On Micro Continuous Function via $\mu\hat{g}\pi$ -closed Set

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Abstract

Recently we introduced $\mu\hat{g}\pi$ -closed set in Micro topological spaces. The aim of this paper is to introduce a new class of Micro continuous function called $\mu\hat{g}\pi$ -continuous function in Micro topological spaces and also discussedtheir properties.

Keywords: $\mu\hat{g}\pi$ -closed set, $\mu\hat{g}\pi$ - open set, $\mu\hat{g}\pi$ -continuous, $\mu\hat{g}\pi$ – irresolute function

1 Introduction

The concept of rough set theory was studied by Pawlak [7] and he introduced the idea of lower approximation, upper approximation and boundary region of a subset of the universe. Carmel Richard and et al. [6] presented the concept of Nano topology in this year 2013. The Micro topology was introduced by Sakkraiveeranan Chandrasekar [8] and he also studied the concepts of Micro pre open and Micro semi-open sets. Ibrahim [4,5] introduced Micro -open sets and Micro -closed sets in Micro topological spaces. Recently Anandhi and Balamani [1] initiated the concept of Micro -generalized closed sets in Micro topological spaces and also they have studied the properties of Micro separation axioms related to Micro -generalized closed sets in Micro topological spaces.

In this paper we introduce a new class of function called $\mu\hat{g}\pi$ –continuous function and study some of their properties.

2. Preliminaries

In this paper, $(\Omega, \mathcal{N}, \mathcal{M})$ denote the Micro topological spaces, where $= \tau_R(X)$, $\mathcal{M} = \mu_R(X)$ and MTS denote micro and micro topological space respectively. For a subset P of a space , $cl_\mu(P)$ and $int_\mu(P)$ denote the closure of P and the interior of P respectively.

Definition 2.1.[8] Let $(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X)\}$ and $\mu \neq \tau_R(X)$ and $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is Micro topological space and Micro open sets termed as the elements of $\mu_R(X)$ and Micro closed set termed as complement of a Micro open set.

Definition 2.2. Let Micro topological space $(\Omega, \mathcal{N}, \mathcal{M})$. A subset of A is

i. Micro-g-closed, if $cl_\mu(A) \subseteq L$, $A \subseteq L$ and L is Micro- open in U.[5]
ii. Micro-αg-closed, if $\alpha cl_\mu(A) \subseteq L$, $A \subseteq L$ and L is Micro- open in U.[3]
iii. Micro-gα-closed, if $\alpha cl_\mu(A) \subseteq L$, $A \subseteq L$ and L is Micro- α- open in U.[3]
iv. Micro-sg-closed, if $scl_\mu(A) \subseteq L$, $A \subseteq L$ and L is Micro- s- open in U.[2]
v. icro-gs-closed, if $scl_\mu(A) \subseteq L$, $A \subseteq L$ and L is Micro-open in U.[2]
vi. Micro-g^*-closed, if $cl_\mu(A) \subseteq L$, $A \subseteq L$ and L is Micro- g-open in U.[9]

Definition 2.3.[10] Let $(X, \tau_R(A), \mu_R(A))$ and $(Y, \tau'_R(A), \mu'_R(A))$ be two Micro topological spaces. A function $f : X \rightarrow Y$ is called Micro-generalized continuous function if $f^{-1}(B)$ is Micro g-closed set in X for every Micro-closed set B in Y.

3 Micro $\mu\hat{g}\pi$ -continuous

Definition 3.1. Let (Ω, N, M) and (Ψ, N', M') be two MTS's . Then a mapping $f : \Omega \rightarrow \Psi$ is $\mu\hat{g}\pi$ -continuous if the inverse image of every Micro closed set in Ψ is closed in Ω.

