(S,d)Magic Labeling of Some Trees

Dr P. Sumathi [1], P. Mala[2]

[1]Department of Mathematics,

C. Kandaswami College for Men, Anna Nagar, Chennai-102.

[2]Department of Mathematics,

St Thomas College of Arts and Science, Koyambedu, Chennai-107.

[1]sumathipaul@gmail.com, [2]mala.vedha@gmail.com

Article Info
Page Number: 1895-1904
Publication Issue:
Vol. 72 No. 1 (2023)

Article History
Article Received: 15 November 2022
Revised: 24 December 2022
Accepted: 18 January 2023

Abstract. Let G (p,q) $be\ a$ connected, undirected, simple and non-trivial graph with p vertices and q edges. Let f be an injective function f: V(G) → $\{\ s, s+d, s+2d, \ldots. s+(q+1)d\ \}$ and g be an injective function g: E(G) → $\{d, 2d, 3d, \ldots 2(q-1)d\}$.Then the graph G is said to be (s, d) magic labeling if $f(u)+g(uv)+f(v)$ is a constant, for all $u, v \in V(G)$. A graph G is called (s,d) magic graph if it admits (s,d) magic labeling. In this paper the existence of (s, d) magic labeling in some trees such as a Coconut tree, Regular bamboo tree, Symmetrical tree, Olive tree, Spider graph are found.

Keywords: Coconut tree, Regular bamboo tree, Symmetrical tree, Olive tree, Spider graph

1. Introduction.

The first magic-type labeling was introduced by Sedlacek in 1963 He labeled edges of a graph with real numbers and required the sum of labels of all edges incident to a vertex to be constant. The brief summaries of definitions which are necessary for the present investigation are provided below

2. Definition

Definition 2.1: A graph $G(p,q)$ is said to be **(s,d) magic graph** if there exists a function

f: V(G) → $\{ s, s+d, s+2d, \ldots. s+(q+1)d \}$ and g :E (G) → {d,2d,3d......2(q-1)d} which are injective such that the sum of the labels on the vertices and the labels of its incident edge is a constant.

Definition 2.2 [5]: A **Coconut tree** $CT(m,n)$ is the graph,for all positive integer n and $m \geq 2$ is obtained from the path Pm by appending 'n' new pendant edges at an end vertex of Pm.

Definition 2.3: A rooted tree in which every level contains vertices of the same degree is called **symmetrical trees.**

Definition 2.4: [3] A **regular bamboo tree** is a rooted tree consisting of one central vertex, and several legs of equal length attached to it, the leaves of which are identified with leaves of stars of equal size.

Definition 2.5: **Olive tree** graph O_n is a rooted tree consisting of n branches and i^{th}branch is a path of length 'i'.

Definition 2.6: [4] **A spider graph** SP$(1^n 2^m)$ is a graph formed from a star $K_{1,n+m}$ and each of its m vertices having degree 1 is joined to a new vertex.

3. Main Result

Theorem 3.1: The coconut tree $CT(m,\eta)$ is (s,d) magic labeling for all positive integer η and $m \geq 2$.

Proof: Let $G = CT(m,\eta)$ be the coconut tree let $u_{1,}u_{2,} \dots\dots u_m;v_{1,}v_{2,} \dots\dots v_\eta$ be the vertices of coconut tree.

Let $u_{1,}u_{2,} \dots\dots u_m$ be the vertices of the path and $v_{1,}v_{2,} \dots\dots v_\eta$ be the pendent vertices attached with the end vertex of the path P_m.

Since $|V(G)| = m + \eta, |E(G)| = m + \eta - 1$

We define $f: V(G) \to \{ s, s + d, s + 2d \dots. s + (q + 1)d \}$ to label the vertices.

Therefore $f(u_1) = s$

$f(u_i) = s + (i - 1)d \, , 2 \leq i \leq m$

$f(u_m) = s + (m - 1)d$

$f(v_1) = s + md$

$f\left(v_{j+1}\right) = s + (m + j)d, 1 \leq j \leq \eta - 1$

We define $g: E(G) \to \{ d, 2d, 3d \dots .2(q - 1)d \}$ to label the edges.

