Root Square Mean Labeling of Theta Related Graphs

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Abstract A graph G with p vertices and q edges is called root square mean graph if it is possible to label the vertices $x \in V$ with distinct lables f(x) from $1,2,\dots,q+1$ in such a way that when each edge $e = uv$ is labeled with

$f(e=uv) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then the resulting edge labels are distinct. In this case f is called a root square mean labeling of G. In this paper we prove that theta related graphs such as θ_α, $\theta_\alpha \odot \overline{K_1}$, $\theta_\alpha \odot \overline{K_2}$, $\theta_\alpha \odot \overline{K_3}$, $\theta_\alpha \odot K_2, \theta_\alpha \cup C_\beta, (\theta_\alpha \odot \overline{K_1}) \cup C_\beta$, $\theta_\alpha \cup (C_\beta \odot \overline{K_1})$, $(\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot \overline{K_1}), \theta_\alpha \cup (C_\beta \odot \overline{K_2}), (\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot \overline{K_2}), \theta_\alpha \cup (C_\beta \odot \overline{K_3}), (\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot \overline{K_3}), \theta_\alpha \cup (C_\beta \odot K_2), (\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot K_2), \theta_\alpha \cup L_\beta, \theta_\alpha \cup T_\beta, \theta_\alpha \cup Q_\beta$ are all root square mean graphs.

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Introduction

Let $G = (V, E)$ be a (p, q) graph with $p = |V(G)|$ vertices and $q = |E(G)|$ edges, where $V(G)$ and $E(G)$ respectively denote the vertex set and edge set of the graph G. Graph labeling is used in many areas of sciences and technology. For all other standard terminology and notations we follow Harary [2]. A lot of graph labeling techniques are discussed in [1]. The concept of root square mean labeling has been introduced by S. S. Sandhya, S. Somasundaram and S. Anusa in 2014 [4]. Meena. S and Mani. R [3] investigated this labeling for some cycle related graphs . Prime cordial labeling for theta graph has been proved by A. Sugumaran and P. Vishnu Prakash [5]. In this paper, we examine about the root square mean labeling of theta θ_α graphs and some new examples are presented and verified. We now give the definitions which are necessary for the present investigation.

Definition 1.1:

The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition 1.2:

A theta graph is a block with two non-adjacent vertices of degree 3 and all other vertices of degree 2 is called a theta graph and it is denoted by θ_α.

Definition 1.3:

The union of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph $G = G_1 \cup G_2$

with vertex set $V = V_1 \cup V_2$ and the edge set $= E_1 \cup E_2$.

Definition 1.4:

A triangular snake T_n is obtained from a path $b_1 b_2 \dots b_n$ by joining b_i and b_{i+1} to

a new vertex c_i for $1 \leq i \leq n-1$.

Definition 1.5:

A quadrilateral snake Q_n is obtained from a path $b_1 b_2 \dots b_n$ by joining b_i and b_{i+1}

to a new vertices c_i and d_i for $1 \leq i \leq n-1$ respectively .

II. Main Results

In this paper, we investigate the root square mean labeling of theta θ_α Graphs.

Theorem 2.1:

Theta θ_α is a root square mean graph.

Proof:

Let $G = \theta_\alpha$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of theta in G.

Let $V(G) = \{a_1 a_2 \dots a_\alpha\}$

$E(G) = \{a_i a_{i+1} \ / \ 1 \leq i \leq \alpha - 1\} \cup \{a_1 a_5,\ a_\alpha a_2\}$

Define a function $f: V(G) \longrightarrow \{1,2, \dots, \alpha + 2\}$

$$f(a_i) = i \quad \text{for} \quad 1 \leq i \leq 5$$
$$f(a_i) = 2i - 5 \quad \text{for} \quad 6 \leq i \leq \alpha$$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.1.1:

The root square mean labeling of θ_7 is given below:

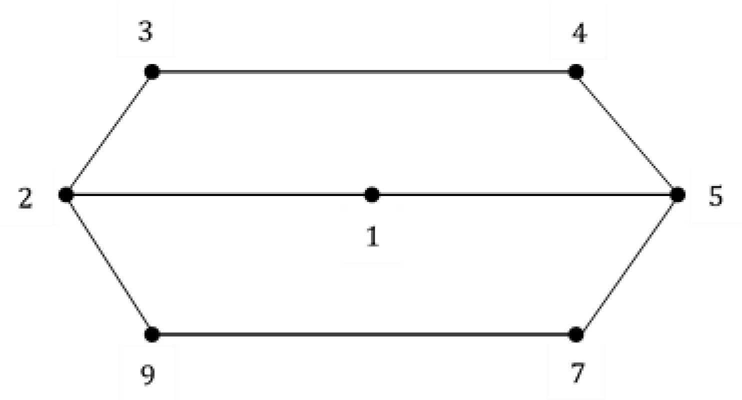

Figure 1

Theorem 2.2:

$\theta_\alpha \odot \overline{K_1}$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \odot \overline{K_1}$

Let $a_1 a_2 \ldots a_\alpha$ be the vertices of theta in G.

Let $b_1 b_2 \ldots b_\alpha$ be the pendant vertices attached at $a_1 a_2 \ldots a_\alpha$ respectively.

Let $V(G) = \{a_1 a_2 \ldots a_\alpha,\ b_1 b_2 \ldots b_\alpha\}$

$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{a_i b_i \,/\, 1 \le i \le \alpha\} \cup \{a_1 a_5,\ a_\alpha a_2\}$

Define a function $f: V(G) \longrightarrow \{1,2,\ldots,2\alpha+1\}$

$f(a_i) = 2i$ for $1 \le i \le 5$

$f(a_i) = 2i + 1$ for $6 \le i \le \alpha$

$f(b_i) = 2i - 1$ for $1 \le i \le 5$

$f(b_i) = 2i$ for $6 \le i \le \alpha$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.2.1:

The root square mean labeling of $\theta_7 \odot \overline{K_1}$ is given below:

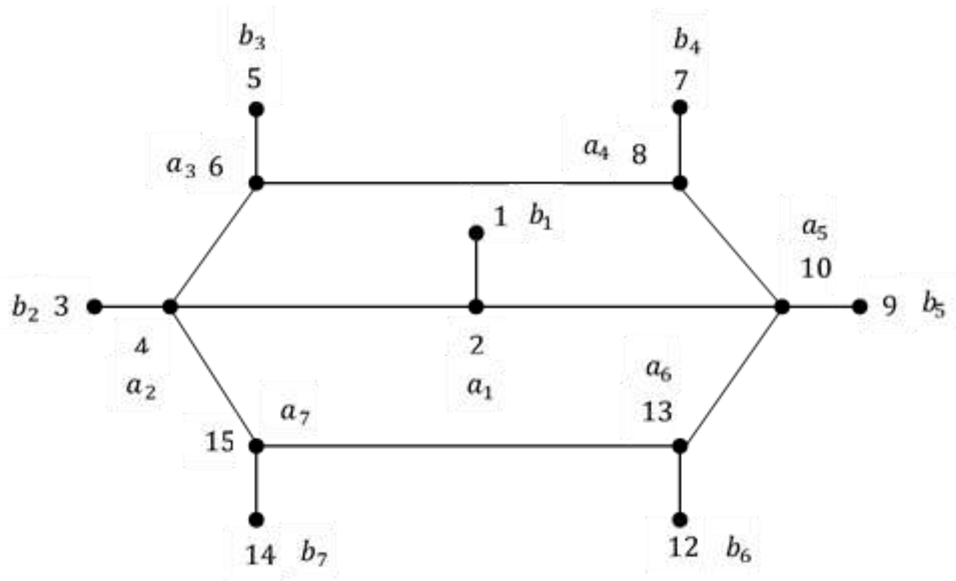

Figure 2

Theorem 2.3:

$\theta_\alpha \odot \overline{K_2}$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \odot \overline{K_2}$

Let $a_1 a_2 \ldots a_\alpha$ be the vertices of the theta θ_α in G.

