Estimators using EWMA Statistic for Estimation of Population Mean

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Abstract: In this study, we have put forward a class of estimators for estimating population mean for the cross-sectional and time scaled surveys. It is revealed in the study conducted by [5] that the use of Exponentially weighted moving average (EWMA) statistic improves the performance of the estimators for the time scaled surveys as it makes the use of past as well as the present information. [5] proposed ratio estimators using EWMA statistic, so we have made improvements by introducing a class of modified difference-ratio and product estimators with the use of tri-mean, mid-range, Gini's mean difference, Downtown's method and probability weighted moments of auxiliary variable. Our estimators showed better results. Results are verified with the simulation.

Keywords: -auxiliary variable; study variable; simulation; simple random sampling.

Introduction

In Survey sampling, the statisticians are ardent to put forward the proficient estimators for estimating population characteristics. Auxiliary information always augments the efficiency of the estimators when sagely selected. For example, to estimate average lifetime of tube lights produced by a plant, money spent on each tube light by the plant to maintain its quality can be considered as the auxiliary variable. [2] propounded the use of ratio estimators for the estimation of population mean when study and auxiliary variable are linear and positively correlated.

$$t_r = \frac{\bar{y}}{\bar{x}}\bar{X} \qquad (1)$$

where $\bar{y}$ is the sample mean of study variable and $\bar{x}$ is the sample mean of auxiliary variable. $\bar{X}$ is the known population mean of auxiliary variable.

When the study variable and auxiliary variable are linear and negatively correlated, [4] suggested the use of product estimator for the estimation of population mean.

$$t_p = \frac{\bar{y}}{\bar{X}}\bar{x} \qquad (2)$$

The approximate mean square error (MSE) expression for estimator in equation (1) and (2) is as:

$$MSE(t_r) \cong f\left(c_y^2 + c_x^2 - 2\rho c_y c_x\right) \qquad (3)$$

$$MSE\left(t_p\right) \cong f\left(c_y^2 + c_x^2 + 2\rho c_y c_x\right) \qquad (4)$$

where $f = \frac{1}{n} - \frac{1}{N}$ (sampling fraction)

c_y and c_x is the coefficient of variation of study variable and auxiliary variable respectively.

ρ is the correlation coefficient between study and auxiliary variable.

Many authors utilized the available auxiliary information for the estimation of population parameters. [8] and [6] made use of coefficient of variation of auxiliary variable for estimating population parameter. Similarly, [11] utilized coefficient of kurtosis of auxiliary variable. [10] proposed ratio-cum-product estimator of population mean using coefficient of variation and coefficient of kurtosis of auxiliary variable. [9] proposed ratio type estimators using quartiles. [3] used Tri-mean, median and quartile deviation of auxiliary variable for estimation purpose. [1] and [3] used conventional and non-conventional measures of auxiliary variable for estimation of population parameter.

Authors have made the improvements by introducing a large number of modified ratio estimators with the use of coefficient of variation, coefficient of kurtosis, skewness, deciles, population tri-mean, mid-range, Hodges-Lehmann estimator, population correlation coefficient etc. All these studies are conducted for cross-sectional surveys.

In the section-I, our study is for cross-sectional surveys. Our proposed class of estimators performs better than the existing ones.

Memory type estimators for time scaled surveys were first time propounded by [5] for estimating population mean using EWMA. To augment the effectiveness of the estimators, [7] was the first to introduce the use of EWMA statistic, which uses former as well as the contemporary information. The EWMA statistic is as:

$Z_i = \lambda \overline{y_i} + (1-\lambda) Z_{i-1}$; where $0 > \lambda > 1$ and $i > 0$ (5)

$\overline{y_i}$ is the mean of sample at time i

Z_{i-1} is the past observation of the statistic

λ is the smoothing constant

When $\lambda = 1$, means that statistic $Z_i = \overline{y_i}$ $(usual\ sample\ mean)$

and $\lambda = 0$, means past information gains all weight and as λ moves from 0 to 1 contemporary information gains more weight than the past information.

$E(Z_i) = \bar{Y}$ and $var(Z_i) = \frac{S_y^2}{n}\left[\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)\right]$

Limiting variance of $var(Z_i) = \frac{S_y^2}{n}\left[\frac{\lambda}{2-\lambda}\right]$

In the section-II, our study is for time scaled surveys which are conducted on regular basis. For example, if we want to estimate unemployment rate of a particular area/region. Then we can take our study variable, Y= Number of unemployed and X= Labour force size as the auxiliary information. Since our study is conducted for time scaled surveys, which makes use of the past information along with the current information. In this we will consider number of unemployed and labour force for different surveys that are conducted on a regular basis and also with a specific time interval. Such as “Survey on household consumer expenditure and employment and unemployment” by NSSO, Survey on Annual Survey of Industries, Rural labour enquiry etc.

