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Optimization of B. O. T Contracts Using a Hybrid Model of Multi-Objective Programming and Game Theory

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Abstract
In the current research, construction, operation and transfer or BOT contracts have been studied, which are of great importance to legal and economic experts today. Different countries, especially developing countries, consider these contracts as an efficient tool for attracting foreign capital and using private sector management in infrastructure projects. In this type of contracts, the public sector entrusts the construction of the design to the private sector, and the private sector is obliged to return the ownership of the design to the public sector after its construction and operation. For the success of BOT, the government must provide the economic, political and especially legal basis for private investment. Contract B. O. TIt is a suitable mechanism for the government and the private sector and the use of proper management of this sector, as well as the use of public funds to carry out construction projects faster. In the presented model and its mechanism, the role of banks and the use of loans were especially considered, because in our country, banks' investment is mainly related to the construction of residential and commercial units, and when the housing market Banks will face serious problems if they go into recession. But in this way, by investing in construction projects, they can earn future and long-term profits, and their funds, which are mainly people's deposits, can be used for the development of the country, and in fact, a win-win game will occur.

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1. Introduction

Contracts Build – operate – transfer or BOT contracts refer to contracts in which the design is assigned to a private company for construction with the permission of the government and on behalf of government agencies [1], The conclusion of BOT contracts as one of the ways to attract foreign capital and take advantage of the efficient management of the private sector is of interest to developing countries. It can be said that the first BOT contract dates back to 1734 and the issue

of the development of the Suez Canal, by which the Egyptian countries tried to expand this project by attracting European funds and using expert personne [2]. In the second half of the 19th century, governments decided to attract private financial aid to expand railways and roads. However, despite the successes that were achieved, Dachal declined after a while because the private sector was willing to invest in this sector only on the condition of a high rate of return of profits, which conflicts with the financial power of the government and also the public interest for public access to this sector [3], It was obvious. This decline continued until the middle of the 20th century. Ernst and Pham Duton, experts in privatization, have considered the 1980s as the serious beginning of a process that paid special attention to the participation of the public and private sectors in launching and operating infrastructure projects in order to provide public benefits, and based on this process, contracts OT is called public-private partnership [4]. Today, financing for the infrastructure projects of countries and privatization as a prerequisite and necessary to achieve economic growth and sustainable development are among the priorities of countries [5].

The B.O.T contract, like any other contract, has disadvantages and advantages, but the advantages of this contract will be more than its disadvantages [6]. One of the most important advantages of using the bot contract format is to launch and carry out large and extensive projects that cannot be done without expensive investments. Some governments, especially in developing countries, do not have the ability to pay huge costs for infrastructure projects in the country [7], Since carrying out these projects is necessary for the growth and development of the country and plays a very important role, governments welcome private and foreign investors.One of the most extensive and effective types of contract packages that lead to the attraction of domestic and foreign investment and guarantee the government's control and authority over its projects or construction, operation and assignment contracts, Being in this scene in Badi requires the examination of the model of the B.O.T contract, based on this,In this article, we are going to present a model for determining the profit of the public and private sectors and use the desired profit of each sector to determine the duration of operation and finally transfer the project from the private sector to the public sector.

2. Research literature

2.1. The concept of contracts: B. O. T

BOT stands for the three words Build-Operate-Transfer. It refers to contracts in which a plan is assigned to a private company for construction with the permission of the government and on behalf of the government agency. After the construction, the privilege of exploiting the plan is given to the construction company for a period of time, and during this period that has been determined in advance; The private company owns the project and its resulting resources, and after the expiration of the contract (which is usually between 25 and 40 years), the ownership of the project is transferred to the government of the contracting party [8]. In this way, the private company (which is usually foreign) undertakes the financing of the infrastructure project, builds it, and after a period of operation, transfers the ownership of the project to the government for free or for a fee (which is usually free). From then on, the government, as the owner, should exploit it in line with the public interest [9].There was no prohibition to hand over a project from the private

sector to another private sector in BOT, but since in most of these contracts, the government is placed as one of the contracting parties. In the presented definition, the government is mentioned as one of the main pillars of the definition. Also, it is not necessary that the investor sector is a foreign sector and in the examples of B. She. The project has been handed over to the domestic private sector. But since the domestic sector usually does not have the necessary technical and financial balance to implement the project, the presence of foreign investors in these types of contracts is very common. Conclusion of contracts B. She. As one of the solutions for attracting foreign capital and exploiting the efficient management of the private sector, it is of special interest to developing countries [10].