$g(u_i u_m) = 2s + 2(q - 1)d - (f(u_i) + f(u_m)), \quad 1 \leq i \leq m - 1$

$g(v_i u_m) = 2s + 2(q - 1)d - (f(v_i) + f(u_m)) \quad 1 \leq i \leq \eta$

Therefore $f(u_i) + g(u_i u_m) + f(u_m)$ and $f(v_i) + g(v_i u_m) + f(u_m)$ are constant equals to $2(s + (q - 1)d)$

Hence the coconut tree $CT(m,\eta)$ admits (s,d) magic labeling.

	Labeling of vertices			Labeling of edges	
Value of $i\ and j$	$f(u_i)$	$f(v_i)$	$f(v_{j+1})$	$g(u_i u_m)$	$g(v_i u_m)$
$i=1$	s	$s+md$	-	-	-
$i=m$	$s+(m-1)d$	-	-	-	-
$1 \le j \le \eta - 1$	-	-	$s+(m+j)d$	-	-
$1 \le i \le \eta$	-	-	-	-	$2(s+(q-1)d)-(f(v_i)+f(u_m))$
$2 \le i \le m$	$s+(i-1)d$	-	-	-	-
$1 \le i \le m-1$	-	-	-	$2(s+(q-1)d)-(f(u_i)+f(u_m))$	-

Table1 Labeling of vertices and edges for the graph $CT(m,\eta)$

Example 3.1.1: The (s,d) magic labeling for the graph $CT(m,\eta)$

Theorem 3.2: All Symmetrical trees are (s,d) magic labeling

Proof: Let $V(G)=\{u_1, u_{i+1}\ \ 1 \le i \le 2^\eta - 1\ \}$ and

$$E(G)=\{u_i u_{2i}\, 1 \le i \le \frac{2^\eta - 2}{2} \cup u_i u_{2i+1}\, 1 \le i \le \frac{2^\eta - 2}{2}\}$$

Since $|V(G)| = 2^\eta - 1, |E(G)| = 2^\eta - 2$

We define $f: V(G) \to \{\ s, s+d, s+2d \ldots . s+(q+1)d\ \}$ to label the vertices.

Therefore $f(u_1) = s$

$f(u_{i+1}) = s + id, 1 \leq i \leq 2^\eta - 2$

We define $g: E(G) \rightarrow \{ d, 2d, 3d \ldots .2(q-1)d \}$ to label the edges.

$$g(u_i u_{2i}) = 2s + 2(q-1)d - (f(u_i) + f(u_{2i})) \quad 1 \leq i \leq \frac{2^\eta - 2}{2}$$

$$g(u_i u_{2i+1}) = 2s + 2(q-1)d - (f(u_i) + f(u_{2i+1})) \quad 1 \leq i \leq \frac{2^\eta - 2}{2}$$

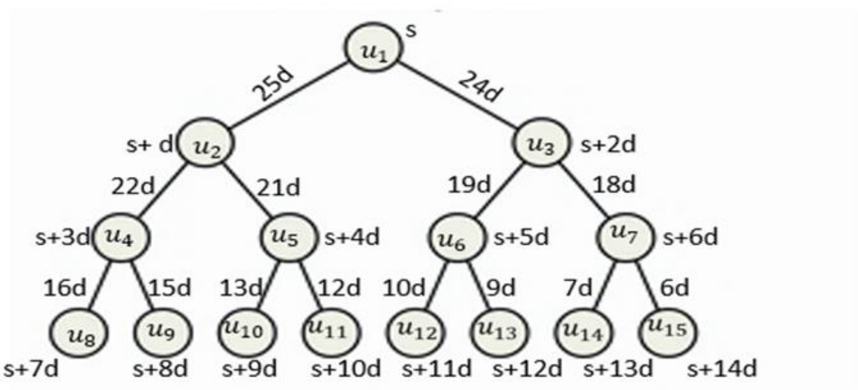
Therefore $f(u_i) + g(u_i u_{2i}) + f(u_{2i})$ and $f(u_i) + g(u_i u_{2i+1}) + f(u_{2i+1})$ are constant equals to $2(s + (q-1)d)$

Hence all symmetrical trees admit (s, d) magic labeling.

