Pythagorean Fuzzy Hx Bi – Ideals of Hx- Near Rings

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ABSTRACT
This article introduces the notion of Pythagorean Fuzzy HX bi-ideals in HX near rings and delves into their relevant properties. Moreover, we examine the homomorphic image and pre-image of these bi-ideals and analyze various related theorems. Throughout our analysis, we find that Pythagorean fuzzy HX bi-ideals play a crucial role in the study of HX near rings, and offer valuable insights into their structure and properties. By exploring the various properties and theorems associated with these bi-ideals, we gain a deeper understanding of their significance and potential applications in a wide range of fields. Overall, our research contributes to the ongoing exploration and development of HX near rings, and opens up exciting new avenues for future research in this area. We hope that our findings will inspire further study and investigation, and contribute to the advancement of this important field of mathematics.

Key words: *HX near ring, HX bi – ideal, Pythagorean fuzzy set, Pythagorean fuzzy HX bi – ideal,Supremum property.*

A. INTRODUCTION:

In his seminal paper [6], L.A. Zadeh introduced fuzzy set theory and its relationship to traditional set theory, providing a framework for generalizing fundamental algebraic concepts. Building on this foundation, subsequent researchers have developed a range of related concepts and structures, including Pythagorean fuzzy subsets [2], intuitionistic Q fuzzy bi-ideals in Near-Rings [1], Pythagorean fuzzy ideal semigroups [5], anti-fuzzy HX bi-ideals of HX rings [3], and intuitionistic fuzzy HX bi-ideals of HX rings [4]. This article aims to add to the ongoing research in algebraic structures by presenting a novel idea - the Pythagorean fuzzy HX bi-ideal of a HX near ring. Our analysis includes an investigation of various related properties and characteristics of this structure, providing a deeper understanding of its potential applications and significance in the field. Additionally, we explore the image and pre-image of Pythagorean fuzzy HX bi-ideals, examining their properties and connections to other algebraic concepts. Through our analysis, we find that Pythagorean fuzzy HX bi-ideals offer valuable insights into the structure and properties of HX near rings, and provide a powerful framework for exploring and generalizing a range of fundamental algebraic concepts. Overall, our research contributes to the ongoing exploration and development of algebraic structures, offering new insights and potential applications in a range of fields. We hope that our findings will inspire further study and investigation, and contribute to the ongoing advancement of this important area of mathematics.

B. PRELIMINARIES

DEFINITION B1: [4]

Consider a ring R and a non-empty subset ϑ ⊂ 2R - {φ} equipped with binary operations '+' and '.'. If ϑ forms a ring under the algebraic operations defined by:

(i) a + b = {a_1 + b_1 / a_1∈ a and b_1 ∈ b}, where Q represents the null element, and –a represents the negative element of a.

(ii) ab = { a_1b_1/ a_1 ∈ a and b_1 ∈ b},

(iii) a (b + c) = ab+ ac and (b + c) a = ba + ca

DEFINITION B2: [4]

Suppose we have a ring R and an intuitionistic fuzzy set H defined on R, given by the expression H = {⟨x, μ(x), η(x)⟩ / x∈R}, where μ : R → [0,1] and η : R → [0,1], and 0 ≤ μ(x) + η(x) ≤ 1 for all x in R. Let 𝔑 be a HX ring, a non-empty subset of 2R − {φ} with binary operations '+' and '.' that satisfy the properties mentioned in the previous article. An intuitionistic fuzzy subset λH = {⟨a, λμ(a), λη(a)⟩ / a∈𝔑 and 0 ≤ λμ(a) + λη(a) ≤ 1} of 𝔑 is called an intuitionistic fuzzy HX bi-ideal or intuitionistic fuzzy bi-ideal induced by H of the HX ring 𝔑 if the following conditions are met for all a, b, and c in 𝔑:

(i) λμ(a−b) ≥ min {λμ(a), λμ(b)}

(ii)λμ(ab) ≥ min {λμ(a), λμ(b)}

(iii) λμ(abc) ≥ min {λμ(a), λμ(c)}

(iv) λη(a−b) ≤ max {λη(a), λη(b)}

(v) λη(ab) ≤ max {λη(a), λη(b)}

(vi) λη(abc) ≤ max {λη(a), λη(c)}

where λμ (a) = max{ μ(x) / for all x∈ a⊆ R} and λη (a) = min{η(x) / for all x∈a ⊆ R}.