2.2. The purpose and importance of the bot contract example

The most important feature of the BOT contract is that it seems to be designed for a time when governments or the public sector do not have the ability to carry out projects or for any reason do not want to invest in different sectors. This is when the bot contract method comes into play. The importance of BOT contract derivatives is determined when governments cannot provide adequate financial resources and investment for the implementation of economic plans or infrastructure projects [11]. This is when the bot contract type is used. This contract is also called a bot partnership contract or a bot rental contract sample, because with this method, the general resources and capabilities of the private and public sectors, as well as the internal and external sectors of governments, are gathered and work to implement infrastructure projects [12].

This cooperation and partnership has benefits for the parties, which has made the example of the bot contract one of the most popular contracts. One of the advantages of the BOT contract, which increases the importance and validity of this contract, is that although it is widely used in joint contracts of the private and public sectors, it can be done in other sectors as well and is not limited to a specific sector [13]. The organization's bot contract has a specific program for use in a variety of projects. Only on the condition that the features of the project, such as its financial benefits, are enough to motivate the investor and the implementer to carry out the project. In simpler terms, the project must provide the interests of the parties to the contract [10].

2.3.Types of contracts BOT

Each part of the bot can be a separate contract that is bundled together in this contract. This integration sometimes causes complications in the contract. The breadth and different angles of the bot contract format have made it very important to know it from a legal point of view. The combination of these three parts can be called bot contract formula [8].

2.3.1. Build in the sample bot contract

This part of the bot contract sample is the first stage of the project. Construction in a general sense includes a set of activities such as project design, applied engineering, supply of goods and equipment needed for construction, installation, implementation and commissioning. All these things are done in order and in continuous steps. At first, the group of engineers undertakes the

design of the project and with their supervision and based on their opinion, the necessary goods and equipment are provided. Finally, with the installation and initial tests, this part of the project is completed and the contract enters the second part, i.e. operation. This part is considered a very important part in the example of the construction bot contract and the construction of large buildings and architectures [14].

2.3.2. Operation in the bot contract definition

Exploitation is the second part of the BOT contract model. This part of the contract is responsible for activities such as guiding and handling and daily implementation of the project, maintaining the facilities and equipment of the buildings, performing repairs and dealing with technical problems, supplying the required parts and replacing them, and supplying the consumables required by the project [7]. In the construction phase and to some extent in this phase, the private sector has invested and pays costs. In the continuation of this stage, the private sector has the right to exploit the project for a certain period of time and secure its own benefits from the profit. This period is written and registered in the text of bot contracts [10]. For example, if a power plant or other infrastructural projects such as roads and bridges and in general the example of a municipal bot contract are carried out, the private sector will operate the constructed project for a period of time. This part of the contract provides the interests of the private sector, which attracts them to execute the contract in the bot way [15].

2.3.3. Transfer and transfer in the BOT contract

The last step in the BOT type of contract is transfer or assignment. In this project, the private sector, which is considered a contractor, operates the built project for a while. After the completion of the specified time for operation, it is required to hand over the project to the employer, which is the government sector [16]. In this assignment, there will be no other rights for the contractor or the private sector, and all rights have been transferred to the government. From then on, the public sector will own all project authority. After completing the assignment stage, the employer can operate the project alone without restrictions and even cooperate with other companies [17].

3. Two-level planning

Two-level planning is related to decision-making processes that put two decision makers in a hierarchical structure. In fact, there were two levels of decision-making in these issues, each level having its own decision-making variables and seeking to optimize its goals in a hierarchical structure. In this planning, the decision maker of the second level optimizes the corresponding objective function according to the values of the variables received from the first level. In return, the decision maker of the first level, using sufficient information about the possible reactions of the second level, chooses the values of the variables that will optimize his objective function [4]. Therefore, the two-level planning problem includes two optimization problems, in which the justified region of the restrictions related to the first level is determined by the second level optimization problem.