Sometimes observation get much affected by the extreme observations and give deceptive results. We have proposed a class of modified class of estimators using estimators of auxiliary information for the cross-sectional and time scaled surveys. Following estimators of auxiliary information have been used for proposing the class of estimators:

1. Tri-mean of Auxiliary variable X:

$$T_m = \frac{1}{2}\left(Q_2 + \frac{Q_1+Q_3}{2}\right)$$

Where Q_i (i=1,2 and 3) is the i^{th} quartile.

2. Mid-Range of auxiliary variable:

$$M_r = \frac{X_{(1)}+X_{(N)}}{2}$$

Where $X_{(i)}$ is the i^{th} observation of auxiliary variable X.

3. Gini's mean difference of the auxiliary variable X:

$$G_{md} = \frac{4}{N-1}\sum_{i=1}^{N}\left(\frac{2i-N-1}{2N}\right)X_{(i)}$$

4. Downtown's method of the auxiliary variable

$$D_m = \frac{2\sqrt{\Pi}}{N(N-1)}\sum_{i=1}^{N}\left(i-\frac{N+1}{2}\right)X_{(i)}$$

5. Probability weighted moments of the auxiliary variable

$$S_{pw} = \frac{\sqrt{\Pi}}{N^2}\sum_{i=1}^{N}(2i-N-1)X_{(i)}$$

Proposed Estimator

Ratio estimator is apt to use when relationship between x and y is linear through the origin. Otherwise, regression estimators give better results. In this article we combined regression and ratio estimator to get the proficient class of estimators.

Section-I

We proposed difference cum ratio and difference cum product class of estimators using auxiliary information for the conventional surveys.

$$t_{rr} = [\theta_1\,\bar{y} + \theta_2\,(\bar{X}\,-\,\bar{x})]\left[\frac{\alpha\bar{X}+\beta C}{\alpha\bar{x}+\beta C}\right] \tag{6}$$

$$t_{rp} = [\theta_1\,\bar{y} + \theta_2\,(\bar{X}\,-\,\bar{x})]\left[\frac{\alpha\bar{x}+\beta C}{\alpha\bar{X}+\beta C}\right] \tag{7}$$

where θ_1 and θ_2 are the suitably chosen constants to minimize MSE of the estimator.

$\alpha\bar{X} + \beta C > 0$ and $\alpha\bar{x} + \beta C > 0$

α and β are constants or functions of parameters of auxiliary variable which are known.

For different values of (θ_1,θ_2,α,β, C), various other members of proposed class of estimators are generated.

Table 1. Members of the proposed class of estimators

θ_1	θ_2	α	β	C	Estimators
θ_1	θ_2	C_x	β	T_m	$t_{rr1} = [\theta_1\,\bar{y} + \theta_2\,(\bar{X}\,-\,\bar{x})]\left[\frac{\alpha\bar{X}+\beta T_m}{\alpha\bar{x}+\beta T_m}\right]$
θ_1	θ_2	C_x	β	M_r	$t_{rr2} = [\theta_1\,\bar{y} + \theta_2\,(\bar{X}\,-\,\bar{x})]\left[\frac{\alpha\bar{X}+\beta M_r}{\alpha\bar{x}+\beta M_r}\right]$
θ_1	θ_2	C_x	β	G_{md}	$t_{rr3} = [\theta_1\,\bar{y} + \theta_2\,(\bar{X}\,-\,\bar{x})]\left[\frac{\alpha\bar{X}+\beta G_{md}}{\alpha\bar{x}+\beta G_{md}}\right]$
θ_1	θ_2	C_x	β	D_m	$t_{rr4} = [\theta_1\,\bar{y} + \theta_2\,(\bar{X}\,-\,\bar{x})]\left[\frac{\alpha\bar{X}+\beta D_m}{\alpha\bar{x}+\beta D_m}\right]$

