

STRONG FORMS OF CONTRA $(1,2)^*g^*$ -CONTINUITY

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Abstract

This paper devotes to introduce and investigate a new class of maps called contra- $(1,2)^*g^*$ -continuous maps which are weaker than contra- $(1,2)^*$ -continuity and stronger than contra- $(1,2)^*g$ -continuity, contra- $(1,2)^*\alpha g$ -continuity, contra- $(1,2)^*gs$ -continuity, contra- $(1,2)^*gsp$ -continuity, contra- $(1,2)^*gp$ -continuity, contra- $(1,2)^*rg$ -continuity, contra- $(1,2)^*\alpha^{**}g$ -continuity and contra- $(1,2)^*gpr$ -continuity. The main results of the paper are that several properties concerning contra- $(1,2)^*g^*$ -continuous maps. Furthermore, the relationships between the contra- $(1,2)^*g^*$ -continuity and some bitopological maps as well as separation axioms are also investigated

Keywords: - contra- $(1,2)^*g^*$ -continuous maps; contra- $(1,2)^*gs$ -bitopological maps.

Introduction

In the literature there are many types of continuities introduced by various authors. Quite recently, Noiri et.al [2,4,5,6,7] introduced and investigated the notions of contra-precontinuity, contra- α -continuity, contra- g -continuity and contra-super-continuity as a continuation of research done by Dontchev [3], and Dontchev and Noiri [4] on the interesting notions of contra-continuity and contra-semi-continuity.

This paper devotes to introduce and investigate a new class of maps called contra- $(1,2)^*g^*$ -continuous maps which are weaker than contra- $(1,2)^*$ -continuity and stronger than contra- $(1,2)^*g$ -continuity, contra- $(1,2)^*\alpha g$ -continuity, contra- $(1,2)^*gs$ -continuity, contra- $(1,2)^*gsp$ -continuity, contra- $(1,2)^*gp$ -continuity, contra- $(1,2)^*rg$ -continuity, contra- $(1,2)^*\alpha^{**}g$ -continuity and contra- $(1,2)^*gpr$ -continuity. The main results of the paper are that several properties concerning contra- $(1,2)^*g^*$ -continuous maps. Furthermore, the relationships between the contra- $(1,2)^*g^*$ -continuity and some bitopological maps as well as separation axioms are also investigated.

Throughout this paper, (X, τ_1, τ_2) , (Y, σ_1, σ_2) and (Z, η_1, η_2) (briefly, X , Y and Z) will denote bitopological spaces.

We recall the following definitions which are useful in the sequel.

Definition 1.1

Let S be a subset of X . Then S is said to be $\tau_{1,2}$ -open [11] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.

The complement of $\tau_{1,2}$ -open set is called $\tau_{1,2}$ -closed.

Notice that $\tau_{1,2}$ -open sets need not necessarily form a topology.

Definition 1.2 [11]

Let S be a subset of a bitopological space X . Then

- (1) the $\tau_{1,2}$ -closure of S , denoted by $\tau_{1,2}\text{-cl}(S)$, is defined as $\bigcap \{F : S \subseteq F \text{ and } F \text{ is } \tau_{1,2}\text{-closed}\}$.
- (2) the $\tau_{1,2}$ -interior of S , denoted by $\tau_{1,2}\text{-int}(S)$, is defined as $\bigcup \{F : F \subseteq S \text{ and } F \text{ is } \tau_{1,2}\text{-open}\}$.

Definition 1.3

A subset A of a bitopological space X is called

- (i) $(1,2)^*\text{-}\alpha$ -open [11] if $A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(A)))$;
- (ii) $(1,2)^*\text{-}\beta$ -open [8] if $A \subseteq \tau_{1,2}\text{-cl}(\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)))$;
- (iii) $(1,2)^*\text{-preopen}$ [11] if $A \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A))$.

The complements of the above mentioned open sets are called their respective closed sets.

The intersection of all $(1,2)^*\text{-pre-closed}$ (resp. $(1,2)^*\text{-}\beta$ -closed and $(1,2)^*\text{-}\alpha$ -closed) sets containing a subset A of X is called the $(1,2)^*\text{-pre-closure}$ (resp. $(1,2)^*\text{-}\beta$ -closure and $(1,2)^*\text{-}\alpha$ -closure) of A and is denoted by $(1,2)^*\text{-pcl}(A)$ (resp. $(1,2)^*\text{-spcl}(A)$ and $(1,2)^*\text{-}\alpha\text{cl}(A)$).

PROPERTIES OF CONTRA $(1,2)^*\text{-g}^*$ -CONTINUOUS MAPS

We introduce the following definition.