DEFINITION B3: [5]

Consider a universe of discourse X and a Pythagorean Fuzzy Set (PFS) P defined as P = {z, ϑ(x), ωp(x) / z ∈ X}, where ϑ: X → [0,1] and ω: X → [0,1] represent the degree of membership and non-membership of the object z ∈ X to the set P, subject to the condition 0 ≤ (ϑp(z)) + $(\omega p(z))^2$ ≤ 1 for all z in X. For simplicity, a PFS can be denoted as P = (ϑ(z), ωp(z)).

C.MAIN RESULTS:

DEFINITION: C1

Let N be a Near Ring, and let $\vartheta \subset 2^N - \{\emptyset\}$ be a non-empty subset of $2^N - \{\emptyset\}$. We say that ϑ is an HX-Near Ring on N if it satisfies the following conditions:

(i)(N,+) is a group (may or may not be Abelian)

(ii) (N, ·) be a semi group

(iii)W+I ={w +i/ w∈ W and i ∈ I} assuming that 'w' is a variable representing an element in the ring R: Let Q denote the null element of R, and let -w denote the negative element of the element w in R.

(iv)WI={wi/ w∈ W and i ∈ I}

(v)w(i + k)= wi+ wk and(i + k)w = iw+ik

DEFINITION: C2

Let X be a universe of discourse, A Pythagorean fuzzy set P={k, $\varrho_p(k), \zeta_p(k)/\ k \in$ w}. Where , ϱ_p: $X \longrightarrow$ [0,1] and ζ_p: $X \longrightarrow$ [0,1] represent the membership and non - membership of the object k∈ w to the subset P such that 0 ≤ w is denoted as P = (ϱ_p(w), ζ_p(w)).

DEFINITION: C3

Let N be a HX – Near ring. Let E = $\{(w, \varrho(w), \zeta(w)/w \in N)\}$ be a Pythagorean Fuzzy Set defined on a HX- Near ring N, where ϱ_p: $X \longrightarrow$ [0,1] and ζ_p: $X \longrightarrow$ [0,1] such that 0 $\leq\left(\varrho_p(w)\right)^2 + \left(\zeta_p(w)\right)^2 \leq 1$ let $\vartheta \subset 2^N - \{\emptyset\}$ be a HX- Near ring in. An Pythagorean fuzzy subset γ_{P_E}=$\left\{\left(\text{w}, \varrho_{P_E}(w), \zeta_{P_E}(w)\right)\right)/\text{w} \in \text{N}\}$andc0 $\leq\left(\varrho_{P_E}(w)\right)^2 + \left(\ \zeta_{P_E}(w)\right)^2 \leq 1$ of N is said that a Pythagorean Fuzzy HX-Bi – ideal or Pythagorean Fuzzy Bi – ideal induced by E of a HX- Near Ring N if Suppose the following conditions hold true: $\forall\ w, i, k \in N$

(PYI) $\varrho_{P_E}\,(w - i) \succcurlyeq min\{\ \varrho_{P_E}(\,(w), \varrho_{P_E}(\,(i)\}$

(PYII) $\varrho_{P_E}(\,(wik) \succcurlyeq min\{\ \varrho_{P_E}\,(w), \varrho_{P_E}(\,(i), \varrho_{P_E}\,(k)\}$

(PYIII) $\zeta_{P_E}\,(w - i) \preccurlyeq max\{\zeta_{P_E}\,(w), \zeta_{P_E}\,(i)\}$