	Labeling of vertices		Labeling of edges	
Value of i	$f(u_i)$	$f(u_{i+1})$	$g(u_i u_{2i})$	$g(u_i u_{2i+1})$
$i = 1$	s	$-$	-	-
, $1 \leq i \leq 2^\eta - 2$	$-$	$s + id$	-	-
$1 \leq i \leq \frac{2^\eta - 2}{2}$	-	-	$2(s + (q-1)d) - (f(u_\iota) + f(u_{2i}))$	$2(s + (q-1)d) - (f(u_i) + f(u_{2i+1}))$

Table 2 Labeling of vertices and edges for the graph symmetrical tree

Example 3.1.2: The (s, d) magic labeling for Symmetrical tree 15 vertices



Theorem 3.3: Every regular bamboo tree is (s, d) magic labeling

Proof: Let $u_j^1, u_j^2 \dots\dots u_j^\eta$ be the vertices of j^{th} path where u_0 be the central vertex.let $v_1, v_2 \dots . v_{mk}$ be the pendent vertices the bamboo tree has $k(\eta+m-1)+1$ vertices and $k(\eta+m-1)$ edges.

We define $f: V(G) \rightarrow \{ s, s+d, s+2d \dots . s+(q+1)d \}$ to label the vertices.

Therefore $f(u_0) = s$

$f\left(u_j^i\right) = s + \left((i-1)k + j\right)d \quad 1 \le i \le \eta; 1 \le j \le \eta$

$f(v_i)$=$s + (\eta k + i)d \quad 1 \le i \le mk$

We define $g: E(G) \rightarrow \{ d, 2d, 3d \dots . 2(q-1)d \}$ to label the edges.

$g\left(u_0 u_j^i\right) = 2s + 2(q-1)d - (f(u_0) + f\left(u_j^i\right)) \ 1 \le j \le k$

$g\left(u_j^i u_j^{i+1}\right) = 2s + 2(q-1)d - (f\left(u_j^i\right) + f(u_j^{i+1}) \ 1 \le j \le k; 1 \le i \le \eta - 1$

$g\left(u_j^n v_i\right) = 2s + 2(q-1)d - \left(f\left(u_j^n\right) + f(v_i)\right) \ 1 \le j \le k; 1 \le i \le m$

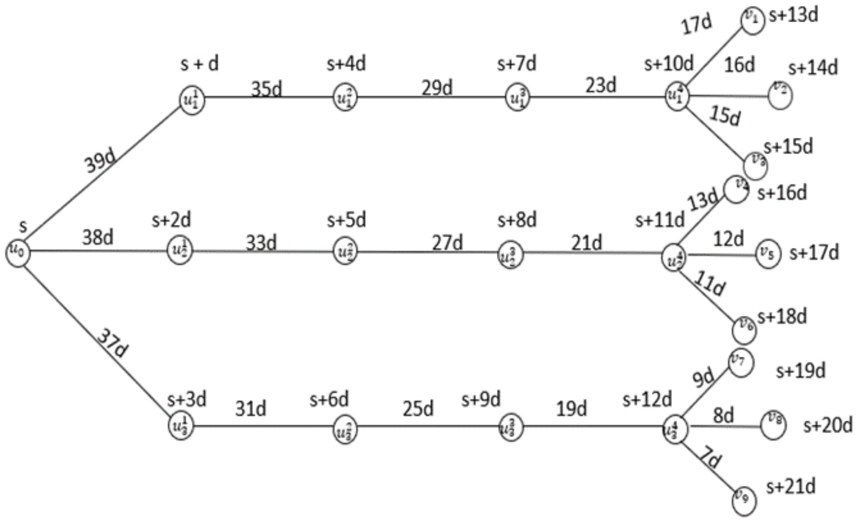
Therefore $f(u_0) + g\left(u_0 u_j^i\right) + f\left(u_j^i\right)$, $f\left(u_j^i\right) + g\left(u_j^i u_j^{i+1}\right) + f\left(u_j^{i+1}\right)$ and $f\left(u_j^n\right) + g\left(u_j^n v_i\right) + f(v_i)$ are constant equals to $2(s+(q-1)d)$