Definition 2.1

A subset A of a bitopological space X is called

- (i) $(1,2)^*\text{-g}$ -closed [9] if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X ;
- (ii) $(1,2)^*\text{-gs}$ -closed if $(1,2)^*\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X ;
- (iii) $(1,2)^*\text{-}\alpha\text{g}$ -closed if $(1,2)^*\text{-}\alpha\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X ;
- (iv) $(1,2)^*\text{-}\alpha^{**}\text{g}$ -closed if $(1,2)^*\text{-}\alpha\text{cl}(A) \subseteq \tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(U))$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X ;
- (v) $(1,2)^*\text{-gsp}$ -closed if $(1,2)^*\text{-spcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X ;
- (vi) $(1,2)^*\text{-rg}$ -closed [8] if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular $(1,2)^*\text{-open}$ in X ;
- (vii) $(1,2)^*\text{-gp}$ -closed if $(1,2)^*\text{-pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$ -open in X ;
- (viii) $(1,2)^*\text{-gpr}$ -closed if $(1,2)^*\text{-pcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is regular $(1,2)^*\text{-open}$ in X and
- (ix) $(1,2)^*\text{-g}^*$ -closed if $\tau_{1,2}\text{-cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^*\text{-g}$ -open in X .

The complements of the above mentioned closed sets are called their respective open sets.

Remark 2.2

- (i) Every $\tau_{1,2}$ -closed set is $(1,2)^*\text{-g}^*$ -closed.
- (ii) Every $(1,2)^*\text{-g}^*$ -closed set is $(1,2)^*\text{-g}$ -closed and hence an $(1,2)^*\text{-}\alpha\text{g}$ -closed, $(1,2)^*\text{-gs}$ -closed, $(1,2)^*\text{-gsp}$ -closed, $(1,2)^*\text{-gp}$ -closed, $(1,2)^*\text{-gpr}$ -closed, $(1,2)^*\text{-}\alpha^{**}\text{g}$ -closed and $(1,2)^*\text{-rg}$ -closed.

Definition 2.3

A bitopological space X is called

- (i) a $(1,2)^*\text{-T}_{1/2}$ space if every $(1,2)^*\text{-g}^*$ -closed set in it is $\tau_{1,2}$ -closed.
- (ii) a $(1,2)^*\text{-T}_{1/2}^*$ space if every $(1,2)^*\text{-g}$ -closed set in it is $(1,2)^*\text{-g}^*$ -closed.
- (iii) a $(1,2)^*\text{-Tc}$ space if every $(1,2)^*\text{-gs}$ -closed set in it is $(1,2)^*\text{-g}^*$ -closed.
- (iv) an $(1,2)^*\text{-}\alpha\text{Tc}$ space if every $(1,2)^*\text{-}\alpha\text{g}$ -closed set in it is $(1,2)^*\text{-g}^*$ -closed.
- (v) $(1,2)^*\text{-locally indiscrete}$ if each $\tau_{1,2}$ -open subset of X is $\tau_{1,2}$ -closed in X .

Definition 2.4

A map $f: X \rightarrow Y$ is called

- (i) contra- $(1,2)^*\text{-g}$ -continuous if $f^{-1}(V)$ is $(1,2)^*\text{-g}$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y ;
- (ii) contra- $(1,2)^*\text{-}\alpha\text{g}$ -continuous if $f^{-1}(V)$ is $(1,2)^*\text{-}\alpha\text{g}$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y ;
- (iii) contra- $(1,2)^*\text{-gs}$ -continuous if $f^{-1}(V)$ is $(1,2)^*\text{-gs}$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y ;
- (iv) contra- $(1,2)^*\text{-rg}$ -continuous if $f^{-1}(V)$ is $(1,2)^*\text{-rg}$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y ;
- (v) contra- $(1,2)^*\text{-gp}$ -continuous if $f^{-1}(V)$ is $(1,2)^*\text{-gp}$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y ;
- (vi) contra- $(1,2)^*\text{-gpr}$ -continuous if $f^{-1}(V)$ is $(1,2)^*\text{-gpr}$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y ;
- (vii) $(1,2)^*\text{-g}^*$ -continuous if $f^{-1}(V)$ is $(1,2)^*\text{-g}^*$ -closed set of X for every $\sigma_{1,2}$ -closed set V of Y ;

- (viii) $(1,2)^*g^*$ -irresolute if $f^{-1}(V)$ is $(1,2)^*g^*$ -closed set of X for every $(1,2)^*g^*$ -closed set V of Y ;
- (ix) pre $(1,2)^*g^*$ -closed if $f(A)$ is a $(1,2)^*g^*$ -closed set of Y for every $(1,2)^*g^*$ -closed set of X and
- (x) $(1,2)^*$ -preclosed if $f(V)$ is $(1,2)^*$ -preclosed in Y for every $\tau_{1,2}$ -closed set V of X .

Definition 2.5

A map $f: X \rightarrow Y$ is called contra- $(1,2)^*g^*$ -continuous if $f^{-1}(V)$ is a $(1,2)^*g^*$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y .

Theorem 2.6

Every contra- $(1,2)^*$ -continuous map is contra $(1,2)^*g^*$ -continuous.