θ_1	θ_2	C_x	β	S_{pw}	$t_{rr5} = [\theta_1\, \bar{y} + \theta_2\, (\bar{X} - \bar{x})] \left[\frac{\alpha\bar{X}+\beta S_{pw}}{\alpha\bar{x}+\beta S_{pw}}\right]$
θ_1	θ_2	C_x	β	T_m	$t_{rp1} = [\theta_1\, \bar{y} + \theta_2\, (\bar{X} - \bar{x})] \left[\frac{\alpha\bar{x}+\beta T_m}{\alpha\bar{X}+\beta T_m}\right]$
θ_1	θ_2	C_x	β	M_r	$t_{rp2} = [\theta_1\, \bar{y} + \theta_2\, (\bar{X} - \bar{x})] \left[\frac{\alpha\bar{x}+\beta M_r}{\alpha\bar{X}+\beta M_r}\right]$
θ_1	θ_2	C_x	β	G_{md}	$t_{rp3} = [\theta_1\, \bar{y} + \theta_2\, (\bar{X} - \bar{x})] \left[\frac{\alpha\bar{x}+\beta G_{md}}{\alpha\bar{X}+\beta G_{md}}\right]$
θ_1	θ_2	C_x	β	D_m	$t_{rp4} = [\theta_1\, \bar{y} + \theta_2\, (\bar{X} - \bar{x})] \left[\frac{\alpha\bar{x}+\beta D_m}{\alpha\bar{X}+\beta D_m}\right]$
θ_1	θ_2	C_x	β	S_{pw}	$t_{rp5} = [\theta_1\, \bar{y} + \theta_2\, (\bar{X} - \bar{x})] \left[\frac{\alpha\bar{x}+\beta S_{pw}}{\alpha\bar{X}+\beta S_{pw}}\right]$

To derive MSE expression of the estimator t_{rr} , we performed the Taylor series expansion using the following terms:

$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{y}}$ and $e_1 = \frac{\bar{x}-\bar{X}}{\bar{x}}$

$E[e_0] = E[e_1] = 0$;

$E(e_0^2) = f\ \frac{var(y_i)}{\bar{Y}^2} = f\ c_y^2;$

$E(e_1^2) = f\ \frac{var(x_i)}{\bar{X}^2} = f\ c_x^2;$

$$E(e_0 e_1) = f\ \frac{Cov(y_i, x_i)}{\bar{Y}\bar{X}} = f\ \rho\ c_y c_x$$

Using the above terms, Equation (6) can be written as:

$$t_{rr} = [\theta_1 \bar{Y}(1 + e_0) - \theta_2 \bar{X} e_1] \left[\frac{\alpha\bar{X}+\beta C}{\alpha\bar{X}(1+e_1)+\beta C}\right] \quad (8)$$

Equation (8) can further be written as:

$$t_{rr} = \bar{Y}[\theta_1(1 + e_0) - \theta_2\, \gamma^* e_1](1 + \gamma\, e_1)^{-1} \quad (9)$$

Where $\gamma = \frac{\alpha\, \bar{X}}{\alpha\, \bar{X}+\beta C}$ and $\gamma^* = \frac{\bar{X}}{\bar{Y}}$

Subtracting μ_y from both the sides of equation (9) and then squaring:

$$(t_{rr} - \bar{Y})^2 = [\bar{Y}\{\theta_1(1 + e_0 - \gamma\, e_1 - \gamma e_0 e_1 + \gamma^2 e_1^2) + \theta_2\, \gamma^*(\gamma e_1^2 - e_1)\} - \bar{Y}]^2 \quad (10)$$

Now taking expectation on both the sides of equation (10), we get MSE expression as:

$$MSE(t_{rr}) = \bar{Y}^2\ E\big[1 + \theta_1^2\, (1 + e_0^2 + 3\, \gamma^2\, e_1^2 - 4\, \gamma\, e_0\, e_1) + \theta_2^2\, \gamma^{*2}\, e_1^2 + 2\theta_1\theta_2\gamma^*(2\gamma e_1^2 - e_0 e_1) - 2\theta_1(1 - \gamma e_0\, e_1 + \gamma^2 e_1^2) - 2\theta_2\gamma\, \gamma^* e_1^2\big] \quad (11)$$

Equation (11) can further be written as:

$$MSE(t_{rr}) \cong\ [1 + \theta_{1r}^2\, A_{1r} + \theta_{2r}^2\, B_{1r} + 2\theta_{1r}\theta_{2r} C_{1r} - 2\theta_{1r} D_{1r} - 2\theta_{2r} E_{1r}] \quad (12)$$

where

$$A_{1r} = 1 + f\big(c_y^2 + 3\, \gamma^2\, c_x^2 - 4\, \gamma\, \rho\, c_y\, c_x\big)$$

$B_{1r} = f\,\gamma^{*2}\,c_y^2$
$C_{1r} = f\gamma^*\left(2\gamma c_x^2 - \rho\, c_y\, c_x\right)$
$D_{1r} = 1 - f\left(\gamma\rho\, c_y\, c_x - \gamma^2 c_x^2\right)$
$E_{1r} = f\gamma\,\gamma^* c_x^2$

To get the Minimum MSE of the estimator t_{rr} , we differentiated equation (12) with respect to θ_{1r} and θ_{2r} and we get MSE of the estimator t_{rr} minimum for the following values:

$$\theta_{1r} = \frac{B_{1r}D_{1r} - C_{1r}E_{1r}}{A_{1r}B_{1r} - C_{1r}^2} = \theta_{1r}^*(say)$$
$$\theta_{2r} = \frac{A_{1r}E_{1r} - C_{1r}D_{1r}}{A_{1r}B_{1r} - C_{1r}^2} = \theta_{2r}^*(say)$$