	Labeling of vertices			Labeling of edges		
Value of $i\ and j$	$f(u_i)$	$f(v_i)$	$f\left(u_j^i\right)$	$g\left(u_0 u_j^i\right)$	$g\left(u_j^i u_j^{i+1}\right)$	$g\left(u_j^n v_i\right)$
$i = 0$	s	-	-	-	-	-
$1 \le i = j \le \eta$	-	-	$s + \left((i-1)k + j\right)d$	-	-	-
$1 \le i \le mk$	-	$s + (nk + i)d$	-	-	-	-
$1 \le j \le k$	-	-	-	$2s + 2(q-1)d - (f(u_0) + f\left(u_j^i\right))$	-	-
$1 \le i \le \eta - 1$	-	-	-	-	$2s + 2(q-1)d - (f\left(u_j^i\right) + f(u_j^{i+1})$	-

$1 \leq i \leq m$	-	-	-	-	-	$2s + 2(q-1) - \left(f\left(u_j^n\right) + f(v_i)\right)$

Table3 Labeling of vertices and edges for the graph regular bamboo tree

Example3.1.3 The (s, d) magic labeling for regular bamboo tree



Theorem 3.4: Olive tree O_η is (s, d) magic labeling

Proof: Let O_η be the olive tree having n paths of length 1,2,… η adjoined at on vertex $v_{1,0}$

Let the vertices of O_η be $\{v_{1,0}, v_{1,1} \ldots v_{1,\eta}, v_{2,1}, \ldots\ldots v_{2(\eta-1)}, v_{3,1}, v_{3,2} \ldots . v_{3(\eta-2)} \ldots . v_{\eta,1}\}$

Here $|V(G)| = \frac{\eta^2+\eta+2}{2}$ and$|E(G)| = \frac{\eta(\eta+1)}{2}$

We define $f: V(G) \rightarrow \{ s, s+d, s+2d \ldots . s+(q+1)d \}$ to label the vertices.

$f(v_{1,0}) = s$

$f(v_{1,j}) = s + jd \quad 1 \leq j \leq \eta$

$f(v_{i,j}) = f(v_{i-1,\eta+2-i}) + jd \quad ,2 \leq i \leq \eta - 2 ; 1 \leq j \leq \eta + 1 - i$

$f(v_{\eta-1,1}) = f(v_{\eta-2,3}) + d$

$f(v_{\eta-1,2}) = f(v_{\eta-2,3}) + 3d$

$f\left(v_{\eta,1}\right) = f\left(v_{\eta-1,1}\right) + d$

We define $g: E(G) \rightarrow \{\, d, 2d, 3d \ldots .2(q-1)d \,\}$ to label the edges.

$g\left(v_{i,j} v_{i+1,j}\right) = 2s + 2(q-1) - \left(f\left(v_{i,j}\right) + f(v_{i+1,j}\right)$,$1 \leq j \leq \eta - 1 : 1 \leq i \leq \eta - j$

$g\left(v_{1,0} v_{1,j}\right) = 2s + 2(q-1) - \left(f\left(v_{1,0}\right) + f(v_{1,j}\right)$,$1 \leq j \leq \eta$