Let $b_1 b_2 \ldots b_\alpha$ and $c_1 c_2 \ldots c_\alpha$ be the pendant vertices attached at $a_1 a_2 \ldots a_\alpha$ respectively.

Let $V(G) = \{a_1 a_2 \ldots a_\alpha,\ b_1 b_2 \ldots b_\alpha,\ c_1 c_2 \ldots c_\alpha\}$

$$E(G) = \{a_i a_{i+1} \,/\, 1 \leq i \leq \alpha - 1\} \cup \{a_i b_i \,/\, 1 \leq i \leq \alpha\} \cup \{a_i c_i \,/\, 1 \leq i \leq \alpha\} \cup \{a_1 a_5,\ a_\alpha a_2\}$$

Define a function $f: V(G) \longrightarrow \{1,2,\ldots,3\alpha+2\}$

$f(a_i) = 3i - 1$ for $1 \leq i \leq 5$

$f(a_i) = 3i + 1$ for $6 \leq i \leq \alpha$

$f(b_i) = 3i - 2$ for $1 \leq i \leq 5$

$f(b_i) = 3i$ for $6 \leq i \leq \alpha$

$f(c_i) = 3i$ for $1 \leq i \leq 5$

$f(c_i) = 3i + 2$ for $6 \leq i \leq \alpha$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.3.1:

The root square mean labeling of $\theta_7 \odot \overline{K_2}$ is given below:

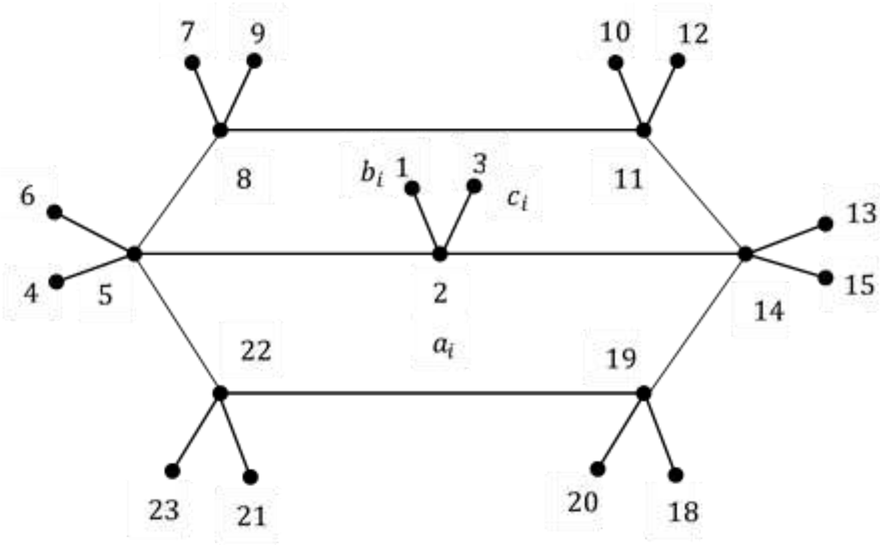

Figure 3

Theorem 2.4:

$\theta_\alpha \odot \overline{K_3}$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \odot \overline{K_3}$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta θ_α in G.

Let $b_1 b_2 \dots b_\alpha$, $c_1 c_2 \dots c_\alpha$ and $d_1 d_2 \dots d_\alpha$ be the pendant vertices attached at $a_1 a_2 \dots a_\alpha$ respectively.

Let $V(G) = \{a_1 a_2 \dots a_\alpha,\ b_1 b_2 \dots b_\alpha,\ c_1 c_2 \dots c_\alpha,\ d_1 d_2 \dots d_\alpha\}$

$$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{a_i b_i \,/\, 1 \le i \le \alpha\} \cup \{a_i c_i \,/\, 1 \le i \le \alpha\} \cup \{a_i d_i \,/\, 1 \le i \le \alpha\} \cup \{a_1 a_5,\ a_\alpha a_2\}$$

Define a function $f: V(G) \longrightarrow \{1,2,\dots,4\alpha + 2\}$

$$f(a_i) = 4i - 2 \quad \text{for} \quad 1 \le i \le 5$$
$$f(a_i) = 4i \quad \text{for} \quad 6 \le i \le \alpha$$

$$f(b_i) = 4i - 3 \quad \text{for} \quad 1 \le i \le 5$$
$$f(b_i) = 4i - 1 \quad \text{for} \quad 6 \le i \le \alpha$$

$$f(c_i) = 4i - 1 \quad \text{for} \quad 1 \le i \le 5$$
$$f(c_i) = 4i + 1 \quad \text{for} \quad 6 \le i \le \alpha$$

$$f(d_i) = 4i \quad \text{for} \quad 1 \le i \le 5$$
$$f(d_i) = 4i + 2 \quad \text{for} \quad 6 \le i \le \alpha$$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.4.1:

The root square mean labeling of $\theta_7 \odot \overline{K_3}$ is given below:

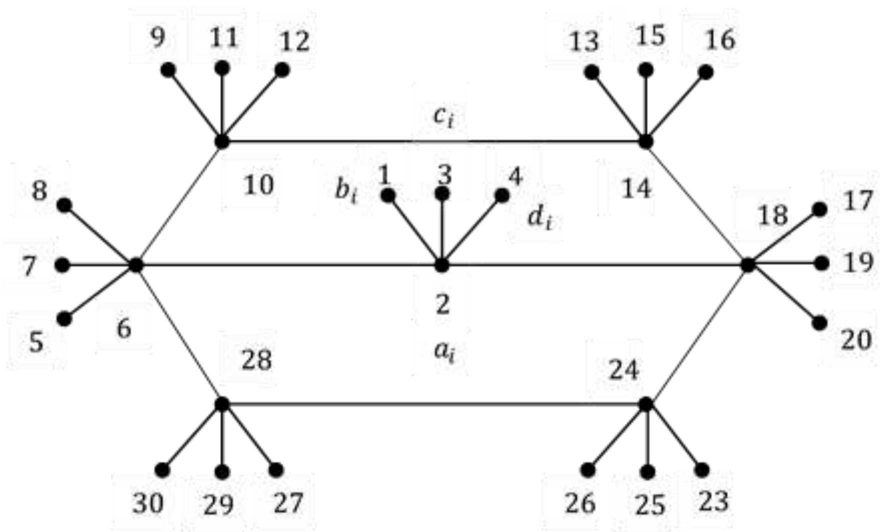

Figure 4

Theorem 2.5:

$\theta_\alpha \odot K_2$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \odot K_2$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of theta in G.

Let $b_1 b_2 \dots b_\alpha$ and $c_1 c_2 \dots c_\alpha$ be the pendant vertices attached at $a_1 a_2 \dots a_\alpha$ respectively.

Let $V(G) = \{a_1 a_2 \dots a_\alpha, \ b_1 b_2 \dots b_\alpha, \ c_1 c_2 \dots c_\alpha\}$

$$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{a_i b_i \,/\, 1 \le i \le \alpha\} \cup \{a_i c_i \,/\, 1 \le i \le \alpha\}$$
$$\cup \{b_i c_i \,/\, 1 \le i \le \alpha\} \cup \{a_1 a_5, \ a_\alpha a_2\}$$

Define a function $f: V(G) \longrightarrow \{1,2,\dots,4\alpha + 2\}$

$f(a_i) = 4i - 2$ for $1 \le i \le 5$
$f(a_i) = 4i$ for $6 \le i \le \alpha$

$f(b_i) = 4i - 3$ for $1 \le i \le 5$
$f(b_i) = 4i - 1$ for $6 \le i \le \alpha$

$f(c_i) = 4i$ for $1 \le i \le 5$
$f(c_i) = 4i + 2$ for $6 \le i \le \alpha$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.5.1:

The root square mean labeling of $\theta_7 \odot K_2$ is given below:

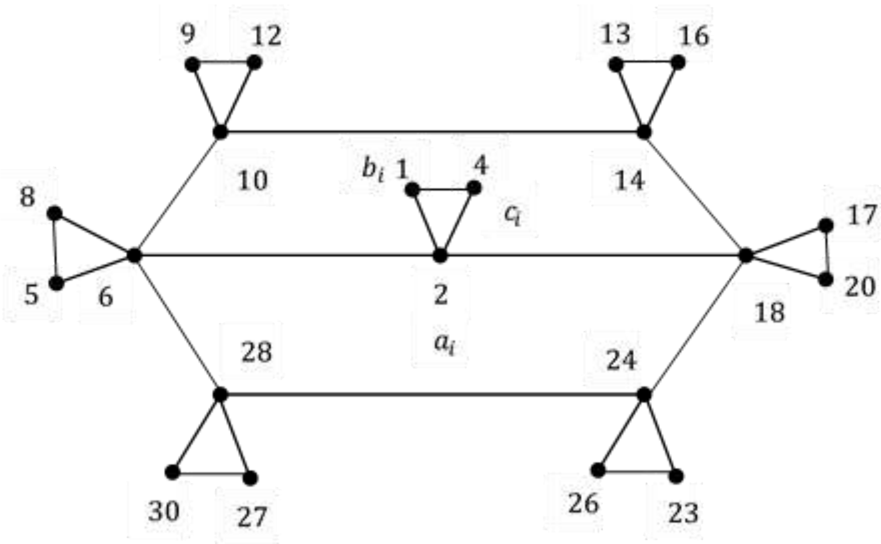

Figure 5

Theorem 2.6:

$\theta_\alpha \cup C_\beta$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup C_\beta$

Let $a_1 a_2 \ldots a_\alpha$ be the vertices of the theta in G.

Let $b_1 b_2 \ldots b_\beta$ be the vertices of cycle in G.

Let $V(G) = \{a_1 a_2 \ldots a_\alpha, b_1 b_2 \ldots b_\beta\}$

$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{b_i b_{i+1} \,/\, 1 \le i \le \beta - 1\}$

$\cup \{a_1 a_5,\ a_2 a_\alpha,\ b_\beta b_1\}$

Define a function $f: V(G) \longrightarrow \{1,2,\ldots,\alpha+\beta+1\}$

$$f(a_i) = i \quad \text{for} \quad 1 \le i \le 5$$
$$f(a_i) = i+1 \quad \text{for} \quad 6 \le i \le \alpha$$
$$f(b_i) = \alpha+i+1 \quad \text{for} \quad 1 \le i \le \beta$$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.6.1:

The root square mean labeling of $(\theta_7 \cup C_5)$ is given below:

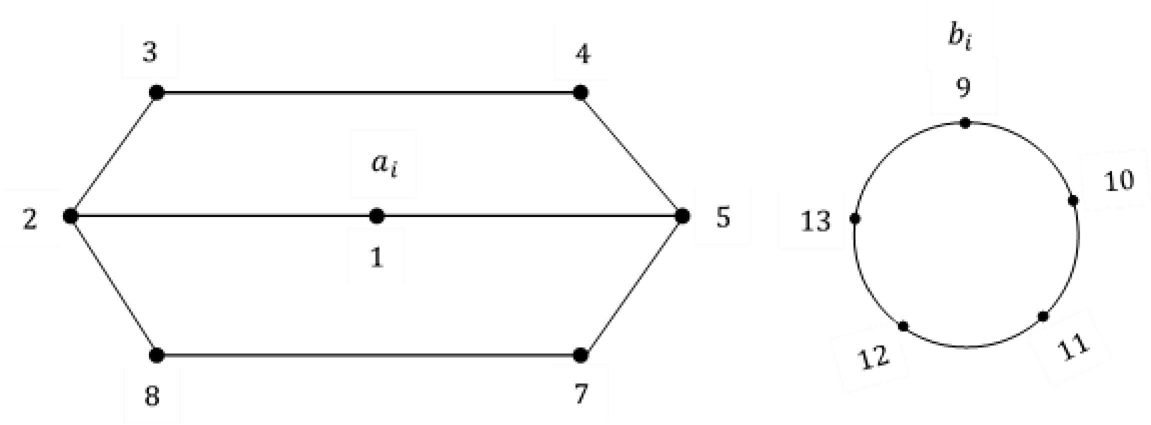

Figure 6

Theorem 2.7:

$\theta_\alpha \cup \left(C_\beta \odot \overline{K_1}\right)$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup \left(C_\beta \odot \overline{K_1}\right)$

Let $a_1a_2 \ldots a_\alpha$ be the vertices of the theta in G.

Let $b_1b_2 \ldots b_\beta$ be the vertices of cycle in G and let $c_1c_2 \ldots c_\beta$ be the pendant vertices attached at $b_1b_2 \ldots b_\beta$ respectively.

Let $V(G) = \left\{a_1a_2 \ldots a_\alpha, b_1b_2 \ldots b_\beta,\ c_1c_2 \ldots c_\beta\right\}$

$$E(G) = \{a_ia_{i+1} \,/\, 1 \le i \le \alpha-1\} \cup \{b_ib_{i+1} \,/\, 1 \le i \le \beta-1\}$$
$$\cup \{b_ic_i \,/\, 1 \le i \le \beta\} \cup \left\{a_1a_5,\ a_2a_\alpha,\ b_\beta b_1\right\}$$

Define a function $f: V(G) \longrightarrow \{1,2,\ldots,\alpha+2\beta+1\}$

$f(a_i) = i$ for $1 \leq i \leq 5$

$f(a_i) = i + 1$ for $6 \leq i \leq \alpha$

$f(b_i) = \alpha + 2i + 1$ for $1 \leq i \leq \beta$

$f(c_i) = \alpha + 2i$ for $1 \leq i \leq \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.7.1:

The root square mean labeling of $\theta_7 \cup \left(C_5 \odot \overline{K_1}\right)$ is given below:

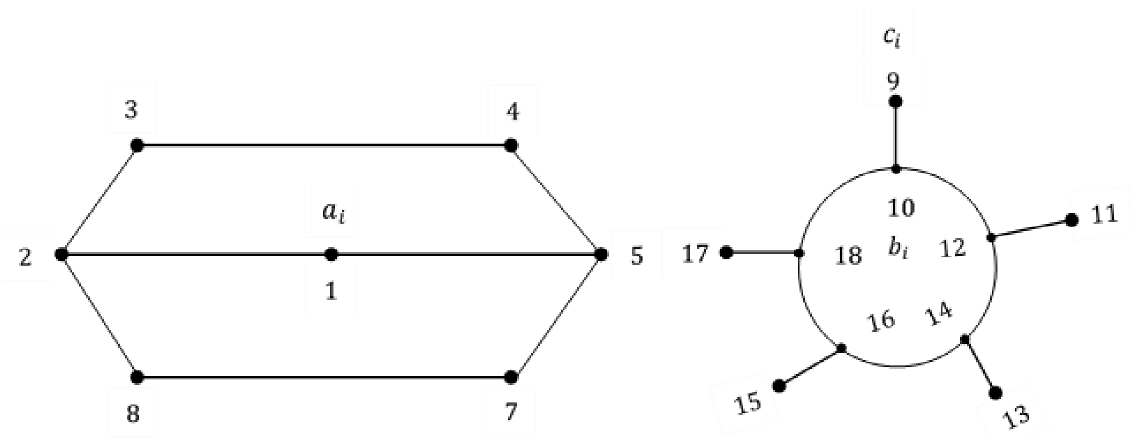

Figure 7

Theorem 2.8:

$\theta_\alpha \cup \left(C_\beta \odot \overline{K_2}\right)$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup \left(C_\beta \odot \overline{K_2}\right)$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta in G.

Let $b_1 b_2 \dots b_\beta$ be the vertices of cycle in G and let $c_1 c_2 \dots c_\beta$ and $d_1 d_2 \dots d_\beta$ be the pendant vertices attached at $b_1 b_2 \dots b_\beta$ respectively.