Therefore,

$$Min.MSE(t_{rr}) \cong \left[1 + \theta_{1r}^{*2}\,A_{1r} + \theta_{2r}^{*2}\,B_{1r} + 2\theta_{1r}^*\theta_{2r}^* C_{1r} - 2\theta_{1r}^* D_{1r} - 2\theta_{2r}^* E_{1r}\right] \quad (13)$$

Similarly, MSE expression for the estimator t_{rp} is derived as:

$$MSE\left(t_{rp}\right) = \bar{Y}^2\; E\left[1 + \theta_{1p}^2\,(1 + e_0^2 + \gamma^2\, e_1^2 + 4\,\gamma\, e_0\, e_1) + \theta_{2p}^2\,\gamma^{*2}\, e_1^2 - 2\theta_{1p}\theta_{2p}\gamma^*(2\gamma e_1^2 + e_0 e_1) - 2\theta_{1p}(1 + \gamma^2 e_1^2) + 2\theta_{2p}\gamma\,\gamma^* e_1^2\right] \quad (14)$$

Equation (14) can further be written as:

$$MSE\left(t_{rp}\right) \approx \left[1 + \theta_{1p}^2\,A_{1p} + \theta_{2p}^2\,B_{1p} + 2\theta_{1p}\theta_{2p}C_{1p} - 2\theta_{1p}D_{1p} - 2\theta_{2p}E_{1p}\right] \quad (15)$$

where

$A_{1p} = 1 + f\left(c_y^2 + \gamma^2\,c_x^2 + 4\,\gamma\,\rho\,c_y\,c_x\right)$
$B_{1p} = f\,\gamma^{*2}\,c_y^2$
$C_{1p} = -f\gamma^*\left(2\gamma c_x^2 + \rho\,c_y\,c_x\right)$
$D_{1p} = 1 + f\gamma\rho\,c_y\,c_x$
$E_{1p} = -f\gamma\,\gamma^* c_x^2$

To get the Minimum MSE of the estimator t_{rp} , we differentiated equation (15) with respect to θ_{1p} and θ_{2p} and we get MSE of the estimator t_{rp} minimum for the following values:

$$\theta_{1p} = \frac{B_{1p}D_{1p} - C_{1p}E_{1p}}{A_{1p}B_{1p} - C_{1p}^2} = \theta_{1p}^*(say)$$
$$\theta_{2p} = \frac{A_{1p}E_{1p} - C_{1p}D_{1p}}{A_{1p}B_{1p} - C_{1p}^2} = \theta_{2p}^*(say)$$

Therefore,

$$Min.MSE\left(t_{rp}\right) \cong \left[1 + \theta_{1p}^{*2}\,A_{1p} + \theta_{2p}^{*2}\,B_{1p} + 2\theta_{1p}^*\theta_{2p}^* C_{1p} - 2\theta_{1p}^* D_{1p} - 2\theta_{2p}^* E_{1p}\right] \quad (16)$$

Section-II

In this section, we utilized EWMA statistic to propose difference-ratio and product estimator using auxiliary information for the time scaled surveys. So, the EWMA statistic for study variable y is as:

$$Z_i = \lambda\,\bar{y} + (1-\lambda)\,Z_{i-1} \quad (17)$$

Similarly, the EWMA statistic for auxiliary variable x is

$$Q_i = \lambda\,\bar{x} + (1-\lambda)\,Q_{i-1} \quad (18)$$

By using statistic Z_i and Q_i , proposed class of estimator t_{rr} is framed as follows:

$$t_{mrr} = [\theta_1\,Z_i + \theta_2\,(\,Q_i - \mu_x)]\left[\frac{\alpha\mu_x + \beta C}{\alpha Q_i + \beta C}\right] \quad (19)$$

$$t_{mrp} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha Q_i + \beta C}{\alpha \mu_x + \beta C}\right] \quad (20)$$

where θ_1 and θ_2 are the suitably chosen constants to minimize MSE of the estimator.

$\alpha\mu_x + \beta C > 0$ and $\alpha Q_i + \beta C > 0$

α and β are constants or functions of parameters of auxiliary variable which are known.

For different values of (θ_1,θ_2,α,β, C), various other members of proposed class of estimators are generated.