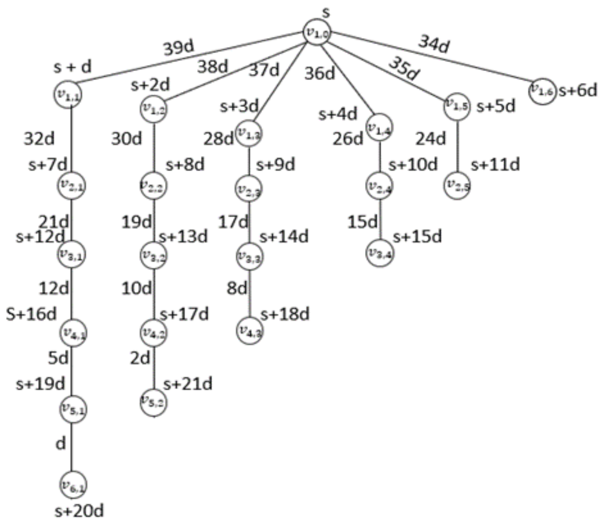
Therefore $f\left(v_{i,j}\right) + g\left(v_{i,j} v_{i+1,j}\right) + f(v_{i+1,j})$and $f\left(v_{1,0}\right) + g\left(v_{1,0} v_{1,j}\right) + f(v_{1,j})$ are constant equals to $2(s + (q-1)d)$

Hence Olive tree admits (s,d) magic labeling.	Labeling of vertices						Labeling of edges	
Value of i and j	$f\left(v_{1,0}\right)$	$f\left(v_{1,j}\right)$	$f\left(v_{i,j}\right)$	$f\left(v_{\eta-1,1}\right)$	$f\left(v_{\eta-1,2}\right)$	$f\left(v_{\eta,1}\right)$	$g\left(v_{i,j} v_{i+1,j}\right)$	$g\left(v_{1,0} v_{1,j}\right)$
$i = 1 \& j = 0$	s	-	$-$	-	-	-	-	-
$i = \eta - 1, j = 1$	-	-	-	$f\left(v_{\eta-2,3}\right) + d$	-	-	-	-
$i = \eta - 1, j = 2$	-	-	-	-	$f\left(v_{\eta-2,3}\right) + 3d$	-	-	-
$i = \eta, j = 1$	-	-	-	-	-	$f\left(v_{\eta-1,1}\right) + d$	-	-
$1 \leq j \leq \eta$	-	$s + jd$	-	-	-	-	-	$2s + 2(q-1) - \left(f\left(v_{1,0}\right) + f(v_{1,j}\right)$

$2 \leq i \leq \eta - 2$ $1 \leq j \leq \eta + 1 - i$	-	-	$f(v_{i-1,\eta+2-i}$ $+ jd$	-	-	-	-	-
$1 \leq j \leq \eta - 1$: $1 \leq i \leq \eta - j$	-	-	-	-	-	-	$2s + 2(q - 1) - (f(v_{i,j}) + f(v_{i+1,j})$	-

Table 4 Labeling of vertices and edges for the graph Olive tree

Example 3.1.4: The (s, d) magic labeling for olive tree



Theorem 3.5: A Spider graph $SP(1^{\eta}, 2^{m})$ is (s, d) magic labeling where n and m be positive integers.

Proof :Let Spider graph $SP(1^{\eta}, 2^{m})$ have a vertex set

$V(SP(1^{n}, 2^{m})) = \{u, v_1, v_2 \dots . v_{\eta}, w_1 . w_2 \dots . w_m, x_1, x_2 \dots x_m \}$

and the edge $E(SP(1^{n}, 2^{m})) = \{uv_1, uv_2, \dots . uv_{\eta}, uw_1, uw_2 \dots . . uw_m, \dots wx_1 \dots . wx_m\}$

Since $p = 2m + \eta + 1$ and $q = 2m + \eta$

We define $f: V(G) \to \{ s, s + d, s + 2d \dots . s + (q + 1)d \}$ to label the vertices.