In general, one of the two-level programming problem is defined as follows:

$$\min_{x,y} \quad F(x,y)$$
$$G(x,y) \leq 0$$
$$\min_{y} \quad F(x,y)$$
$$s.t \quad g\,(x,y) \leq 0$$

where $x \in R^n$ and $y \in R^n$ The variables of the above problem are divided into two categories:

a) $x \in R^n$ high-level variables

b) $y \in R^n$ low-level variables

Similarly, $F: R^n \times R^m \to R$ & $F: R^n \times R^m \to R$ are also the upper and lower level objective functions of vector functions $g: R^n \times R^m \to R^{p2}$ & $G: R^n \times R^m \to R^{p1}$ are called upper level and lower level constraints, respectively. In two-level planning, the first-level decision-maker, i.e. the decision-maker at the highest level, is called the leader, and the second-level decision-maker, i.e. the decision-maker at the lowest level, is called the follower. Each decision maker tries to optimize his objective function regardless of the decision maker's goal of the other level, but the decision of each level affects the value of the objective function and the decision space of the other level. If we consider the set $\Omega(x)$ as follows.

$$\Omega(x) = g/g(x,y) \leq 0$$

When the second problem is given by minimizing the function $F(x,y)$ on the set $\Omega(x)$ we put it.

$$M(X) = y: y \in argminf(x,y): y \in \Omega(x)$$

So the problem can be rewritten as follows.

$$\min_{x,y} \quad F(x,y)$$
$$s.tG(x,y) \leq 0, y \in M(x)$$

The feasible region of the above problem is as follows:

$$IR = (x,y)G(x,y) \leq 0, y \in M(x)$$

This area is called the reachable area. The reachable area is often non-convex. Two-level planning problems can be classified as follows.

A) Linear two-level programming problems

b) Non-linear two-level programming problems

In this article, we examine and solve the linear type.

The solvability of the problem is the answer that applies to all the constraints of the problem.

The optimal solution is the feasible solution that has the lowest value of the objective function in the minimization problem and the highest value in the maximization problem.

If x^* is the optimal solution of the minimization problem, we have

$$\forall x \in X: f(x^*) \geq f(x)$$

The definition of a set X in R^n is called convex, when

$$\forall x1, x2 \in X, \forall \lambda \in [o,1]: \lambda x1 + (1 - \lambda x2) \in X$$

The function $F: R^n \to R$ is assumed, we say that the function $F(x)$has a global minimizer at $x = x^*$ if

$$\forall x \in R^n: F(x^*) \leq F(x)$$

or equivalently

$$x^* = argmin_{x \in R^n} F(x)$$

Now let's look at an example of a two-dimensional.
Suppose a two-level problem is defined with the following conditions:

$$max \quad f_1: 2x_1 \ - x_2$$

$$max \quad f_1: x_1 \ + \ 2\, x_2$$

$$\text{s.t}\ \forall x \in X\{$$
$$3x_1 - 5x_2 \leq 15$$
$$3x_1 - x_2 \leq 21$$
$$3x_1 - x_2 \leq 27$$
$$3x_1 - 4x_2 \leq 15$$
$$x_1 - 3x_2 \leq 30$$

The corner points of this issue are:

$(5,0)$	(1)
$(7/5, 1/5)$	(2)
$(8,3)$	(3)
$(7,6)$	(4)
$(3,9)$	(5)
$(0,10)$	(6)
$(0,0)$	(7)

By putting the above values in the objective functions f_1 and f_2, we will have the following results in relation (12) respectively:

$(f_1, f_2) = (10,5)$	(1)
$(f_1, f_2) = (13/5, 10/5)$	(2)
$(f_1, f_2) = (13, 14)$	(3)
$(f_1, f_2) = (8, 19)$	(4)
$(f_1, f_2) = (-3, 21)$	(5)
$(f_1, f_2) = (-10, 2)$	(6)
$(f_1, f_2) = (0, 0)$	(7)

Therefore, point (2) with coordinates $(1.5, 7.5)$ is the optimal solution and we have:

$$f_1^* = 13/5$$
$$f_2^* = 10/5$$

4. Assumptions of the model

Based on the existing research about the bargaining game, the authors have presented assumptions for their models about the players' project information, rational behavior, bargaining price. Following these issues, the following two hypotheses have been presented as model evolution:

1. The government is interested in the private sector conducting a B.O.T contract with reasonable behavior during negotiations. The meaning of rational behavior is that both groups calculate all possible outcomes and compare them sufficiently to protect their profit and follow it.
2. The parties share complete and similar information about the B.O.T project with each other to ensure that they can respond clearly to each other.