Let $V(G) = \{a_1 a_2 \dots a_\alpha, b_1 b_2 \dots b_\beta, c_1 c_2 \dots c_\beta, d_1 d_2 \dots d_\beta\}$

$E(G) = \{a_i a_{i+1} / 1 \leq i \leq \alpha - 1\} \cup \{b_i b_{i+1} / 1 \leq i \leq \beta - 1\}$

$\cup \{b_i c_i / 1 \leq i \leq \beta\} \cup \{b_i d_i / 1 \leq i \leq \beta\} \cup \{a_1 a_5, a_2 a_\alpha, b_\beta b_1\}$

Define a function $f: V(G) \longrightarrow \{1,2,\dots,\alpha + 3\beta + 1\}$

$f(a_i) = i$ for $1 \leq i \leq 5$

$f(a_i) = i + 1$ for $6 \leq i \leq \alpha$

$f(b_i) = \alpha + 3i$ for $1 \leq i \leq \beta$

$f(c_i) = \alpha + 3i - 1$ for $1 \leq i \leq \beta$

$f(d_i) = \alpha + 3i + 1$ for $1 \leq i \leq \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.8.1:

The root square mean labeling of $\theta_7 \cup \left(C_4 \odot \overline{K_2}\right)$ is given below:

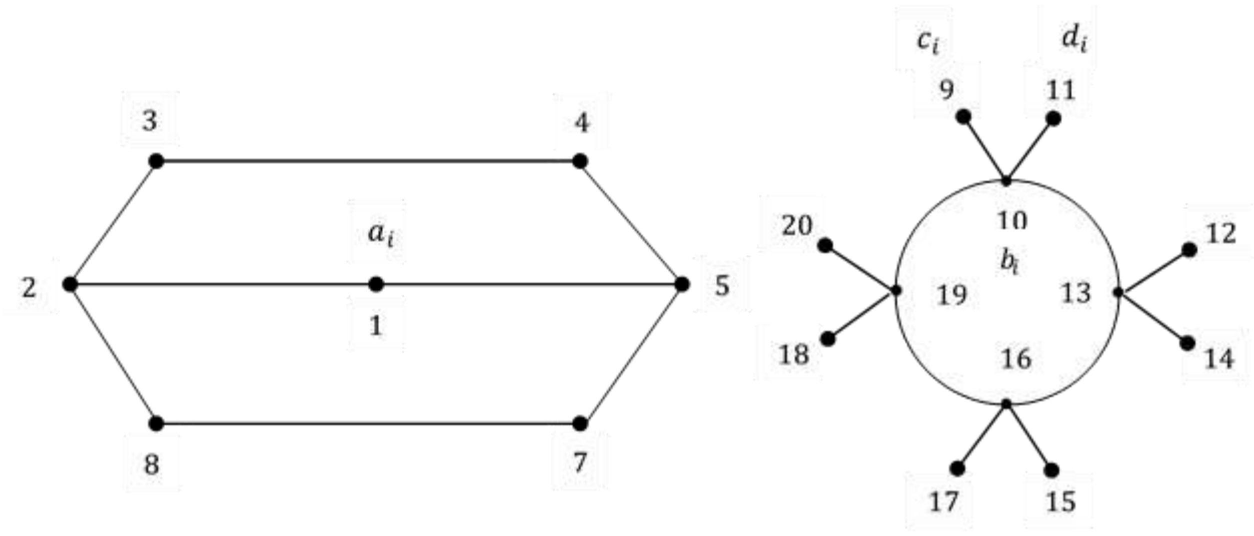

Figure 8

Theorem 2.9:

$\theta_\alpha \cup \left(C_\beta \odot \overline{K_3}\right)$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup \left(C_\beta \odot \overline{K_3}\right)$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta in G.

Let $b_1 b_2 \dots b_\beta$ be the vertices of cycle in G and let $c_1 c_2 \dots c_\beta, d_1 d_2 \dots d_\beta$ and $d_1' d_2' \dots d_\beta'$ be the pendant vertices attached at $b_1 b_2 \dots b_\beta$ respectively.

Let $V(G) = \{a_0 a_1 a_2 \dots a_6, b_1 b_2 \dots b_\beta, c_1 c_2 \dots c_\beta, d_1 d_2 \dots d_\beta, d_1' d_2' \dots d_\beta'\}$

$E(G) = \{a_i a_{i+1} \,/\, 1 \leq i \leq \alpha - 1\} \cup \{b_i b_{i+1} \,/\, 1 \leq i \leq \beta - 1\}$

$\cup \{b_i c_i \ / \ 1 \le i \le \beta\} \cup \{b_i d_i \ / \ 1 \le i \le \beta\} \cup \{b_i d_i' \ / \ 1 \le i \le \beta\}$

$\cup \{a_1 a_5,\ a_2 a_\alpha,\ b_\beta b_1\}$

Define a function $f: V(G) \longrightarrow \{1,2,\ldots,\alpha + 4\beta + 1\}$

$f(a_i) = i$ for $1 \le i \le 5$

$f(a_i) = i + 1$ for $6 \le i \le \alpha$

$f(b_i) = \alpha + 4i - 1$ for $1 \le i \le \beta$

$f(c_i) = \alpha + 4i - 2$ for $1 \le i \le \beta$

$f(d_i) = \alpha + 4i$ for $1 \le i \le \beta$

$f(d_i') = \alpha + 4i + 1$ for $1 \le i \le \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.9.1:

The root square mean labeling of $\theta_7 \cup (C_4 \odot \overline{K_3})$ is given below:

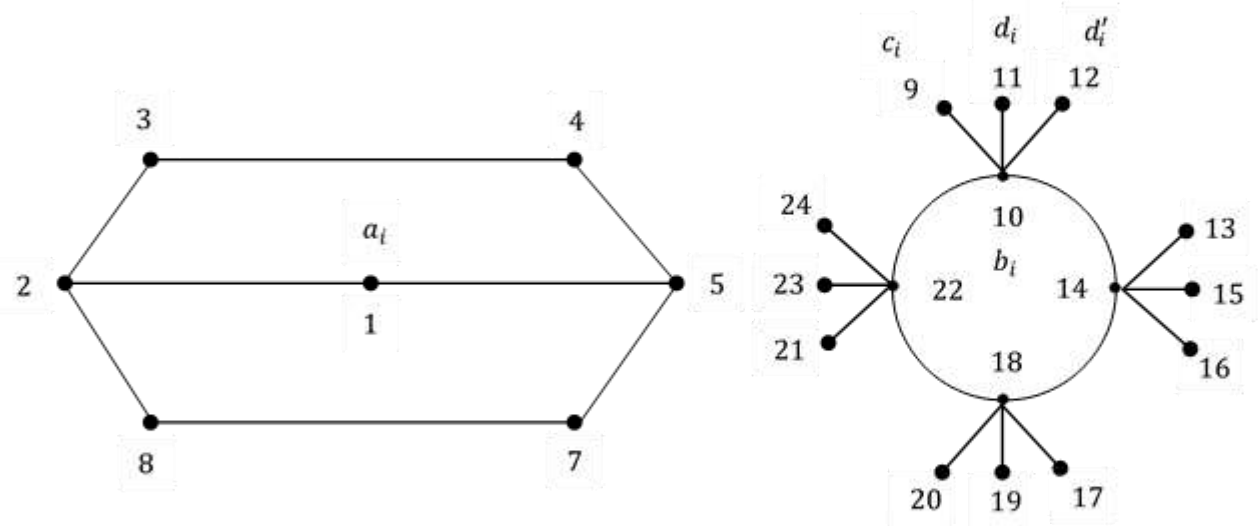

Figure 9

Theorem 2.10:

$\theta_\alpha \cup (C_\beta \odot K_2)$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup (C_\beta \odot K_2)$

Let $a_1 a_2 \ldots a_\alpha$ be the vertices of the theta in G.

Let $b_1b_2 \dots b_\beta$ be the vertices of cycle in G and let $c_1c_2 \dots c_\beta$ and $d_1d_2 \dots d_\beta$ be the pendant vertices attached at $b_1b_2 \dots b_\beta$ respectively.