Example 3.2. Let $\Omega = \{p_1, p_2, p_3, p_4\}$ with $/R = \{\{p_1\}, \{p_3\}, \{p_2 p_4\}\}$. Let $X = \{p_1, p_2\} \subseteq \Omega$, then $\tau_R(X) = \{U, \varphi, \{p_1\}, \{p_1, p_2, p_4\}, \{p_2, p_4\}\}$. If $= \{p_3\}$, then the micro topology $\mu_R(X) = \{\Omega, \varphi, \{p_1\}, \{p_3\}, \{p_1, p_3\}, \{p_2, p_4\}, \{p_2, p_3, p_4\}, \{p_1, p_2, p_4\}$. Let $\Psi = \{q_1, q_2, q_3, q_4\}$ with $\Psi/R = \{\{q_1\}, \{q_3\}, \{q_2 q_4\}\}$. Let $X' = \{q_1, q_2\} \subseteq \Psi$, then $\tau'_R(X) = \{\Psi, \varphi, \{q_1\}, \{q_1, q_2, q_4\}, \{q_2, q_4\}\}$. If $\mu' = \{q_3\}$, then $\mu'_R(X) = \{\Omega, \varphi, \{q_1\}, \{q_3\}, \{q_1, q_3\}, \{q_2, q_4\}, \{q_2, q_3, q_4\}, \{q_1, q_2, q_4\}$. Let $f : \Omega \rightarrow \Psi$ be a function define as: $f(p_1) = q_1$, $f(p_2) = q_2$, $f(p_3) = q_3$, $f(p_4) = q_4$ is $\mu\hat{g}\pi$ -continuous.

Theorem 3.3. Every Micro- π -continuous is $\mu\hat{g}\pi$ -continuous but not conversely.

Proof. Let $f : \Omega \rightarrow \Psi$ is Micro- π -continuous. let V be Micro-closed in Ψ. Then $f^{-1}(V)$ is Micro- π -closed in Ω and therefore $f^{-1}(V)$ is $\mu\hat{g}\pi$ -closed in Ω. Hence f is $\mu\hat{g}\pi$ -continuous.

Example 3.4. In Example 3.2, Let $f : \Omega \rightarrow \Psi$ be a function define as: $f(p_1) = q_1$, $f(p_2) = q_2$, $f(p_3) = q_3$, $f(p_4) = q_4$ is $\mu\hat{g}\pi$ –continuous not Micro π- continuous because $f^{-1}\{q_1, q_3, q_4\} = \{p_1, p_3, p_4\}$ not in Ω .

Theorem 3.5. Every $\mu\hat{g}\pi$ -continuous is Micro- g -continuous but not conversely.

Proof. Let $f : \Omega \rightarrow \Psi$ is $\mu\hat{g}\pi$-continuous. let V be closed in Ψ . Then $f^{-1}(V)$ is $\mu\hat{g}\pi$ -closed in Ω and therefore $f^{-1}(V)$ is Micro- g -closed in Ω. Hence f is Micro- g -continuous.

Example 3.6. In Example 3.2, Let $f : \Omega \rightarrow \Psi$ be a function define as: $f(p_1) = q_1$, $f(p_2) = q_2$, $f(p_3) = q_3$, $f(p_4) = q_4$ is Micro-g-continuous not $\mu\hat{g}\pi$ –continuous because $f^{-1}\{q_1, q_2\} = \{p_1, p_2\}$ not in Ω .

Theorem 3.7. : A function $f : \Omega \rightarrow \Psi$ is $\mu\hat{g}\pi$ –continuous if and only if the inverse image of every Micro-closed set in Ψ is $\mu\hat{g}\pi$ -closed in Ω.

Proof. Suppose that the function $f : \Omega \rightarrow \Psi$ is $\mu\hat{g}\pi$ -continuous. Let Q be a Micro-closed set in Ψ. Then the complement $\Psi - Q$ is Micro open set in Ψ. Since f is $\mu\hat{g}\pi$ -continuous, $f^{-1}(\Psi - Q)$ is $\mu\hat{g}\pi$ -open set in Ω. But $f^{-1}(\Psi - Q) = \Psi - f^{-1}(Q)$ is $\mu\hat{g}\pi$ -open set in Ω. So $f^{-1}(Q)$ is $\mu\hat{g}\pi$ -closed in Ω.

Conversely, assume that the inverse image of every Micro closed set in Ψ is $\mu\hat{g}\pi$ -closed in Ω. Consider a Micro open set P in Ψ. Then $\Psi - P$ is Micro closed set in Ψ. By hypothesis $f^{-1}(\Psi - P)$ is $\mu\hat{g}\pi$ -closed in Ω. But $f^{-1}(\Psi - P) = \Psi - f^{-1}(P)$ is $\mu\hat{g}\pi$ -closed in Ω. Therefore $f^{-1}(P)$ is $\mu\hat{g}\pi$ -open in Ω. Hence f is $\mu\hat{g}\pi$-continuous function.