$f(u) = s$

$f(w_i) = s + id \ ,1 \leq i \leq m$

$f(v_i) = w_m + id, 1 \leq i \leq \eta$

$f(x_i) = v_\eta + id, 1 \leq i \leq m$

We define $g: E(G) \rightarrow \{\ d, 2d, 3d \ldots .2(q-1)d\ \}$ to label the edges.

$g(uv_i) = 2s + 2(q-1)d - (f(u) + f(v_i)\ ,1 \leq i \leq \eta$

$g(uw_i) = 2s + 2(q-1)d - (f(u) + f(w_i)\ ,1 \leq i \leq m$

$g(ux_i) = 2s + 2(q-1)d - (f(u) + f(x_i)\ ,1 \leq i \leq m$

Therefore $f(u) + g(uv_i) + f(v_i)$, $f(u) + g(uw_i) + f(w_i)$ and

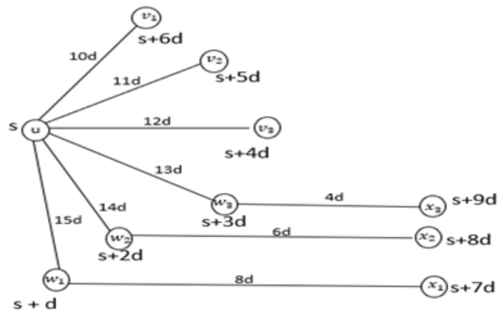
$f(u) + g(ux_i) + f(x_i)$ are constant equals to $2(s + (q-1)d)$

Hence Spider graph admits (s, d) magic labeling.

	Labeling of vertices				Labeling of edges		
Value of i	$f(u)$	$f(w_i)$	$f(v_i)$	$f(x_i)$	$g(uv_i)$	$g(uw_i)$	$g(ux_i)$
	s	-	-	-	-	-	-
$1 \leq i \leq m$	-	$s + id$	-	$v_\eta + id$	-	$2s + 2(q-1)d - (f(u) + f(w_i)$	$2s + 2(q-1)d - (f(u) + f(x_i)$
$1 \leq i \leq \eta$	-	-	$w_m + id$	-	$2s + 2(q-1)d - (f(u) + f(v_i)$	-	-

Table 5 Labeling of vertices and edges for the graph Spider graph

Example 3.1.5 The (s, d) magic labeling for the graph Spider graph



Conclusion

In this paper, the (s,d)magic labeling number for some trees is determined. Our future work will involve calculating the (s,d)magic labeling number for more families of graphs in trees.

References

1. J. A. Gallian, Dynamic Survey of Graph Labeling, Electronic Journal of Combinatorics 18, DS6 (2015).
2. Udhayakumar, N., & Ramya, N. On prime labeling of some trees. Malaya Journal of Matematik, Vol. S, No. 2, 4042-4045, 2020 https://doi.org/10.26637/MJM0S20/1049
3. Dhabliya, M. D. (2019). Uses and Purposes of Various Portland Cement Chemical in Construction Industry. Forest Chemicals Review, 06–10.
4. Dhabliya, M. D. (2018). A Scientific Approach and Data Analysis of Chemicals used in Packed Juices. Forest Chemicals Review, 01–05.
5. Dhabliya, D. (2021a). AODV Routing Protocol Implementation: Implications for Cybersecurity. In Intelligent and Reliable Engineering Systems (pp. 144–148). CRC Press.
6. Sekar, C., & Ramachandran, V. (2014). One modulo N gracefulness of regular bamboo tree and coconut tree. *International journal on applications of graph theory in wireless ad hoc networks and sensor networks (GRAPH-HOC)*, *6*(2), 1-10.
7. Khasanah, I. N. (2021, May). Super total graceful labeling of some trees. In *Journal of Physics: Conference Series* (Vol. 1872, No. 1, p. 012004). IOP Publishing.
8. Prime Labeling of Split Graph of CoconutTree CT(m,n) LAVANYA. S1 and GANESAN.V2 International Journal of Advanced Scientific Research and Management, Volume 5 Issue 4, Apr 2020