(PYIV)$\zeta_{P_E}\,(wik) \preccurlyeq max\{\zeta_{P_E}\,(w), \zeta_{P_E}\,(i), \zeta_{P_E}\,(k)\}$

where $\varrho_{P_E}(w)$ = sup$\{\ \varrho_p\,(w)/\forall w \in A \subseteq N\}$ and $\zeta_{P_E}\,(w)$=$inf\{\ \zeta_p\,(w)/\forall w \in A \subseteq N\}$ for $'\succcurlyeq'$ means that $'\geq'$ and $'\preccurlyeq'$ means that $'\leq'$

EXAMPLE: C4

Let $N = \{a, b, c, d\}$ be set with two binary operations as follows: Then $(N, +,\cdot)$ is HX- Near Ring. We define

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

·	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	b
d	a	a	a	b

$\varrho_{P_E}(a) = sup\ \{\ \varrho_p\,(w)/w \in a \subseteq N\} = 0.8$, $\varrho_{P_E}(b) = sup\{\varrho_p\,(w)/w \in b \subseteq N\} = 0.7$

$\varrho_{P_E}(c) = sup\ \{\varrho_p(w)/w \in c \subseteq N\} = 0.5$, $\varrho_{P_E}(d) = sup\ \{\varrho_p\,(w)/w \in d \subseteq N\} = 0.5$

$\zeta_{P_E}\,(a) = inf\ \{\ \zeta_p\,(w)/w \in a \subseteq N\} = 0.1$, $\zeta_{P_E}(b) = inf\{\zeta_p\,(w)/w \in b \subseteq N\} = 0.2$

$\zeta_{P_E}(c) = inf\ \{\zeta_p\ (w)/w \in c \subseteq N\} = 0.4,\ \zeta_{P_E}(d) = in\{\zeta_p\ (w)/w \in d \subseteq N\} = 0.4$

Clearly N is a HX bi – ideal of a HX Near Ring.

THEOREM: C5

Suppose ɢ and ʜ are Pythagorean fuzzy sets on a HX Near ring N. Let $\varrho_{P_ɢ}$ and $\zeta_{P_ʜ}$ be Pythagorean Fuzzy HX-Bi-ideals in N. Then, their union, denoted by $\varrho_{P_ɢ} \cup \zeta_{P_ʜ}$ is also a Pythagorean Fuzzy HX-Bi-ideal in N.

PROOF:

Suppose ɢ ={< w, $\eta_p(w)$, $\gamma_p(w)$ >/$w \in N$} and ʜ={< w, $\varrho_p\ (w)$, $\zeta_p\ (w)$ >/$w \in N$} be any two Pythagorean Fuzzy Sets defined on a HX Near Ring N . Then $\varrho_{P_ɢ}$ = {< w, ϱ_{P_γ} (w), ϱ_{P_δ} (w)/w $\in N$} and $\zeta_{P_ʜ}$ = {< w, ζ_{P_α} (w),ζ_{P_β} (w)/w $\in N$} be any two Pythagorean fuzzy HX bi – ideal of a HXnear ring N. Then $\varrho_{P_ɢ} \cup \zeta_{P_ʜ}$ = {w, ($\varrho_{P_\gamma}\ \zeta_{P_\alpha}$)(w), ($\varrho_{P_\delta} \cap \zeta_{P_\beta}$)(w)/w $\in N$}

Let

(i) $(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})\ (w - i) = max\ \{\ \varrho_{P_\gamma}\ (w - i), \zeta_{P_\alpha}\ (w - i)\}$

$\succcurlyeq\ max\ \{min\{\ \varrho_{P_\gamma}\ (w), \varrho_{P_\gamma}\ (i)\}, min\{\zeta_{P_\alpha}\ (w), \zeta_{P_\alpha}\ (i)\}\}$

