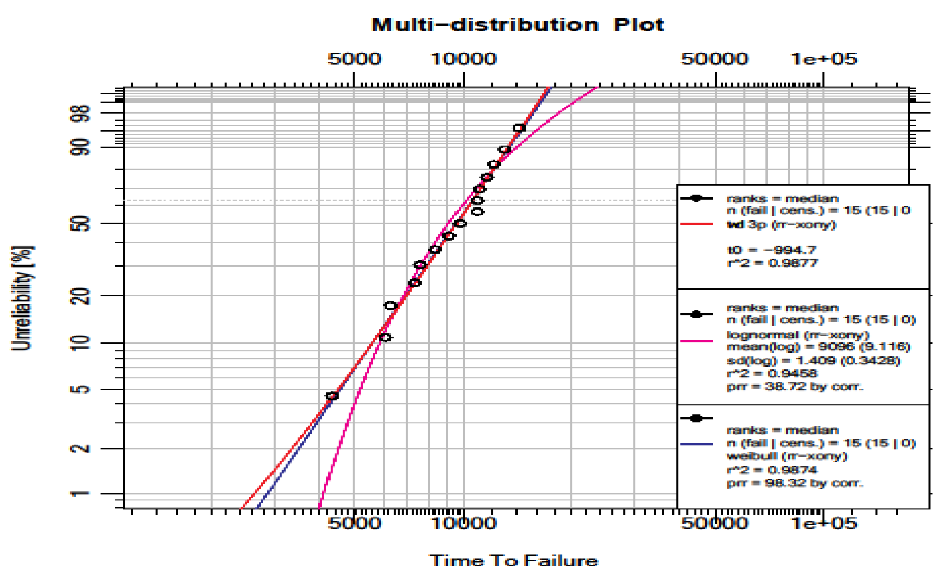


Figure 10: multi-distribution with transformed Weibull two, three parameters, and lognormal.

The precursor computation for establishing bounds on the likelihood ratio involves creating a likelihood contour at a specified confidence level for a given model. These contours represent horizontal sections through the peaked likelihood mound centered around the maximum likelihood estimate. The contour slices are generated at ratio values determined by the following relationship[12]:

$$\text{ratio test} = \text{mle} - \frac{qchisq(CL, def)}{2},$$

where mle is the maximum log-likelihood estimate, CL is the confidence limit, and def represents the degrees of freedom. The degrees of freedom are set to 1 when comparing the model fit itself and 2 when making comparisons against other data. we can show that in Figure 11.

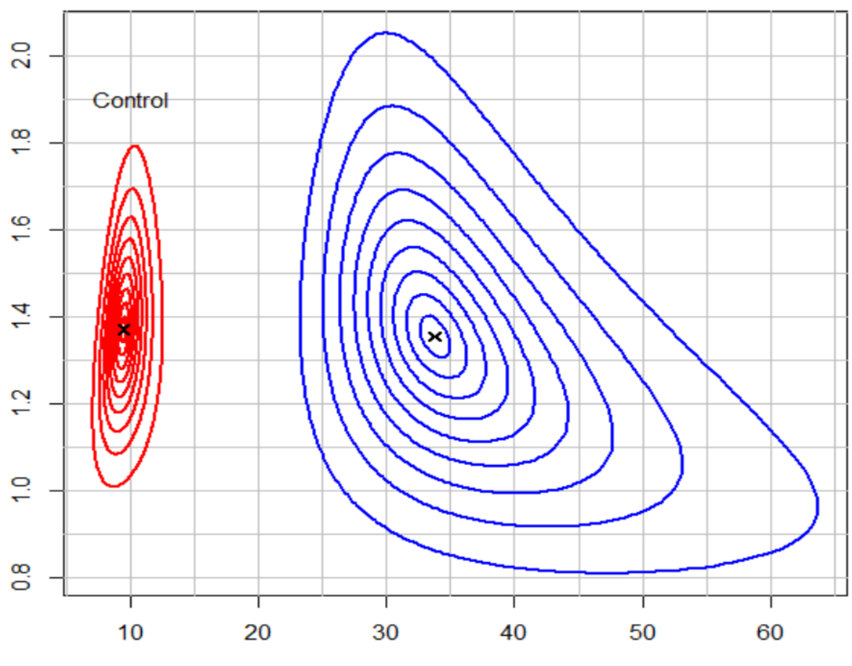


Figure 11: Comparison of datasets by likelihood contour based on a submitted data set with 3 failure points and approximately 30,000 right-censored suspension, values.

The points on a specific confidence level contour are used to define confidence interval bounds. Figure 12 shows how the extreme Beta value points form asymptotes for the bouns on a 2-parameter mode.