Let $V(G) = \{a_1a_2 \dots a_\alpha, b_1b_2 \dots b_\beta,\ c_1c_2 \dots c_\beta, d_1d_2 \dots d_\beta\}$

$E(G) = \{a_ia_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{b_ib_{i+1} \,/\, 1 \le i \le \beta - 1\}$

$\cup \{b_ic_i \,/\, 1 \le i \le \beta\} \cup \{b_id_i \,/\, 1 \le i \le \beta\} \cup \{c_id_i \,/\, 1 \le i \le \beta\}$

$\cup \{a_1a_5,\ a_2a_\alpha,\ b_\beta b_1\}$

Define a function $f: V(G) \longrightarrow \{1,2, \dots, \alpha + 4\beta + 1\}$

$f(a_i) = i$ for $1 \le i \le 5$

$f(a_i) = i + 1$ for $6 \le i \le \alpha$

$f(b_i) = \alpha + 4i - 1$ for $1 \le i \le \beta$

$f(c_i) = \alpha + 4i - 2$ for $1 \le i \le \beta$

$f(d_i) = \alpha + 4i + 1$ for $1 \le i \le \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.10.1:

The root square mean labeling of $\theta_7 \cup (C_4 \odot K_2)$ is given below:

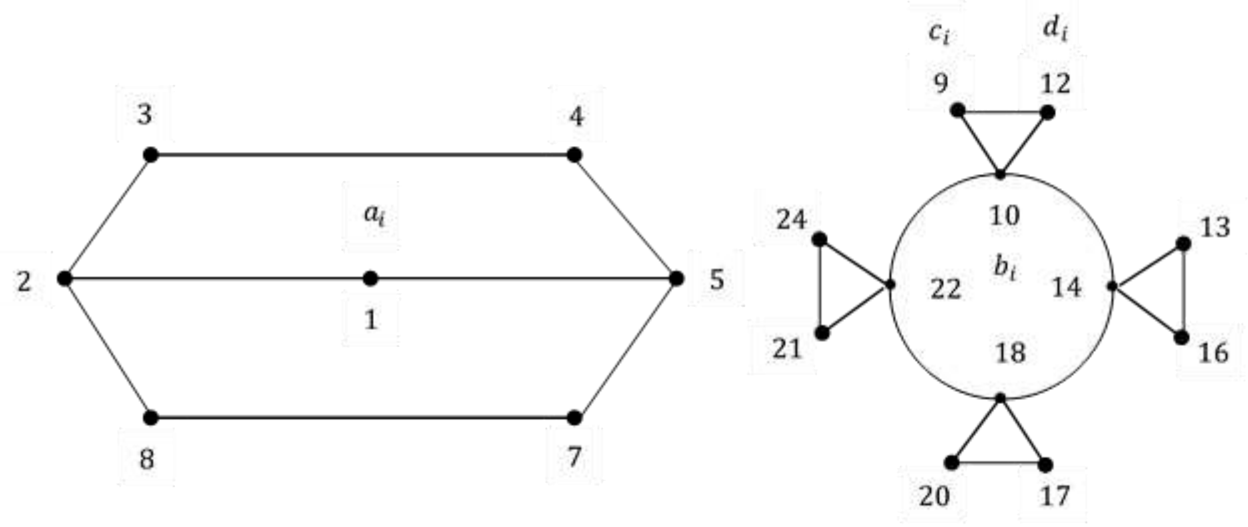

Figure 10

Theorem 2.11:

$\left(\theta_\alpha \odot \overline{K_1}\right) \cup C_\beta$ is a root square mean graph.

Proof:

Let $G = (\theta_\alpha \odot \overline{K_1}) \cup C_\beta$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta in G and let $b_1 b_2 \dots b_\alpha$ be the pendant vertices attached at $a_1 a_2 \dots a_\alpha$ respectively.

Let $c_1 c_2 \dots c_\beta$ be the vertices of cycle in G.

Let $V(G) = \{a_1 a_2 \dots a_\alpha,\ b_1 b_2 \dots b_\alpha,\ c_1 c_2 \dots c_\beta\}$

$$E(G) = \{a_i a_{i+1} \ / \ 1 \leq i \leq \alpha - 1\} \cup \{a_i b_i \ / \ 1 \leq i \leq \alpha\} \cup \{c_i c_{i+1} \ / \ 1 \leq i \leq \beta - 1\}$$

$$\cup \{a_1 a_5,\ a_2 a_\alpha,\ c_\beta c_1\}$$

Define a function $f: V(G) \longrightarrow \{1, 2, \dots, 2\alpha + \beta + 1\}$

$f(a_i) = 2i$ for $1 \leq i \leq 5$
$f(a_i) = 2i + 1$ for $6 \leq i \leq \alpha$

$f(b_i) = 2i - 1$ for $1 \leq i \leq 5$
$f(b_i) = 2i$ for $6 \leq i \leq \alpha$

$f(c_i) = 2\alpha + i + 1$ for $1 \leq i \leq \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.11.1:

The root square mean labeling of $(\theta_7 \odot \overline{K_1}) \cup C_5$ is given below:

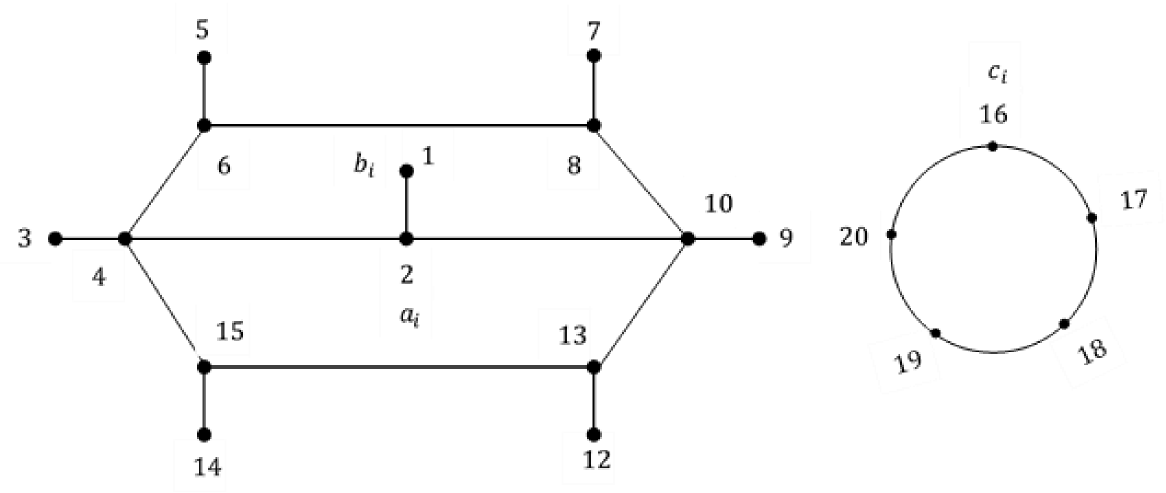

Figure 11

Theorem 2.12:

$(\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot \overline{K_1})$ is a root square mean graph.

Proof:

Let $G = (\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot \overline{K_1})$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta in G and let $b_1 b_2 \dots b_\alpha$ be the pendant vertices attached at $a_1 a_2 \dots a_\alpha$ respectively.

Let $c_1 c_2 \dots c_\beta$ be the vertices of cycle in G and let $d_1 d_2 \dots d_\beta$ be the pendant vertices attached at $c_1 c_2 \dots c_\beta$ respectively.