$= max\{min\{\ \varrho_{P_\gamma}\ (w), \zeta_{P_\alpha}\ (w)\}, min\{\ \ \varrho_{P_\gamma}\ (i), \zeta_{P_\alpha}\ (i)\}\}$

$=min\{max\{\{\ \varrho_{P_\gamma}(w), \zeta_{P_\alpha}(w)\},max\{\ \varrho_{P_\gamma}(i), \zeta_{P_\alpha}(i)\}\}\}$

Therefore$(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})\ (w - i) \succcurlyeq min\{(\ \varrho_{P_\gamma} \cup \zeta_{P_\alpha})(w), (\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(i)\}$

(ii) $(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})\ (wik) = max\ \{\ \varrho_{P_\gamma}\ (wik), \zeta_{P_\alpha}\ (wik)\}$

$\succcurlyeq\ max\ \{min\{\ \varrho_{P_\gamma}\ (w), \varrho_{P_\gamma}\ (i), \varrho_{P_\gamma}\ (k)\}, min\{\zeta_{P_\alpha}\ (w), \lambda_p^{\alpha}\ (i), \zeta_{P_\alpha}(k)\}\}$

$=max\{min\{\ \varrho_{P_\gamma}\ (w), \zeta_{P_\alpha}\ (w)\}, min\{\ \ \varrho_{P_\gamma}\ (i), \zeta_{P_\alpha}\ (i)\}, min\{\ \varrho_{P_\gamma}\ (k), \zeta_{P_\alpha}\ (k)\}\}$

$=\ min\ \{max\ \{\{\ \varrho_{P_\gamma}\ (w), \zeta_{P_\alpha}(w)\},\ max\{\ \varrho_{P_\gamma}\ (i),\ \zeta_{P_\alpha}\ (i)\},\ max\{\ \varrho_{P_\gamma}\ (k),\ \zeta_{P_\alpha}\ (k)\}\}\}$

Therefore $(\varrho_{P_\gamma} \cup \zeta_{P_\alpha})\ (wik) \succcurlyeq min\{(\ \varrho_{P_\gamma} \cup \zeta_{P_\alpha})(w), (\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(i), (\varrho_{P_\gamma} \cup \zeta_{P_\alpha})(k)\}$

(iii) $(\varrho_{P_\delta} \cap \zeta_{P_\beta})(w - i) =\ min\{\varrho_{P_\delta}\ (w - i), \zeta_{P_\beta}(w - i)\ \}$

$\preccurlyeq\ min\ \{\ max\ \{\varrho_{P_\delta}\ (w), \varrho_{P_\delta}\ (i)\}, max\{\zeta_{P_\beta}\ (w), \zeta_{P_\beta}\ (i)\}\}$

$= min\{max\{\varrho_{P_\delta}\ (w), \zeta_{P_\beta}\ (w)\}, max\{\varrho_{P_\delta}\ (i), \zeta_{P_\beta}\ (i)\}\}$ $\preccurlyeq$ $min\{max\{\varrho_{P_\delta}\ (w),\ \zeta_{P_\beta}\ (w)\},\ max\{\varrho_{P_\delta}\ (i), \zeta_{P_\beta}\ (i)\} \preccurlyeq max\{min\{\varrho_{P_\delta}\ (w),\ \zeta_{P_\beta}\ (w)\},\ min\{\varrho_{P_\delta}\ (i), \zeta_{P_\beta}\ (i)\}\}$

Therefore $(\ \varrho_{P_\delta} \cap \zeta_{P_\beta}\ )\ (w - i) \preccurlyeq max\{((\varrho_{P_\delta} \cap \zeta_{P_\beta}\ ))(w),\ (\varrho_{P_\delta} \cap \zeta_{P_\beta}))(i)\}$

(iv)$(\varrho_{P_\delta} \cap \zeta_{P_\beta})(wik) =\ min\{\varrho_{P_\delta}\ (wik), \zeta_{P_\beta}(wik)\ \}$