4.1. Model

The Chang Kang model, which used two-level planning in 2008 to obtain the concession period, was used as a basic article to find the ideal investment amount and the economic life of the project and the concession period in B contracts [7]. She. We use t. And according to the topics presented, the following model is suggested. In this model, we get closer to the real world by assuming that the economic life span and the required capital are constant. In this model, two-level planning is used. In two-level planning, first the player makes a decision at the first level, and then the follower or follower makes a decision according to the performance of the leading player. Points were initially given by the government in the documents proposed by B. She. T is announced, then the private sector makes a decision after seeing the government's proposal and negotiates with the government about the royalties of the project. The first level will include the objective function of the government or the leader and its limitations, and the second level will include the objective function of the private sector or the follower and its limitations.

In the implementation of any project, the government always thinks of increasing the net financial flow resulting from it. Therefore, the objective function based on two-level programming in the first level is as follows.

$$max\ NPV_G(T_f) = \sum_{T=0}^{T0} \frac{-I(t)}{(1+rwacc)^t} + \sum_{T=T_L}^{T_{PR}} \frac{-(1-a)IR_t}{(1+rwacc)^t} + \int_{T_0}^{T_f} \left[\frac{V_t(I)+\mathrm{Q}_t(I)-C_t(I)}{(1+rawacc)^t}\right] dt \qquad (1)$$

Also, the private sector is trying to increase the net financial flow from the project. The objective function of the second level is as follows:

$$max\ NPV_G(T_f) = \sum_{T=0}^{T0} \frac{-I(t)}{(1+rwacc)^t} + \sum_{T=T_L}^{T_{PR}} \frac{-(1-a)IR_t}{(1+rwacc)^t} + \int_{T_0}^{T_f} [\frac{\mathrm{Q}_t(I)-C_t(I)}{(1+rawacc)^t}]dt \qquad (2)$$

And the outline of the model is as follows:

$$max\ z = NPV_G(T_f) \qquad (3)$$

$$NPV \geq \sum_{T=0}^{T_0} \frac{I(t)\times(1+R_1+R_2)}{(1+rwacc)^t} \qquad (4)$$

$$V_t(I) \geq 0 \forall t \in [T_0, T_f] \quad (5)$$

$$T_f = inf\{V_t(I) + \mathrm{Q}_t(I) - C_t(I) \leq 0\} \quad (6)$$

$$T_f \geq T_0 \quad (7)$$

$$\max \mathrm{z} = \mathrm{NPV_P} \quad (8)$$

$$NPV_P \geq \sum_{T=0}^{T_0} \frac{I(t) \times (1 + R_2)}{(1 + rwacc)^t} \quad (9)$$

$$\frac{\int_{t0}^{T_f} \left[\frac{Q_t(I) - C_t(I)}{(1 + rwacc)^t}\right] dt}{\sum_{T=0}^{T_0} \frac{I(t)}{(1 + rwacc)^t} + \sum_{T=T_L}^{T_{PR}} \frac{-(1 - a)IR_t}{(1 + rwacc)^t}} \geq 1 + R_1 \quad (10)$$

$$0.2 \sum_{T=0}^{T_0} I(t) \leq I(0) \leq 0.3 \sum_{T=0}^{T_0} I(t) \quad (11)$$

$$T_f = \inf\{Q_t(I) - C_t(I) \leq 0\} \quad (12)$$

$$T_f \geq T_0 \quad (13)$$

4.2. Parameters used in the model:

The length of the construction period of the project is influenced by the amount of investment in the construction period and the quality of the construction of the project.

I: represents the expected investment amount.

$Q_t(I)$: indicates the amount of expected annual income.

$C_t(I)$: represents the cost of operation and maintenance during the operation period.

$V_t(I)$: social benefits resulting from the project

Q_t: expected annual income, which usually at the beginning of the project's operation and at the end of the economic life of Q_t is lower than its middle range and is like a concave function with respect to time.

C_t: The cost of maintenance and operation, which at the beginning of operation and at the end of the economic life of the project C_t has a lower amount than the range of bases and is a function of time like Q.

V_t: represents social benefits. Large-scale infrastructure projects are one of the most important issues in serving the government to the people. It is very difficult to calculate its amount, and with the passage of time, its amount gradually decreases.

R_1: Rate of the government's capital return, the government expects a minimum capital increase of R_1 percent during the project operation period.

R_2 : rate of return on investment of the private sector, every investor expects a minimum capital increase of R2 percent during the operation of the project.

R_0 : is the cost slope of time reduction. By compressing the duration of the construction period, the costs in this period increase. We assume that this increase in cost is a linear function of the decrease in time. R0 represents the cost slope of time reduction.

Rwacc: represents the weighted average cost of capital considering the risk factor.