Theorem 3.8. Let $(\Omega, N, M), (\Psi, N', M')$and (Υ, N'', M'') be three MTS. If $f : \Omega \rightarrow \Psi$ is a $\mu\hat{g}\pi$ -continuous function and $g : \Psi \rightarrow \Upsilon$ be a Micro continuous function then $g \circ f : \Omega \rightarrow \Upsilon$ is $\mu\hat{g}\pi$ -continuous function.

Proof. Let Q be a Micro closed set in Υ. Since by g is Micro continuous function, then $g^{-1}(Q)$ is Micro closed set in Ψ. Since f is $\mu\hat{g}\pi$ -continuous function, then $f^{-1}(g^{-1}(Q))$ is $\mu\hat{g}\pi$ -closed set in Ω but $(g \circ f)^{-1}(Q) = (f^{-1} \circ g^{-1}(Q) = f^{-1}(g^{-1}(Q))$. Thus $(g \circ f)^{-1}(Q)$ is $\mu\hat{g}\pi$ closed set in Ω. Hence $g \circ f$ is $\mu\hat{g}\pi$ continuous function.

Theorem 3.9. Let $f : \Omega \rightarrow \Psi$ be a $\mu\hat{g}\pi$ -continuous function, then for every subset P of Ω, $f(\mu\hat{g}\pi\ cl(P)) \subseteq cl(f(P))$.

Proof. Let $f : \Omega \rightarrow \Psi$ be a $\mu\hat{g}\pi$ -continuous function and P be any subset of Ω. Then $cl_{\mu}(f(P))$ is a Micro closed set in Ψ. Since f is $\mu\hat{g}\pi$ -continuous, $f^{-1}(cl_{\mu}(f(P)))$ is $\mu\hat{g}\pi$ -closed in Ω . Since $f(P) \subseteq cl\mu(f(P))$, then $P \subseteq f^{-1}(cl_{\mu}(f(P)))$. Therefore, $f^{-1}(cl_{\mu}(f(P)))$ is Micro closed set containing P. By the definition of $\mu\hat{g}\pi$ -closure, $\mu\hat{g}\pi\ cl_{\mu}(P) \subseteq f^{-1}(cl_{\mu}(f(P)))$ which implies that $f(\mu\hat{g}\pi cl_{\mu}(P)) \subseteq cl_{\mu}(f(P))$.

Definition 3.10. Let (Ω, N, M) and (Ψ, N', M') be two Micro-topologicalspaces. A function $f : \Omega \rightarrow \Psi$ is called $\mu\hat{g}\pi$ -continuous at a point $p \in \Omega$ if for every Micro open set K containing f(p) in Ψ , there exist a $\mu\hat{g}\pi$ -open set L containing p in Ψ, such that $f(K) \subseteq L$.

Theorem 3.11. $f : \Omega \rightarrow \Psi$ is $\mu\hat{g}\pi$ -continuous iff f is $\mu\hat{g}\pi$ continuous at each point of Ω .

Proof. Let $f : \Omega \rightarrow \Psi$ be $\mu\hat{g}\pi$ continuous, $a \in \Omega$ and H be a Micro open set in Ω containing f(a). Since f is Micro-continuous, $f^{-1}(H)$ is Micro open in Ω containing a. Let $= f^{-1}(H)$, then $f(G) \subseteq H$ and $f(a) \subseteq G$. Hence f is continuous at a.

conversely, suppose that f is micro- continuous at each point of H . let H be a Micro -open in Ω , if $f^{-1}(H) = \varphi$ then it is Micro-open. So let $f^{-1}(H) \neq \varphi$. Take any $a \in f^{-1}(H)$, then $f(a) \subseteq H$. Since f is Micro-continuous at each point, then there exist a Micro-open set G_a containing a such that $f(G_a) \subseteq H$, let $G = (G_a : a \in f^{-1}(H))$. Claim : $G = f^{-1}(H)$ if $x \in f^{-1}(H)$ then $x \in G_x \subseteq G$. hence $G = f^{-1}(H)$. Since G_x is Micro-open, by definition

3.10., G is micro open and hence $G = f^{-1}(H)$ is micro-open for every Micro-open set H in Ω. Hence f is Micro-continuous.