$\preccurlyeq\ min\ \{max\{\varrho_{P_\delta}\ (w), \varrho_{P_\delta}\ (i), \varrho_{P_\delta}\ (k)\}, max\{\zeta_{P_\beta}\ (w), \zeta_{P_\beta}\ (i), \zeta_{P_\beta}\ (k)\}\}$

$= max\{min\{\varrho_{P_\delta}\ (w),\ \zeta_{P_\beta}\ (w)\},\ min\{\varrho_{P_\delta}\ (i), \zeta_{P_\beta}\ (i)\},\ min\{\varrho_{P_\delta}\ (k), \zeta_{P_\beta}\ (k)\}\}$

Therefore

$(\varrho_{P_\delta} \cap \zeta_{P_\beta})\ (wik) \preccurlyeq max\{(\varrho_{P_\delta} \cap \zeta_{P_\beta})(w), (\varrho_{P_\delta} \cap \zeta_{P_\beta})(i), (\varrho_{P_\delta} \cap \zeta_{P_\beta})(k)\}$

Hence

$\varrho_{P_{G}} \cup \zeta_{P_{H}}$ is a Pythagorean fuzzy HX- bi – ideal of a HX Near ring N.

THEOREM: C6

Let N_1 and N_2 be any two HX near rings on the HX Near Ring N_1 and N_2 respectively. Let f: $N_1 \rightarrow N_2$ be an homomorphism onto HX Near Rings. Let ҥ be a Pythagorean Fuzzy Subset of N_1. Let $\zeta_{P_{H}}$ be a Pythagorean Fuzzy HX bi – ideal of N_1. Then f $(\zeta_{P_{H}})$ be a Pythagorean fuzzy HXbi – ideal of N_2, if $\zeta_{P_{H}}$ has a supremum property and $\zeta_{P_{H}}$ is a f-invariant.

PROOF:

Let ҥ={< w, ϱ_p (w), ζ_P (w) >/$w \in N$} be a Pythagorean fuzzy sets defined on a HX near ring $N1$. Then $\zeta_{P_{H}}$={w, $\zeta_{P_{\alpha}}$ (w), $\zeta_{P_{\beta}}$ (w)/$w \in N1$} be Pythagorean fuzzy HX bi – ideal of a HX near ring $N1$. Then $f(\zeta_{P_{H}})$ = {$f(w)$, $f(\zeta_{P_{\alpha}})(f(w))$, $f(\zeta_{P_{\beta}})(f(w))$/$w \in N1$} there exist $w, i, k \in N_1$ such that $f(w)$, $f(i)$, $f(k) \in N_2$

(i) $(f\ (\zeta_{P_{\alpha}}))\ (f\ (w) - f\ (i)) = (f\ (\zeta_{P_{\alpha}}))\ ((w-i)) = \zeta_{P_{\alpha}}\ (w-i) \geqslant m\{\zeta_{P_{\alpha}}\ (w), \zeta_{P_{\alpha}}(i)\}$

Therefore

$(f(\zeta_{P_{\alpha}}))\ (f(w) - f(i)) \geqslant min\{f(\zeta_{P_{\alpha}})(f(w)),\ f(\zeta_{P_{\alpha}})(f(i))\}$

(ii) $(f\ (\zeta_{P_{\alpha}}))\ (f\ (w)\ f\ (i)\ (k)) = (f\ (\zeta_{P_{\alpha}}))\ ((wik)) = \zeta_{P_{\alpha}}\ (wik) \geqslant min\{\zeta_{P_{\alpha}}(w), \zeta_{P_{\alpha}}(i), \zeta_{P_{\alpha}}(k)\}$

Therefore

$(f(\zeta_{P_{\alpha}}))\ (f(w)f(i)\ f(k)) \geqslant min\{f(\zeta_{P_{\alpha}})(f(w)),\ f(\zeta_{P_{\alpha}})(f(i)), f(\zeta_{P_{\alpha}})(f(k))\}$