Table 2. Members of the proposed class of estimators

θ_1	θ_2	α	β	C	Estimators
θ_1	θ_2	C_x	β	T_m	$t_{mr1} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha\mu_x + \beta T_m}{\alpha Q_i + \beta G_{md}}\right]$
θ_1	θ_2	C_x	β	M_r	$t_{mr2} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha\mu_x + \beta M_r}{\alpha Q_i + \beta M_r}\right]$
θ_1	θ_2	C_x	β	G_{md}	$t_{mr3} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha\mu_x + \beta G_{md}}{\alpha Q_i + \beta G_{md}}\right]$
θ_1	θ_2	C_x	β	D_m	$t_{mr4} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha\mu_x + \beta D_m}{\alpha Q_i + \beta D_m}\right]$
θ_1	θ_2	C_x	β	S_{pw}	$t_{mr5} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha\mu_x + \beta S_{pw}}{\alpha Q_i + \beta S_{pw}}\right]$
θ_1	θ_2	C_x	β	T_m	$t_{mp1} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha Q_i + \beta T_m}{\alpha\mu_x + \beta G_{md}}\right]$
θ_1	θ_2	C_x	β	M_r	$t_{mp2} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha Q_i + \beta M_r}{\alpha\mu_x + \beta M_r}\right]$
θ_1	θ_2	C_x	β	G_{md}	$t_{mp3} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha Q_i + \beta G_{md}}{\alpha\mu_x + \beta G_{md}}\right]$
θ_1	θ_2	C_x	β	D_m	$t_{mp4} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha Q_i + \beta D_m}{\alpha\mu_x + \beta D_m}\right]$
θ_1	θ_2	C_x	β	S_{pw}	$t_{mp5} = [\theta_1 \, Z_i + \theta_2 \, (\, Q_i - \mu_x)] \left[\frac{\alpha Q_i + \beta S_{pw}}{\alpha\mu_x + \beta S_{pw}}\right]$

To derive MSE expression of the estimator t_{mr} , we performed the Taylor series expansion using the following terms:

$$e_0 = \frac{Z_i - \mu_y}{\mu_y} \text{ and } e_1 = \frac{Q_i - \mu_x}{\mu_x}$$

$E[e_0] = E[e_1] = 0$; $E(e_0^2) = f \, \frac{var(Z_i)}{\bar{Y}^2} = f \, c_y^2 \left[\frac{\lambda}{2-\lambda}\right]$; $E(e_1^2) = f \, \frac{var(Q_i)}{\bar{X}^2} = f \, c_x^2 \left[\frac{\lambda}{2-\lambda}\right]$;

$E(e_0 e_1) = f \, \frac{Cov(Z_i,Q_i)}{\bar{Y}\bar{X}} = f \, \rho \, c_y c_x \left[\frac{\lambda}{2-\lambda}\right]$

Using the above terms, Equation (19) can be written as:

$$t_{mr} = \left[\theta_{1mr} \, \mu_y(1 + e_0) - \theta_{2mr}\mu_x e_1\right] \left[\frac{\alpha\mu_x + \beta C}{\alpha \, \mu_x(1+e_1) + \beta C}\right] \quad (21)$$

Equation (21) can further be written as:

$$t_{mr} = \mu_y[\theta_{1mr}(1+e_0) - \theta_{2mr}\,\gamma'^{*}e_1](1+\gamma'\,e_1)^{-1} \qquad (22)$$

Where $\gamma' = \frac{\alpha\,\mu_x}{\alpha\,\mu_x+\beta C}$, $\gamma'^{*} = \frac{\mu_x}{\mu_y}$ and $\beta = 1$

Subtracting μ_y from both the sides of equation (22) and then squaring:

$$\left(t_{mr} - \mu_y\right)^2 = \left[\,\mu_y\{\theta_{1mr}(1+e_0-\gamma'\,e_1-\gamma' e_0 e_1+\gamma'^2 e_1^2)+\theta_{2mr}\,\gamma'^{*}(\gamma' e_1^2-e_1)\} - \mu_y\right]^2$$
(23)

Now taking expectation on both the sides of equation (23), we get MSE expression as:

$$MSE(t_{mr}) = \mu_y{}^2\ E\big[1+\theta_{1mr}^2\,(1+e_0^2+3\,\gamma'^2\,e_1^2-4\,\gamma'\,e_0\,e_1)+\theta_{2mr}^2\,{\gamma'^{*}}^2\,e_1^2 + 2\theta_{1mr}\theta_{2mr}\gamma'^{*}(2\gamma' e_1^2-e_0e_1)-2\theta_{1mr}(1-\gamma' e_0\,e_1+\gamma'^2 e_1^2)-2\theta_{2mr}\gamma'\,\gamma'^{*}e_1^2\big]$$
(24)