4 Micro- $\mu\hat{g}\pi$ irresolute function

Definition 4.1. Let (Ω, N, M) and (Ψ, N', M') be two MTS's. Then mapping $f : \Omega \rightarrow \Psi$ is $\mu\hat{g}\pi$ - irresolute if the inverse image of every $\mu\hat{g}\pi$-closed set in Ψ is $\mu\hat{g}\pi$-closed in Ω.

Example 4.2. Let $\Omega = \{p_1, p_2, p_3, p_4\}$ with $/R = \{\{p_1\}, \{p_2\}, \{p_3 p_4\}\}$. Let $X = \{p_1, p_3\} \subseteq \Omega$, then $\tau_R(X) = \{U, \varphi, \{p_1\}, \{p_1, p_3, p_4\}, \{p_3, p_4\}\}$. If $= \{p_2\}$, then the micro topology $\mu_R(X) = \{\Omega, \varphi, \{p_1\}, \{p_2\}, \{p_1, p_2\}, \{p_3, p_4\}, \{p_2, p_3, p_4\}, \{p_1, p_3, p_4\}$. Let $\Psi = \{q_1, q_2, q_3, q_4\}$ with $\Psi/R = \{\{q_1\}, \{q_3\}, \{q_2 q_4\}\}$. Let $X' = \{q_1, q_3\} \subseteq \Psi$, then $\tau'_R(X) = \{\Psi, \varphi, \{q_1\}, \{q_1, q_3, q_4\}, \{q_3, q_4\}\}$. If $\mu' = \{q_2\}$, then $\mu'_R(X) = \{\Omega, \varphi, \{q_1\}, \{q_2\}, \{q_1, q_2\}, \{q_3, q_4\}, \{q_2, q_3, q_4\}, \{q_1, q_3, q_4\}$. Let $f : \Omega \rightarrow \Psi$ be a function define as: $f(p_1) = q_1$, $f(p_2) = q_2$, $f(p_3) = q_3$, $f(p_4) = q_4$ is $\mu\hat{g}\pi$ –irresolute function.

Theorem 4.3. Let $f : \Omega \rightarrow \Psi$ and $g : \Psi \rightarrow \Theta$ be two $\mu\hat{g}\pi$ –irresolute functions. Then their composition $g \circ f : \Omega \rightarrow \Theta$ is a $\mu\hat{g}\pi$ -irresolute function.

Proof. Follows from the definitions.

Theorem 4.4. Let $f : \Omega \rightarrow \Psi$ be a $\mu\hat{g}\pi$ -irresolute function and $g : \Psi \rightarrow \Theta$ be a $\mu\hat{g}\pi$ - continuous function. Then their composition $g \circ f : \Omega \rightarrow \Theta$ is a $\mu\hat{g}\pi$ -continuous function.

Proof. Let V be any closed set in Θ. Since g is $\mu\hat{g}\pi$ -continuous, $g^{-1}(V)$ is $\mu\hat{g}\pi$ -closed in Ψ. Since f is $\mu\hat{g}\pi$ -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $\mu\hat{g}\pi$ -closed in Ω. Hence $g \circ f : \Omega \rightarrow \Theta$ is a $\mu\hat{g}\pi$ -continuous function.

Theorem 4.5. If $f : \Omega \rightarrow \Psi$ is bijective, Micro open and $\mu\hat{g}\pi$-continuous, then f is $\mu\hat{g}\pi$ - irresolute.

Proof. Let P be a $\mu\hat{g}\pi$ -closed set in Ψ. Let $f^{-1}(P) \subseteq G$, where G is Micro open in Ω. Therefore, $P \subseteq f(G)$ holds. Since f(G) is Micro open and P is $\mu\hat{g}\pi$ -closed in Ψ, then $\overline{P} \subseteq f(G)$. Hence $f^{-1}(\overline{P}) \subseteq G$. Since f is $\mu\hat{g}\pi$ –continuous and $\overline{P}$ is Micro closed in Ψ, $\overline{f^{-1}(\overline{P})} \subseteq G$. That is., $\mu\hat{g}\pi$ -closed in Ω. Hence f is $\mu\hat{g}\pi$ -irresolute.

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