(iii) $(f(\zeta_{P_{\beta}}))\ (f(w) - f(i)) = (f\ (\zeta_{P_{\beta}}))\ (f(w-i)) = \zeta_{P_{\beta}}\ (w-i) \leqslant max\{\ \zeta_{P_{\beta}}\ (w), \zeta_{P_{\beta}}$ $(i)\}$

Therefore

$(f\ (\zeta_{P_{\alpha}}))\ (f\ (w) - f\ (i)) \leqslant max\ \{(\zeta_{P_{\alpha}})(f(w)),\ f(\zeta_{P_{\alpha}})(f(i))\}$

(iv) $(f\ (\zeta_{P_{\beta}}))\ (f\ (w)\ f\ (i)\ f\ (k)) = (f\ (\zeta_{P_{\beta}}))\ (f\ (wik)) = \zeta_{P_{\beta}}\ (wik) \leqslant max\{\ \zeta_{P_{\beta}}\ (w), \zeta_{P_{\beta}}$ $(i), \zeta_{P_{\beta}}\ (k)\}$

Therefore

$(f(\zeta_{P_{\beta}}))\ (f(w)f(i)\ f(k)) \leqslant max\ \{f(\zeta_{P_{\beta}})(f(w)),\ f(\zeta_{P_{\beta}})(f(i)), f(\zeta_{P_{\beta}})(f(k))\}$

Hence

f $(\zeta_{P_{H}})$ is a Pythagorean Fuzzy HX bi – ideal of N_2.

THEOREM:C7

Let ҥ $=$ <$\varrho_{P_{H}}, \zeta_{P_{H}}$> be a Pythagorean Fuzzy HX bi – ideal of a HX Near Ring N and f: [0,1] $\rightarrow$ $\underset{p}{[0,1]}$ be an increasing function then the Pythagorean fuzzy set $ҥ_P^f = < \varrho_{H}^f, \zeta_{H}^f >$ defined by $\varrho_{P_{H}}^{f(w)} = f\left(\varrho_{P_{H}}(w)\right)$ and $\zeta_{P_{H}}^{f(w)} = f\left(\zeta_{P_{H}}(w)\right)$ is a Pythagorean fuzzy HX bi – ideal of N.

PROOF:

Let ҥ $= < \varrho_{P_{H}}, \zeta_{P_{H}} >$ be a pythagorean fuzzy HX bi – ideal of a HX near ring N for any $w, i, k \in N$

(i) $\varrho^{f}_{P_{Ꞃ}}(w-i) = f\left[\varrho_{P_{Ꞃ}}(w-i)\right] \geqslant f\left[min\left\{\varrho_{P_{Ꞃ}}(w), \varrho_{P_{Ꞃ}}(i)\right\}\right]$

$=min\left\{f\left(\varrho_{P_{Ꞃ}}(w)\right), f\left(\varrho_{P_{Ꞃ}}(i)\right)\right\}$

$= min\left\{\varrho^{f}_{P_{Ꞃ}}(w), \varrho^{f}_{P_{Ꞃ}}(i)\right\}$

Therefore

$\varrho^{f}_{P_{Ꞃ}}(w-i) \geqslant min\left\{\varrho^{f}_{P_{Ꞃ}}(w), \varrho^{f}_{P_{Ꞃ}}(i)\right\}$

(ii) $\varrho^{f}_{P_{Ꞃ}}(wik) = f\left[\varrho_{P_{Ꞃ}}(wik)\right]$

$\geqslant f\left[min\left\{\varrho_{P_{Ꞃ}}(w), \varrho_{P_{Ꞃ}}(i), \varrho_{P_{Ꞃ}}(k)\right\}\right]$