Equation (24) can further be written as:

$$MSE(t_{mr}) = \mu_y{}^2\ [1+\theta_{1mr}^2\,A_{1mr}+\theta_2^2\,B_{1mr}+2\theta_{1mr}\theta_{2mr}C_{1mr}-2\theta_{1mr}D_{1mr} - 2\theta_{2mr}E_{1mr}] \qquad (25)$$

where

$$A_{1mr} = 1+f\left(\frac{\lambda}{2-\lambda}\right)\left(c_y^2+3\,\gamma'^2\,c_x^2-4\,\gamma'\,\rho\,c_y\,c_x\right)$$

$$B_{1mr} = f\left(\frac{\lambda}{2-\lambda}\right)\,{\gamma'^{*}}^2\,c_y^2$$

$$C_{1mr} = f\left(\frac{\lambda}{2-\lambda}\right)\gamma'^{*}\left(2\gamma' c_x^2-\rho\,c_y\,c_x\right)$$

$$D_{1mr} = 1-f\left(\frac{\lambda}{2-\lambda}\right)\left(\gamma'\rho\,c_y\,c_x-\gamma'^2 c_x^2\right)$$

$$E_{1mr} = f\left(\frac{\lambda}{2-\lambda}\right)\gamma'\,\gamma'^{*}c_x^2$$

To get the Minimum MSE of the estimator t_{mrr} , we differentiated equation (25) with respect to θ_{1mr} and θ_{2mr} and we get MSE of the estimator t_{mrr} minimum for the following values:

$$\theta_{1mr} = \frac{B_{1mr}D_{1mr}-C_{1mr}E_{1mr}}{A_{1mr}B_{1mr}-C_{1mr}^2} = \theta_{1mr}^{*}(say)$$

$$\theta_{2mr} = \frac{A_{1mr}E_{1mr}-C_{1mr}D_{1mr}}{A_{1mr}B_{1mr}-C_{1mr}^2} = \theta_{2mr}^{*}(say)$$

Therefore,

$$Min.MSE(t_{mr}) \approx \begin{bmatrix} 1+\theta_{1mr}^{*2}\,A_{1mr}+\theta_{2mr}^{*2}\,B_{1mr}+2\theta_{1mr}^{*}\theta_{2mr}^{*}C_{1mr}-2\theta_{1mr}^{*}D_{1mr} \\ -2\theta_{2mr}^{*}E_{1mr} \end{bmatrix} \qquad (26)$$

Similarly, MSE expression for the product estimator is derived as:

$$MSE\left(t_{mp}\right) \approx E\big[1+\theta_{1mp}^2\,(1+e_0^2+\gamma'^2\,e_1^2+4\,\gamma'\,e_0\,e_1)+\theta_{2mp}^2\,{\gamma'^{*}}^2\,e_1^2 - 2\theta_{1mp}\theta_{2mp}{\gamma'}^{*}(2\gamma'\,e_1^2+e_0e_1)-2\theta_{1mp}\left(1+{\gamma'}^2 e_1^2\right)+2\theta_{2mp}\gamma'\,\gamma'^{*}e_1^2\big] \quad (27)$$

Equation (23) can further be written as:

$$MSE\left(t_{mp}\right) \approx \left[1+\theta_{1mp}^2\,A_{1mp}+\theta_{2mp}^2\,B_{1mp}+2\theta_{1mp}\theta_{2mp}C_{1mp}-2\theta_{1mp}D_{1mp} - 2\theta_{2mp}E_{1mp}\right] \qquad (28)$$

where

$A_{1mp} = 1 + f\left(\frac{\lambda}{2-\lambda}\right)\left(c_y^2 + \gamma'^2 c_x^2 + 4\gamma' \rho c_y c_x\right)$

$B_{1mp} = f\left(\frac{\lambda}{2-\lambda}\right) \gamma'^{*^2} c_y^2$

$C_{1mp} = -f\left(\frac{\lambda}{2-\lambda}\right)\gamma'^{*}\left(2\gamma' c_x^2 + \rho c_y c_x\right)$

$D_{1mp} = 1 + f\left(\frac{\lambda}{2-\lambda}\right)\gamma'\rho c_y c_x$

$E_{1mp} = -f\left(\frac{\lambda}{2-\lambda}\right)\gamma' \gamma'^{*} c_x^2$

To get the Minimum MSE of the estimator t_{mp} , we differentiated equation (28) with respect to θ_{1mp} and θ_{2mp} and we get MSE of the estimator t_{mp} minimum for the following values:

$$\theta_{1mp} = \frac{B_{1mp}D_{1mp} - C_{1mp}E_{1mp}}{A_{1mp}B_{1mp} - C_{1mp}^2} = \theta_{1mp}^*(say)$$

$$\theta_{2mp} = \frac{A_{1mp}E_{1mp} - C_{1mp}D_{1mp}}{A_{1mp}B_{1mp} - C_{1mp}^2} = \theta_{2mp}^*(say)$$