Let $V(G) = \{a_1 a_2 \dots a_\alpha,\ b_1 b_2 \dots b_\alpha,\ c_1 c_2 \dots c_\beta, d_1 d_2 \dots d_\beta\}$

$$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{c_i c_{i+1} \,/\, 1 \le i \le \beta - 1\} \cup \{a_i b_i \,/\, 1 \le i \le \alpha\}$$

$$\cup \{c_i d_i \,/\, 1 \le i \le \beta\} \cup \{a_1 a_5,\ a_2 a_\alpha,\ c_\beta c_1\}$$

Define a function $f: V(G) \longrightarrow \{1, 2, \dots, 2\alpha + 2\beta + 1\}$

$f(a_i) = 2i$ for $1 \le i \le 5$
$f(a_i) = 2i + 1$ for $6 \le i \le \alpha$

$f(b_i) = 2i - 1$ for $1 \le i \le 5$
$f(b_i) = 2i$ for $6 \le i \le \alpha$

$f(c_i) = 2\alpha + 2i + 1$ for $1 \le i \le \beta$

$f(d_i) = 2\alpha + 2i + 1$ for $1 \le i \le \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.12.1:

The root square mean labeling of $(\theta_7 \odot \overline{K_1}) \cup (C_5 \odot \overline{K_1})$ is given below:

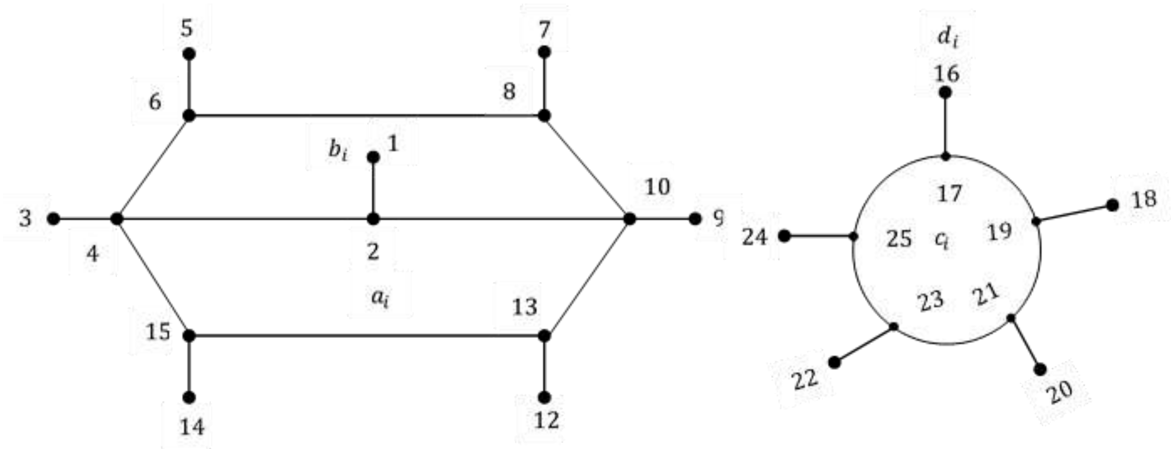

Figure 12

Theorem 2.13:

$\left(\theta_\alpha \odot \overline{K_1}\right) \cup \left(C_\beta \odot \overline{K_2}\right)$ is a root square mean graph.

Proof:

Let $G = \left(\theta_\alpha \odot \overline{K_1}\right) \cup \left(C_\beta \odot \overline{K_2}\right)$

Let $a_1 a_2 \ldots a_\alpha$ be the vertices of the theta in G and let $b_1 b_2 \ldots b_\alpha$ be the pendant vertices attached at $a_1 a_2 \ldots a_\alpha$ respectively.

Let $c_1 c_2 \ldots c_\beta$ be the vertices of cycle C_β in G and let $d_1 d_2 \ldots d_\beta$ and $d_1' d_2' \ldots d_\beta'$ be the pendant vertices attached at $c_1 c_2 \ldots c_\beta$ respectively.

Let $V(G) = \{a_1 a_2 \ldots a_\alpha,\ b_1 b_2 \ldots b_\alpha,\ c_1 c_2 \ldots c_\beta, d_1 d_2 \ldots d_\beta, d_1' d_2' \ldots d_\beta'\}$

$$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{c_i c_{i+1} \,/\, 1 \le i \le \beta - 1\} \cup \{a_i b_i \,/\, 1 \le i \le \alpha\}$$

$$\cup \{c_i d_i \,/\, 1 \le i \le \beta\} \cup \{c_i d_i' \,/\, 1 \le i \le \beta\} \cup \{a_1 a_5,\ a_2 a_\alpha,\ c_\beta c_1\}$$

Define a function $f: V(G) \longrightarrow \{1, 2, \ldots, 2\alpha + 3\beta + 1\}$

$f(a_i) = 2i$ for $1 \le i \le 5$

$f(a_i) = 2i + 1$ for $6 \le i \le \alpha$

$f(b_i) = 2i - 1$ for $1 \le i \le 5$

$f(b_i) = 2i$ for $6 \le i \le \alpha$

$f(c_i) = 2\alpha + 3i$ for $1 \le i \le \beta$

$f(d_i) = 2\alpha + 3i - 1$ for $1 \le i \le \beta$

$f(d_i') = 2\alpha + 3i + 1$ for $1 \le i \le \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.13.1:

The root square mean labeling of $(\theta_7 \odot \overline{K_1}) \cup (C_4 \odot \overline{K_2})$ is given below:

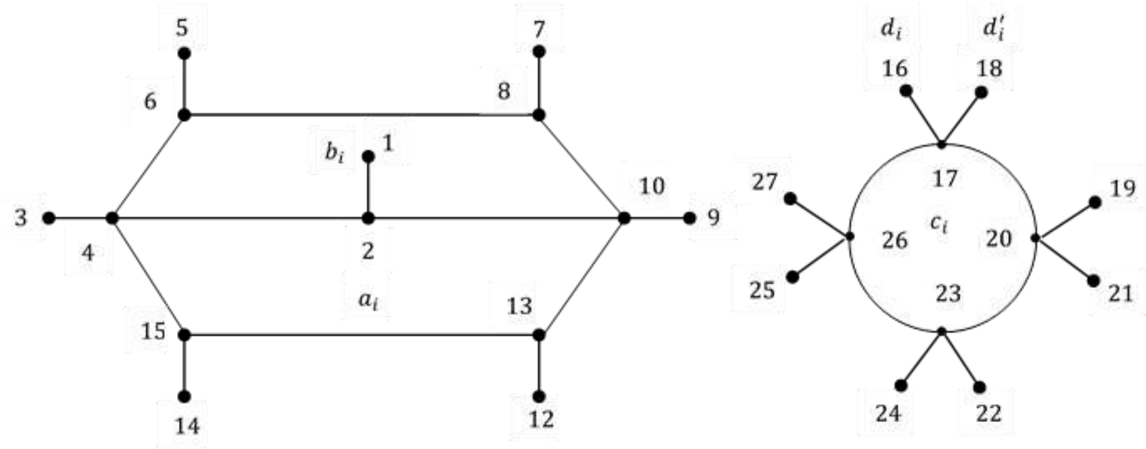

Figure 13

Theorem 2.14:

$(\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot \overline{K_3})$ is a root square mean graph.

Proof:

Let $G = (\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot \overline{K_3})$

Let $a_1a_2 \dots a_\alpha$ be the vertices of the theta in G and let $b_1b_2 \dots b_\alpha$ be the pendant vertices attached at $a_1a_2 \dots a_\alpha$ respectively.

Let $c_1c_2 \dots c_\beta$ be the vertices of cycle C_β in G and let $c_1'c_2' \dots c_\beta'$, $d_1d_2 \dots d_\beta$ and $d_1'd_2' \dots d_\beta'$ be the pendant vertices attached at $c_1c_2 \dots c_\beta$ respectively.