$=min\left\{f\left(\varrho_{P_{Ꞃ}}(w)\right), f\left(\varrho_{P_{Ꞃ}}(i)\right), f\left(\varrho_{P_{Ꞃ}}(k)\right)\right\}$

$= min\{\mu_{p}{}^{Ꞃf}(w), \mu_{p}{}^{Ꞃf}(i), \mu_{p}{}^{Ꞃf}(k)\}$

Therefore

$\varrho^{f}_{P_{Ꞃ}}(w-i) \geqslant min\left\{\varrho^{f}_{P_{Ꞃ}}(w), \varrho^{f}_{P_{Ꞃ}}(i), \varrho^{f}_{P_{Ꞃ}}(k)\right\}$

(iii) $\zeta^{f}_{P_{Ꞃ}}(w-i) = f\left[\zeta_{P_{Ꞃ}}(w-i)\right] \leqslant f\left[max\left\{\zeta_{P_{Ꞃ}}(w), \zeta_{P_{Ꞃ}}(i)\right\}\right]$

$=max\left\{f\left(\zeta_{P_{Ꞃ}}(w)\right), f\left(\zeta_{P_{Ꞃ}}(i)\right)\right\}$

$= max\left\{\zeta^{f}_{P_{Ꞃ}}(w), \zeta^{f}_{P_{Ꞃ}}(i)\right\}$

Therefore

$\zeta^{f}_{P_{Ꞃ}}(w-i) \leqslant max\left\{\zeta^{f}_{P_{Ꞃ}}(w), \zeta^{f}_{P_{Ꞃ}}(i)\right\}$

(iv) $\zeta^{f}_{P_{Ꞃ}}(wik) = f\left[\zeta_{P_{Ꞃ}}(wik)\right]$

$\leqslant f\left[max\left\{\zeta_{P_{Ꞃ}}(w), \zeta_{P_{Ꞃ}}(i),\right\}\zeta_{P_{Ꞃ}}(k)\right]$

$=max\left\{f\left(\zeta_{P_{Ꞃ}}(w)\right), f\left(\zeta_{P_{Ꞃ}}(i)\right), f\left(\zeta_{P_{Ꞃ}}(k)\right)\right\}$

$= max\left\{\zeta^{f}_{P_{Ꞃ}}(w), \zeta^{f}_{P_{Ꞃ}}(i), \zeta^{f}_{P_{Ꞃ}}(k)\right\}$

Therefore

$\zeta^{f}_{P_{Ꞃ}}(w-i) \leqslant max\left\{\zeta^{f}_{P_{Ꞃ}}(w), \zeta^{f}_{P_{Ꞃ}}(i), \zeta^{f}_{P_{Ꞃ}}(k)\right\}$

THEOREM: C8

If a Pythagorean fuzzy set $Ꞃ =< \varrho_{P_{Ꞃ}}, \zeta_{P_{Ꞃ}} >$ satisfies the conditions

(i) $\varrho_{P_{Ꞃ}}(w-i) \geqslant min\left\{\varrho_{P_{Ꞃ}}(w), \varrho_{P_{Ꞃ}}(i)\right\}$

(ii) $\zeta_{P_{Ꞃ}}(w-i) \leqslant max\left\{\zeta_{P_{Ꞃ}}(w), \zeta_{P_{Ꞃ}}(i)\right\}$ then $\varrho_{P_{Ꞃ}}(0) \geqslant \varrho_{P_{Ꞃ}}(w)$ and $\zeta_{P_{Ꞃ}}(0) \geqslant \zeta_{P_{Ꞃ}}(w) \forall w \in N$

PROOF:

Let $w, i \in N$

$\varrho_{P_Ꟛ}(w-i) \geqslant min\left\{\varrho_{P_Ꟛ}(w), \varrho_{P_Ꟛ}(i)\right\}$ $=min\left\{\varrho_{P_Ꟛ}(0), \varrho_{P_Ꟛ}(0)\right\}$ $\therefore \varrho_{P_Ꟛ}(w-i) \geqslant \varrho_{P_Ꟛ}(0)$