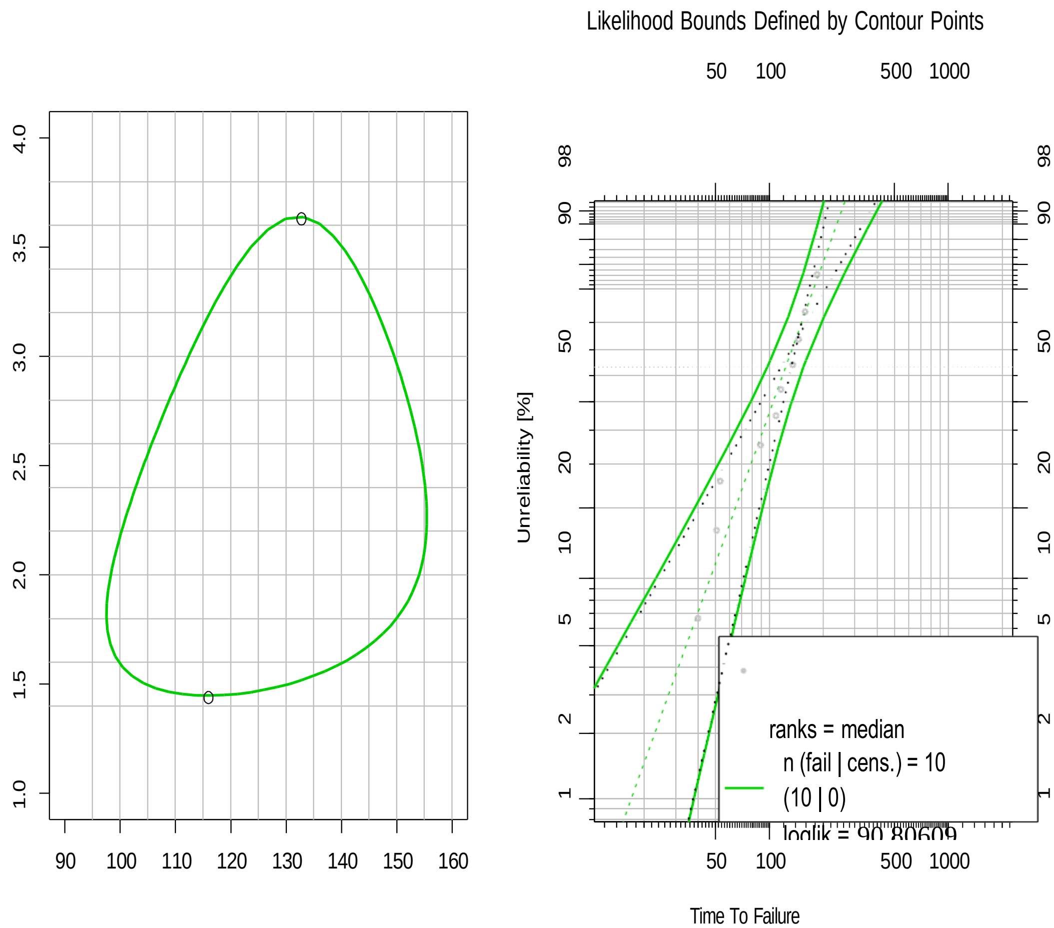


Figure 12: Likelihood ratio bounds formed by confidence level contour.

The Fisher Matrix bounds including uncertainty in the third parameter. The data used for Figure 10 have been applied to form these bounds as bold purple lines in figure 13

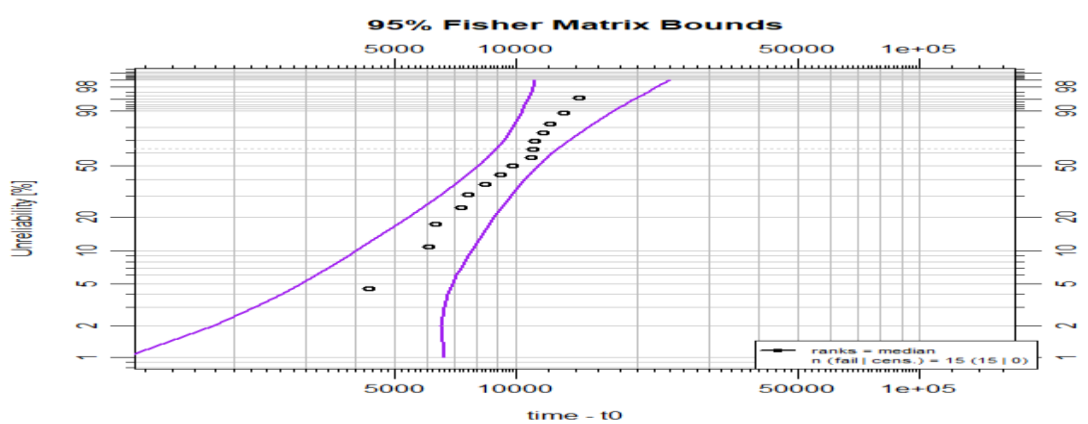


Figure 13: Unusually formed Fisher Matrix bounds on a 3-parameter model.

5. Conclusion

In this study, both real and simulated datasets were utilized to assess the performance of the transmuted Weibull distribution, in comparison to the 2-parameter Weibull distribution. The focus was specifically on modeling gear data, with the use of ratios to identify contour slices. Evaluation metrics included the standard error, coefficient of determination, and both the Bayesian and Akaike information criteria. The transmuted Weibull distribution demonstrated enhanced adaptability and a more precise representation of real data. Its versatility was underscored through numerical analyses and practical applications, thereby highlighting its potential applicability in the realm of reliability engineering. The study culminates in a succinct yet comprehensive exploration of the distribution's effectiveness, offering valuable insights for practitioners in the industry.

Availability of Data: The datasets that support the paper's results are included in the paper.

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