Let $V(G) = \{a_1a_2 \dots a_\alpha, b_1b_2 \dots b_\alpha, c_1c_2 \dots c_\beta, c_1'c_2' \dots c_\beta', d_1d_2 \dots d_\beta, d_1'd_2' \dots d_\beta'\}$

$$E(G) = \{a_ia_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{c_ic_{i+1} \,/\, 1 \le i \le \beta - 1\} \cup \{a_ib_i \,/\, 1 \le i \le \alpha\}$$

$$\cup \{c_id_i \,/\, 1 \le i \le \beta\} \cup \{c_id_i' \,/\, 1 \le i \le \beta\} \cup \{c_ic_i' \,/\, 1 \le i \le \beta\}$$

$$\cup \{a_1a_5,\ a_2a_\alpha,\ c_\beta c_1\}$$

Define a function $f: V(G) \longrightarrow \{1,2,\dots,2\alpha + 4\beta + 1\}$

$f(a_i) = 2i$ for $1 \le i \le 5$
$f(a_i) = 2i + 1$ for $6 \le i \le \alpha$

$f(b_i) = 2i - 1$ for $1 \le i \le 5$
$f(b_i) = 2i$ for $6 \le i \le \alpha$

$f(c_i) = 2\alpha + 4i - 1$ for $1 \le i \le \beta$

$f(c_i') = 2\alpha + 4i - 2$ for $1 \le i \le \beta$

$f(d_i) = 2\alpha + 4i$ for $1 \le i \le \beta$

$f(d_i') = 2\alpha + 4i + 1$ for $1 \le i \le \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.14.1:

The root square mean labeling of $(\theta_7 \odot \overline{K_1}) \cup (C_4 \odot \overline{K_3})$ is given below:

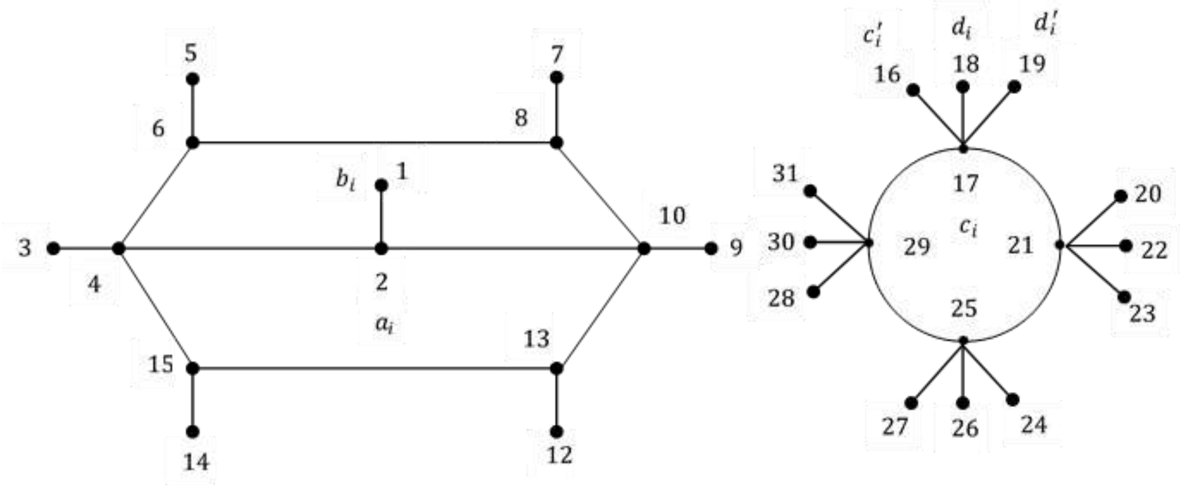

Figure 14

Theorem 2.15:

$(\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot K_2)$ is a root square mean graph.

Proof:

Let $G = (\theta_\alpha \odot \overline{K_1}) \cup (C_\beta \odot K_2)$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta in G and let $b_1 b_2 \dots b_\alpha$ be the pendant vertices attached at $a_1 a_2 \dots a_\alpha$ respectively.

Let $c_1 c_2 \dots c_\beta$ be the vertices of cycle in G and let $d_1 d_2 \dots d_\beta$ and $d_1' d_2' \dots d_\beta'$ be the pendant vertices attached at $c_1 c_2 \dots c_\beta$ respectively.

Let $V(G) = \{a_1 a_2 \dots a_\alpha,\ b_1 b_2 \dots b_\alpha,\ c_1 c_2 \dots c_\beta, d_1 d_2 \dots d_\beta, d_1' d_2' \dots d_\beta'\}$

$E(G) = \{a_i a_{i+1} / 1 \le i \le \alpha - 1\} \cup \{c_i c_{i+1} / 1 \le i \le \beta - 1\} \cup \{a_i b_i / 1 \le i \le \alpha\}$

$\cup \{c_i d_i / 1 \le i \le \beta\} \cup \{c_i d_i' / 1 \le i \le \beta\} \cup \{d_i d_i' / 1 \le i \le \beta\}$

$\cup \{a_1a_5,\ a_2a_\alpha,\ c_\beta c_1\}$.

Define a function $f: V(G) \longrightarrow \{1,2,\dots,2\alpha+4\beta+1\}$

$f(a_i) = 2i$ for $1 \le i \le 5$

$f(a_i) = 2i + 1$ for $6 \le i \le \alpha$

$f(b_i) = 2i - 1$ for $1 \le i \le 5$

$f(b_i) = 2i$ for $6 \le i \le \alpha$

$f(c_i) = 2\alpha + 4i - 1$ for $1 \le i \le \beta$

$f(d_i) = 2\alpha + 4i - 2$ for $1 \le i \le \beta$

$f(d_i') = 2\alpha + 4i + 1$ for $1 \le i \le \beta$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.15.1:

The root square mean labeling of $\left(\theta_7 \odot \overline{K_1}\right) \cup (C_4 \odot K_2)$ is given below:

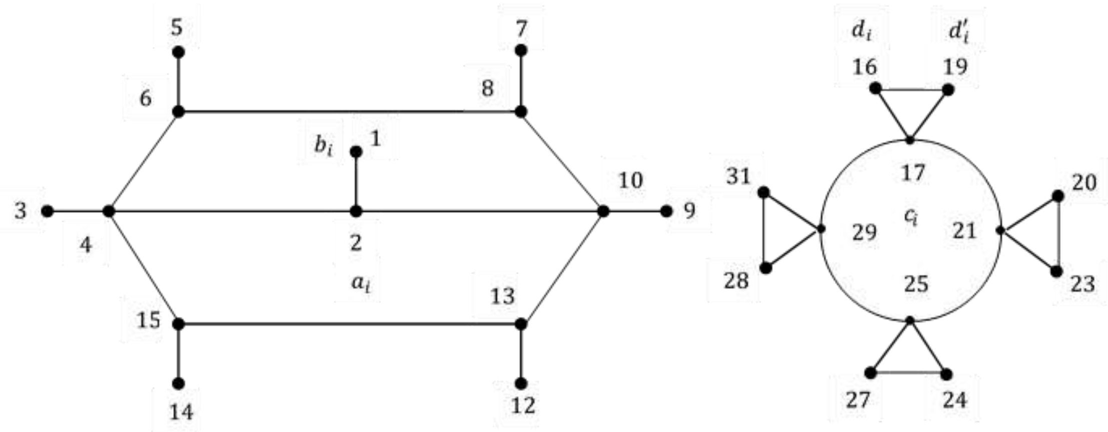

Figure 15

heorem 2.16:

$\theta_\alpha \cup L_\beta$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup L_\beta$

Let $a_1a_2 \dots a_\alpha$ be the vertices of the theta in G.

Let $b_1b_2b_3 \dots b_\beta$ and $c_1c_2c_3 \dots c_\beta$ be the vertices of the ladder in G.

Let $V(G) = \{a_1a_2 \dots a_\alpha,\ \ b_1b_2b_3 \dots b_\beta,\ \ c_1c_2c_3 \dots c_\beta\}$

$$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{b_i b_{i+1} \,/\, 1 \le i \le \beta - 1\}$$

$$\cup \{c_i c_{i+1} \,/\, 1 \le i \le \beta - 1\} \cup \{b_i c_i \,/\, 1 \le i \le \beta\} \cup \{a_1 a_5,\ a_2 a_\alpha\}$$

Define a function $f: V(G) \longrightarrow \{1,2,\dots,\alpha + 3\beta\}$

$f(a_i) = i$ for $1 \le i \le 5$

$f(a_i) = i + 1$ for $6 \le i \le \alpha$

$f(b_i) = \alpha + i + 2$ for $1 \le i \le 2$

$f(b_i) = \alpha + 3i - 2$ for $3 \le i \le \beta$

$f(c_i) = \alpha + 4i - 2$ for $1 \le i \le 2$

$f(c_i) = \alpha + 3i$ for $3 \le i \le \beta$

Then the edge labels are distinct .

Hence f is a root square mean labeling of G.