T_{RP}: indicates the loan repayment period.

T_I: is the time to meet the conditions for getting a loan or selling bonds.

a : It is the ratio of the initial capital of the private sector to the total capital required for the construction of the project.

4.3. Decision variables

λ : The ratio of required investment to expected investment. Using this ratio, the optimal investment amount can be obtained.

T_c : It is time to transfer the project from the private sector to the government. At this time, the project is completely transferred to the government with all facilities and equipment.

T_f : It represents the economic life of the project from the government's point of view, which includes the construction period, the private sector's operation period, and the government's operation period. In most of the researches, the economic life of the project is presented as a default data, if the economic life of the project is influenced by the amount of investment in the construction period. Therefore, it is assumed here that the economic life of the project is a variable.

T'_f : It represents the economic life of the project from the point of view of the private sector (investor).

In this model, clauses (4) and (7) control the assumption of the rationality of the project implementation, which means that if the net financial flow of the public or private sector resulting from the project is less than the value on the right side of the limitation or initial costs, the project implementation is not rational. be For the implementation of any mega-infrastructure project, the people of the community must share in the benefits obtained from the project in a way that this issue is evaluated by applying the expression (3). If $V_t(I)$ has a negative value for $T > T_0$ the government will abandon its implementation to prevent the society from suffering.

From terms (4) and (5), we can understand the economic life of the project from the government's point of view, and from terms (10) and (11) we calculate the economic life from the point of view of the private sector. Expression (8) examines the rate of return on investment with the implementation of the project. The expression (9) can be considered as one of the most important limitations because with this limitation, the initial capital of the private sector is set at a minimum of 20% and a maximum of 30%, so that the rest of the capital required to complete the project is provided through bank loans or partnership bonds. to be By doing this, the common people of the

society can participate in the profit of the project by buying the bonds, and the banks also benefit from long-term favorable profits by investing in these projects.

4.4. Solution method

In the presented model, first, the expected functions of income, maintenance cost and social benefit should be estimated by considering the type of project and the amount of income predicted for it and based on factors such as inflation and standard salaries of employees, etc. for each year. which is done by statistical data simulation methods. The estimation of the social benefit number is obtained by collecting questionnaires and services provided in similar projects and determining the amount of reduction in public and welfare costs by carrying out the desired project. The expected distribution of income is in the form of a concave function because at the beginning of the project the trend is upward and at the end of the economic life due to the exhaustion of the project, the income decreases, so the best estimate for it is a quadratic function. The expected function of the cost from the beginning of the cost was lower than the beginning of the project and the demand started and increases with the increase of demand and erosion of the project and is concave like the income curve. The expected function of social advantage is also similar to the income function and for the same reasons, it first increases and at the end of the economic life, it faces a decrease, which shows that this function is also concave and a quadratic perspective is the best estimate for it.

*I*is the amount of transitory capital during the construction period. In this article, using the relationships in Bao and Wang's article, the relationship between the optimal investment amount in the construction period and the three variables of annual income, maintenance and operation cost, and social benefits is given, which is as follows:

$$Q_t(I) = Q_t\left(\frac{I}{I_0}\right)0/5$$

$$C_t(I) = C_t\left(\frac{I}{I_0}\right)0/5$$

$$V_t(I) = V_t\left(\frac{I}{I_0}\right)0/5$$

$$\lambda = \frac{I}{I_0} \qquad (14)$$

Then, from the constraints (6) and (12) and the relation (14), we obtain the upper limit of the integral in the equations (1) and (2) or the economic life of the project from the point of view of the government and the private sector based on the variable λ .

Economic life in terms of the public sector

In order to obtain the economic life from the point of view of the government sector and according to relation (14) and placing it in relation (6), we arrive at the following relation:

$$Q_t(I) + V_t(I) - C_t(I) \leq 0$$

$$\rightarrow Q_t(I) + V_t(I) - C_t(I) = 0$$
$$\rightarrow Q_t(\frac{I}{I_0})^{0.5} + V_t(\frac{I}{I_0})^{0.5} - C_t(\frac{I}{I_0})^{0.5} = 0$$
$$\rightarrow \lambda(Q_t + V_t) - C_t = 0$$

By solving the above relationship, we get a quadratic equation with the variable T, whose solutions are obtained based on λ.