$\zeta_{P_Ꟛ}(w-i) \leqslant max\left\{\zeta_{P_Ꟛ}(w), \zeta_{P_Ꟛ}(i)\right\}$ $=max\left\{\zeta_{P_Ꟛ}(0), \zeta_{P_Ꟛ}(0)\right\}$ $\therefore \zeta_{P_Ꟛ}(w-i) \leqslant \zeta_{P_Ꟛ}(0)$

$\therefore \varrho_{P_Ꟛ}(w-i) \geqslant \varrho_{P_Ꟛ}(0)$ and $\zeta_{P_Ꟛ}(w-i) \leqslant \zeta_{P_Ꟛ}(0). \forall w, i \in N$

THEOREM :C9

Let $Ꟛ =< \varrho_{P_Ꟛ}, \zeta_{P_Ꟛ} >$ be a Pythagorean fuzzy HX bi – ideal of N then the sets $N_{Ꟛ_{\varrho P}} = \left\{w \in N / \varrho_{P_Ꟛ} = \varrho_{P_Ꟛ}(0)\right\}$ and $N_{Ꟛ_{\zeta P}} = \left\{w \in N / \zeta_{P_Ꟛ}(w) = \zeta_{P_Ꟛ}(0)\right\}$ are HX bi – ideals of N.

PROOF:

Let $w, i \in N_{Ꟛ_{\varrho P}}$ then $\varrho_{P_Ꟛ}(w) = \varrho_{P_Ꟛ}(0)$ and $\zeta_{P_Ꟛ}(i) = \zeta_{P_Ꟛ}(0)$ since $Ꟛ =< \varrho_{P_Ꟛ}, \zeta_{P_Ꟛ} >$ be a Pythagorean fuzzy HX bi – ideal of N we get $\varrho_{P_Ꟛ}(w-i) \geqslant min\left\{\varrho_{P_Ꟛ}(w), \varrho_{P_Ꟛ}(i)\right\} = \varrho_{P_Ꟛ}(0)$ we get $\varrho_{P_Ꟛ}(w-i) = \varrho_{P_Ꟛ}(0)$ hence $w - i \in N_{Ꟛ_{\varrho P}}$ thus $N_{Ꟛ_{\varrho P}}$ is a subgroup of N

Let $w, i, k \in N_{Ꟛ_{\varrho P}}$ then $\varrho_{P_Ꟛ}(w) = \varrho_{P_Ꟛ}(0)$, $\zeta_{P_Ꟛ}(i) = \zeta_{P_Ꟛ}(0)$ and $\zeta_{P_Ꟛ}(k) = \zeta_{P_Ꟛ}(0)$ Since $Ꟛ =< \varrho_{P_Ꟛ}, \zeta_{P_Ꟛ} >$ be a Pythagorean fuzzy HX bi – ideal of N we get

$\varrho_{P_Ꟛ}(wik) \geqslant min\left\{\varrho_{P_Ꟛ}(w), \varrho_{P_Ꟛ}(i), \varrho_{P_Ꟛ}(k)\right\} = \varrho_{P_Ꟛ}(0)$ $\varrho_{P_Ꟛ}(w-i) = \varrho_{P_Ꟛ}(0)$

Hence $wik \in N_{Ꟛ_{\varrho P}}$ thus $N_{Ꟛ_{\varrho P}}$ is a bi – ideal of N.

$\zeta_{P_Ꟛ}(wik) \leqslant max\left\{\zeta_{P_Ꟛ}(w), \zeta_{P_Ꟛ}(i), \zeta_{P_Ꟛ}(k)\right\} = \zeta_{P_Ꟛ}(0)$ $\zeta_{P_Ꟛ}(w-i) = \zeta_{P_Ꟛ}(0)$

Hence $wik \in N_{Ꟛ_{\zeta P}}$

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