Example 2.16.1:

The root square mean labeling of $\theta_7 \cup L_5$ is given below:

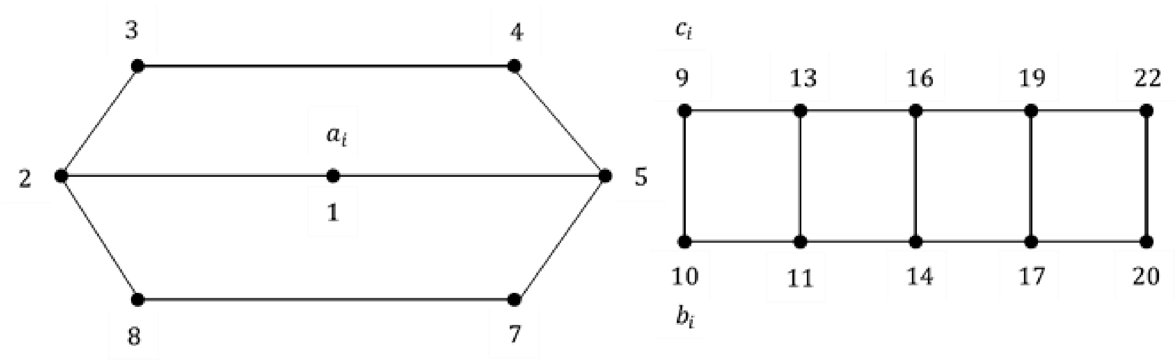

Figure 16

Theorem 2.17:

$\theta_\alpha \cup T_\beta$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup T_\beta$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta in G.

Let $b_1 b_2 b_3 \dots b_\beta$ and $c_1 c_2 c_3 \dots c_\beta$ be the vertices of the ladder in G.

Let $V(G) = \{a_1 a_2 \dots a_\alpha,\ \ b_1 b_2 b_3 \dots b_\beta,\ \ c_1 c_2 c_3 \dots c_\beta\}$

$$E(G) = \{a_i a_{i+1} \;/\; 1 \le i \le \alpha - 1\} \cup \{b_i b_{i+1} \;/\; 1 \le i \le \beta - 1\}$$

$$\cup \{b_i c_i \;/\; 1 \le i \le \beta - 1\} \cup \{c_i b_{i+1} \;/\; 1 \le i \le \beta - 1\} \cup \{a_1 a_5,\ a_2 a_\alpha\}$$

Define a function $f: V(G) \longrightarrow \{1,2,\dots,\alpha + 3\beta - 1\}$

$f(a_i) = i$ for $1 \le i \le 5$

$f(a_i) = i + 1$ for $6 \le i \le \alpha$

$f(b_i) = \alpha + 3i - 1$ for $1 \le i \le \beta$

$f(c_i) = \alpha + 3i$ for $1 \le i \le \beta - 1$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.17.1:

The root square mean labeling of $\theta_7 \cup T_4$ is given below:

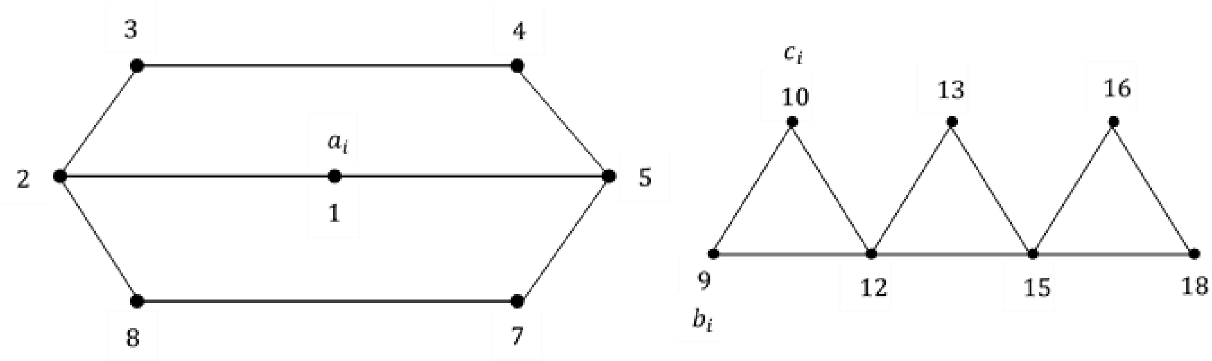

Figure 17

Theorem 2.18:

$\theta_\alpha \cup Q_\beta$ is a root square mean graph.

Proof:

Let $G = \theta_\alpha \cup Q_\beta$

Let $a_1 a_2 \dots a_\alpha$ be the vertices of the theta in G.

Let $b_1 b_2 b_3 \dots b_\beta$ be the path P_β. Let Q_β be the quadrilateral snake obtained from P_β by joining b_i and b_{i+1} to two new vertices c_i and d_i, $1 \le i \le \beta - 1$ respectively.

Let $V(G) = \{a_1 a_2 \dots a_\alpha,\ \ b_1 b_2 b_3 \dots b_\beta,\ \ c_1 c_2 c_3 \dots c_\beta,\ \ d_1 d_2 d_3 \dots d_\beta\}$

$E(G) = \{a_i a_{i+1} \,/\, 1 \le i \le \alpha - 1\} \cup \{b_i b_{i+1} \,/\, 1 \le i \le \beta - 1\} \cup \{b_i c_i \,/\, 1 \le i \le \beta - 1\} \cup \{c_i d_i \,/\, 1 \le i \le \beta - 1\} \cup \{d_i b_{i+1} \,/\, 1 \le i \le \beta - 1\} \cup \{a_1 a_5,\ a_2 a_\alpha\}$

Define a function $f: V(G) \longrightarrow \{1,2,\dots,\alpha + 4\beta - 2\}$

$f(a_i) = i$ for $1 \le i \le 5$

$f(a_i) = i + 1$ for $6 \le i \le \alpha$

$f(b_i) = \alpha + 4i - 2$ for $1 \le i \le \beta$

$f(c_i) = \alpha + 4i - 1$ for $1 \le i \le \beta - 1$

$f(d_i) = \alpha + 4i$ for $1 \le i \le \beta - 1$

Then the edge labels are distinct.

Hence f is a root square mean labeling of G.

Example 2.18.1:

The root square mean labeling of $\theta_7 \cup Q_4$ is given below:

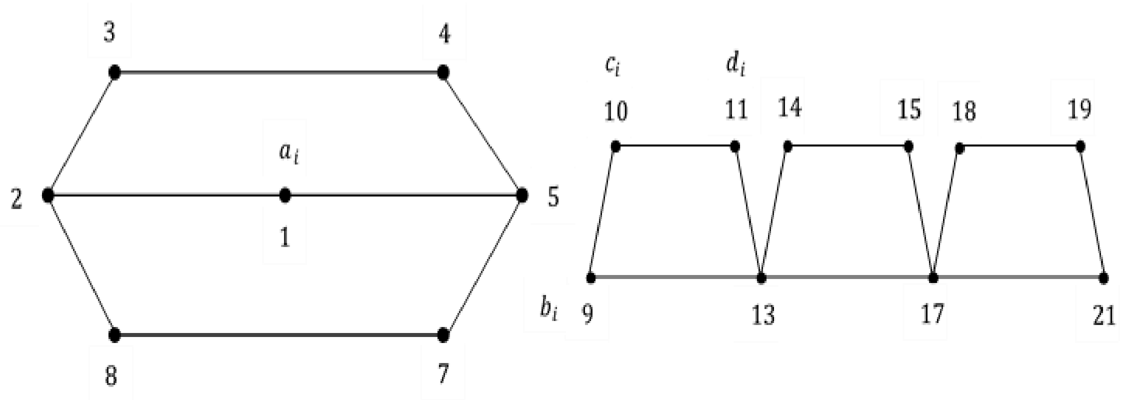

Figure 18

III Conclusion

It is very interesting to find whether a graph admits root square mean labeling or not. We present eighteen new results on root square mean labeling of theta related graphs. The investigation about similar results for various graphs families is an open area of research.

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