Therefore,

$Min.MSE\left(t_{mp}\right) \approx \left[1 + \theta_{1mp}^{*2} A_{1mp} + \theta_{2mp}^{*2} B_{1mp} + 2\theta_{1mp}^*\theta_{2mp}^* C_{1mp} - 2\theta_{1mp}^* D_{1mp} - 2\theta_{2mp}^* E_{1mp}\right]$ (29)

Results

For the comparison purpose, we have calculated Relative efficiency (RE) of the estimators. RE is calculated as:

RE=$\frac{MSE\ of\ proposed\ estimator}{MSE\ of\ the\ usual\ estimator}$

Table 2(a) and 2(b) shows the RE of the ratio and product estimators for the different values of the correlation coefficient respectively. It is found that:

- Our proposed estimator $t_{ri}, t_{rpi} (i = 1,2, \dots ,5)$ perform better than the other existing estimators.
- Our proposed estimators $t_{r1}, t_{r2}, t_{r3}, t_{r4}, t_{r5}$ have the almost same RE, so anyone of the estimator can be used for the estimation purpose depending on the availability of particular auxiliary information. Since $t_{r1}, t_{r2}, t_{r3}, t_{r4}\ and\ t_{r5}$ have almost same RE, this is the reason we have written RE for t_{ri} only. Same is for product estimators $t_{rp1}, t_{rp2}, t_{rp3}, t_{rp4}\ and\ t_{rp5}$.
- As λ increases from 0.1 to 1 (i.e., weightage to current information increases), RE of the estimators decreases.
- For $\lambda = 1$, estimator t_{mr} and t_{mri} $(i = 1, 2, \dots ,5)$ are same as estimator t_r and t_{ri}.
- As we have also calculated RE for the different values of correlation coefficient. On varying ρ from 0.05 to 0.95, RE of the estimators increases. Same is the case when we varied from -0.05 to -0.95.
- RE of the proposed estimators is higher than the existing ones, this shows that proposed estimators are better than the existing ones for any value of ρ.
- We have also studied the change in RE for the different sample values as we varied n from 20 to 500. As the size of n increases RE decreases.

Conclusion

In this paper, we have studied the behaviour of our proposed modified ratio estimators for the cross-sectional and time scaled survey using estimates of auxiliary estimators. We found that our proposed estimators perform better than the existing ones. So, it is recommended to use our estimator for the time scaled surveys.

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Table 2(a) : RE of the ratio estimators for the different positive values of correlation coefficient.

ρ	n			λ=0.1		λ=0.25		λ=0.5		λ=0.75		λ=1	
		t_r	t_{ri}	t_{mr}	t_{mri}	t_{mr}	t_{mri}	t_{mr}	t_{mri}	t_{mr}	t_{mri}	t_{mr}	t_{mri}
	20	0.958	1.074	18.208	20.112	6.708	7.420	2.875	3.189	1.597	1.779	0.958	1.074
	40	0.955	1.036	18.154	19.556	6.688	7.209	2.866	3.094	1.592	1.722	0.955	1.036

0.05	80	0.955	1.019	18.142	19.303	6.684	7.114	2.865	3.051	1.591	1.696	0.955	1.019
	100	0.955	1.016	18.143	19.257	6.684	7.096	2.865	3.043	1.591	1.692	0.955	1.016
	500	0.956	1.006	18.167	19.103	6.693	7.038	2.868	3.017	1.594	1.67	0.956	1.006
0.25	20	1.068	1.143	20.291	21.432	7.476	7.906	3.204	3.397	1.780	1.895	1.068	1.143
	40	1.069	1.104	20.305	20.853	7.481	7.687	3.206	3.299	1.781	1.836	1.069	1.104
	80	1.068	1.086	20.298	20.575	7.478	7.582	3.205	3.251	1.780	1.808	1.068	1.086
	100	1.068	1.083	20.301	20.527	7.479	7.564	3.205	3.243	1.781	1.803	1.068	1.083
	500	1.070	1.073	20.323	20.376	7.488	7.507	3.209	3.218	1.783	1.788	1.070	1.073
0.50	20	1.246	1.424	23.683	26.764	8.725	9.871	3.739	4.239	2.077	2.362	1.246	1.424
	40	1.253	1.381	23.798	26.108	8.768	9.623	3.758	4.128	2.088	2.297	1.253	1.381
	80	1.253	1.358	23.812	25.744	8.773	9.487	3.760	4.068	2.089	2.261	1.253	1.358
	100	1.254	1.354	23.820	25.685	8.776	9.465	3.761	4.058	2.089	2.255	1.254	1.354
	500	1.255	1.342	23.836	25.490	8.782	9.391	3.764	4.025	2.091	2.236	1.255	1.342
0.75	20	1.502	2.423	28.540	45.760	10.515	16.869	4.506	7.239	2.504	4.028	1.502	2.423
	40	1.514	2.364	28.772	44.787	10.600	16.505	4.543	7.078	2.524	3.935	1.514	2.364
	80	1.518	2.326	28.834	44.131	10.623	16.261	4.553	6.971	2.529	3.874	1.518	2.326
	100	1.518	2.320	28.847	44.032	10.628	16.224	4.555	6.955	2.530	3.865	1.518	2.320
	500	1.519	2.299	28.870	43.663	10.628	16.235	4.555	6.965	2.530	3.876	1.519	2.299
0.95	20	1.806	10.786	34.317	204.653	12.643	75.408	5.418	32.327	3.010	17.966	1.806	10.786
	40	1.823	10.582	34.646	200.930	12.764	74.031	5.470	31.732	3.039	17.632	1.823	10.582
	80	1.830	10.417	34.779	197.856	12.813	72.896	5.491	31.243	3.051	17.359	1.830	10.417
	100	1.832	10.393	34.801	197.414	12.821	72.733	5.495	31.173	3.053	17.319	1.832	10.393
	500	1.835	10.292	34.863	195.533	12.844	72.039	5.505	30.874	3.058	17.152	1.835	10.292