$$Q_t = at^2 + bt + c$$
$$Q_t = a't^2 + b't + c'$$
$$Q_t = a''t^2 + b''t + c''$$

$$\rightarrow Q_t(I) + V_t(I) - C_t(I) = 0$$
$$\rightarrow Q_t(\frac{I}{I_0})^{0.5} + V_t(\frac{I}{I_0})^{0.5} - C_t(\frac{I}{I_0})^{0.5} = 0$$
$$\rightarrow \lambda(Q_t + V_t) - C_t = 0$$
$$T_{f=}\frac{-(\lambda b = \lambda b' - b') \pm \sqrt{(\lambda b + \lambda b' - b'')^2 - 4(\lambda a + \lambda a' - a'')(\lambda c + \lambda c' - c'')}}{2(\lambda a + \lambda a' - a'')} \quad (15)$$

As we can see, the above equation has two roots in terms of λ of which only one of the roots is acceptable, by putting relation (15) in relation (1) and solving the integral, a function based on λ is obtained which After derivation, the maximum value of net financial flow is obtained for λ which is determined by placing its value in relation (15) of the economic life of the project from the point of view of the government sector.

Economic life in terms of the private sector

To obtain the economic life from the point of view of the private sector and according to the relation (14) and placing it in the relation (12), we will have the following relation.

$$Q_t = at^2 + bt + c$$
$$C_t = a't^2 + b't + c'$$
$$Q_t(I) - C_t(I) \leq 0$$

$$\rightarrow Q_t(I) - C_t(I) = 0$$
$$\rightarrow Q_t(\frac{I}{I_0})^{0.5} - C_t(\frac{I}{I_0})^{0.5} = 0$$
$$\rightarrow \lambda Q_t - C_t = 0$$
$$T_{f=}\frac{-(\lambda b - b') \pm \sqrt{(\lambda b + b')^2 - 4(\lambda a + a')(\lambda c + c')}}{2(\lambda a - a')} \quad (16)$$

In this part, by putting the obtained λ from the point of view of the public sector in relation (16), the economic life is determined from the point of view of the private sector. By doing this, the government actually uses its right to be superior. In two-level planning, first the player decides on

the first level. And then the next player makes a decision according to the performance of the previous player. Therefore, in this λ model, in order to favor the government, it is determined that we use relations (4), (7), (9), (10), (13) to control and ensure the reasonableness of the amount.

Determining the exclusive privilege period for the private sector

As mentioned in the introduction of decision variables, we call the connection point T_c The period of time that the private sector is allowed to use the facilities of the project's revenues after the end of the construction period. It is called a bonus period. In this article, to get the transfer point, we first consider a range for the transfer point, which we will discuss further.

The upper limit of the monopoly period is called T_{cu} and is obtained from the following relationship.

$$T_{cu} = sup\left\{t < T_f : NPV(T_{cu}) \leq NPV_P\left(T_f\right)\right\} \qquad (17)$$

And its lower limit is called T_{cl} and is obtained from the following relationship.

$$T_{cl} = \inf\{t < T_0 : R_t \geq R_1\} \qquad (18)$$

And finally we have:

$$T_c \in [T_{cl}\,, T_{cu}]$$

Now we use the Robinshtein model to determine the transition point. In Robinstein's model, using the rules of game theory and Nash equilibrium, there are two players, each of them seeks to increase their share in the negotiation. Each player has a discount rate that is obtained from the following relationship.

$$\delta_1 = \frac{1}{1+R_1}\,, \quad \delta_2 = \frac{1}{1+R_1} \qquad (19)$$

After negotiations, the result obtained in the Robinshtein model is as follows.

$$(\delta_1, \delta_2) = \left(\frac{1-\delta_2}{1-\delta_1\delta_2}\,, \frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}\right. \qquad (20)$$

In relation (19), R_1 and R_2 are respectively the minimum expected profit of the public and private sectors that expect from the project and in relation (20), δ_1 and δ_2 are respectively the percentage of use by the government and the private sector of the economic life of the project in terms of It is a private sector.

5. Conclusion

Contract B. O. TIt is a suitable mechanism for the government and the private sector and the use of proper management of this sector, as well as the use of public funds to carry out construction projects faster. In the presented model and its mechanism, the role of banks and the use of loans were especially considered, because in our country, banks' investment is mainly related to the construction of residential and commercial units, and when the housing market Banks will face serious problems if they go into recession. But in this way, by investing in construction projects, they can earn future and long-term profits, and their funds, which are mainly people's deposits, can be used for the development of the country, and in fact, a win-win game will occur.

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