Table 2 (b): RE of the r estimators for the different negative values of correlation coefficient.

ρ	n			λ=0.1		λ=0.25		λ=0.5		λ=0.75		λ=1	
		t_r	t_{rpi}	t_{mrp}	t_{mrpi}	t_{mrp}	t_{mrpi}	t_{mrp}	t_{mrpi}	t_{mrp}	t_{mrpi}	t_{mrp}	t_{mrpi}
	20	0.950	1.072	18.043	20.082	6.647	7.408	2.849	3.184	1.583	1.776	0.950	1.072
	40	0.950	1.035	18.044	19.536	6.648	7.202	2.849	3.091	1.583	1.720	0.950	1.035
	80	0.949	1.018	18.035	19.283	6.644	7.106	2.848	3.048	1.582	1.695	0.949	1.018

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-0.05	100	0.949	1.015	18.030	19.236	6.642	7.089	2.847	3.040	1.582	1.690	0.949	1.015
	500	0.947	1.004	17.995	19.070	6.630	7.026	2.841	3.012	1.579	1.673	0.947	1.004
-0.25	20	1.060	1.136	20.141	21.309	7.420	7.860	3.180	3.378	1.767	1.883	1.060	1.136
	40	1.063	1.100	20.204	20.770	7.444	7.657	3.190	3.285	1.772	1.828	1.063	1.100
	80	1.063	1.082	20.198	20.493	7.442	7.552	3.189	3.239	1.772	1.801	1.063	1.082
	100	1.063	1.078	20.195	20.440	7.440	7.532	3.189	3.230	1.772	1.795	1.063	1.078
	500	1.061	1.066	20.163	20.238	7.429	7.456	3.184	3.196	1.769	1.776	1.061	1.066
-0.50	20	1.244	1.414	23.633	26.596	8.707	9.808	3.731	4.212	2.073	2.347	1.244	1.414
	40	1.251	1.374	23.767	25.988	8.756	9.579	3.753	4.109	2.085	2.286	1.251	1.374
	80	1.252	1.351	23.783	25.617	8.762	9.440	3.755	4.048	2.086	2.250	1.252	1.351
	100	1.252	1.347	23.788	25.547	8.764	9.414	3.756	4.036	2.087	2.243	1.252	1.347
	500	1.252	1.331	23.788	25.272	8.764	9.311	3.756	3.991	2.087	2.217	1.252	1.331
-0.75	20	1.505	2.413	28.588	45.576	10.532	16.801	4.514	7.209	2.508	4.012	1.505	2.413
	40	1.517	2.356	28.824	44.632	10.619	16.448	4.551	7.053	2.528	3.921	1.517	2.356
	80	1.521	2.317	28.901	43.955	10.648	16.196	4.563	6.943	2.535	3.859	1.521	2.317
	100	1.522	2.309	28.922	43.828	10.655	16.149	4.567	6.922	2.537	3.847	1.522	2.309
	500	1.526	2.282	28.988	43.348	10.680	15.971	4.577	6.845	2.543	3.803	1.526	2.282
-0.95	20	1.806	10.781	34.310	204.563	12.640	75.375	5.417	32.312	3.010	17.958	1.806	10.781
	40	1.826	10.566	34.702	200.641	12.785	73.924	5.479	31.686	3.044	17.606	1.826	10.566
	80	1.836	10.397	34.885	197.488	12.852	72.761	5.508	31.185	3.060	17.326	1.836	10.397
	100	1.838	10.366	34.927	196.915	12.868	72.549	5.515	31.094	3.064	17.276	1.838	10.366
	500	1.846	10.256	35.075	194.852	12.922	71.788	5.538	30.766	3.077	17.093	1.846	10.256