The following example supports that the converse of the above theorem is not true in general.

Example 2.7

Let $X = Y = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y, \{b\}\}$ and $\sigma_2 = \{\phi, Y, \{b\}, \{a, b\}, \{b, c\}\}$. Define $f: X \rightarrow Y$ as the identity map. Then f is contra- $(1,2)^*g^*$ -continuous map but it is not contra- $(1,2)^*$ -continuous.

Definition 2.8

A map $f: X \rightarrow Y$ is called

- (i) contra $(1,2)^*\alpha^{**}g$ -continuous if $f^{-1}(V)$ is $(1,2)^*\alpha^{**}g$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y .
- (ii) contra- $(1,2)^*gsp$ -continuous if $f^{-1}(V)$ is $(1,2)^*gsp$ -closed set of X for every $\sigma_{1,2}$ -open set V of Y .

Theorem 2.9

Every contra- $(1,2)^*g^*$ -continuous map is contra- $(1,2)^*g$ -continuous and hence contra- $(1,2)^*\alpha g$ -continuous, contra- $(1,2)^*\alpha^{**}g$ -continuous, contra- $(1,2)^*gs$ -continuous, contra- $(1,2)^*gsp$ -continuous, contra- $(1,2)^*gp$ -continuous, contra- $(1,2)^*rg$ -continuous and contra- $(1,2)^*gpr$ -continuous.

The following example shows that the converse of the above theorem is not true in general.

Example 2.10

Let $X = Y = \{a, b, c\}$. Let $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, Y, \{b\}\}$. Define $f: X \rightarrow Y$ as the identity map. Then f is contra- $(1,2)^*g$ -continuous map and hence contra- $(1,2)^*\alpha g$ -continuous map and contra- $(1,2)^*\alpha^{**}g$ -continuous map, contra- $(1,2)^*gs$ -continuous map, contra- $(1,2)^*gsp$ -continuous map, contra- $(1,2)^*gp$ -continuous map, contra- $(1,2)^*rg$ -continuous map and contra- $(1,2)^*gpr$ -continuous map but it is not contra- $(1,2)^*g^*$ -continuous.

The following example supports that the composition of two contra- $(1,2)^*g^*$ -continuous maps need not be a contra- $(1,2)^*g^*$ -continuous map.

Example 2.11

Let $X = Y = Z = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y\}$, $\sigma_2 = \{\phi, Y, \{b\}\}$, $\eta_1 = \{\phi, Z\}$ and $\eta_2 = \{\phi, Z, \{a, c\}\}$. Let $f: X \rightarrow Y$ be the identity map and $g: Y \rightarrow Z$ be the identity map. Then f is contra- $(1,2)^*g^*$ -continuous map and g is contra- $(1,2)^*g^*$ -continuous map. But their composition $g \circ f$ is not contra- $(1,2)^*g^*$ -continuous.

Theorem 2.12

Let $f: X \rightarrow Y$ be a map. Then the following statements are equivalent.

- (i) f is contra- $(1,2)^*g^*$ -continuous .
- (ii) The inverse image of each $\sigma_{1,2}$ -open set of Y is $(1,2)^*g^*$ -closed in X .
- (iii) The inverse image of each $\sigma_{1,2}$ -closed set of Y is $(1,2)^*g^*$ -open in X .

Proof

- (i) \Rightarrow (ii): Let V be any $\sigma_{1,2}$ -open set in Y . By the assumption of (i), $f^{-1}(V)$ is $(1,2)^*g^*$ -closed in X .
- (ii) \Rightarrow (iii) : Let V be a $\sigma_{1,2}$ -closed set in Y . Then $Y-V$ is $\sigma_{1,2}$ -open in Y . By the assumption of (ii), $f^{-1}(Y-V) = X-f^{-1}(V)$ is $(1,2)^*g^*$ -closed in X . Thus, $f^{-1}(V)$ is $(1,2)^*g^*$ -open in X .
- (iii) \Rightarrow (i) : Let V be an $\sigma_{1,2}$ -open set in Y . Then $Y-V$ is $\sigma_{1,2}$ -closed in Y . By the assumption of (iii), $f^{-1}(Y-V) = X-f^{-1}(V)$ is $(1,2)^*g^*$ -open in X . Therefore $f^{-1}(V)$ is $(1,2)^*g^*$ -closed in X . Hence f is contra- $(1,2)^*g^*$ -continuous map.

SEPARATION AXIOMS

Theorem 3.1

Let $f: X \rightarrow Y$ be a contra-(1,2)*-g*-continuous map. If X is (1,2)*-T_{1/2}* space, then f is contra-(1,2)*-continuous map.

Proof

Let V be an $\sigma_{1,2}$ -open set in Y . Since f is contra-(1,2)*-g*-continuous, $f^{-1}(V)$ is (1,2)*-g*-closed in X . But X is (1,2)*-T_{1/2}* space, $f^{-1}(V)$ is $\tau_{1,2}$ -closed in X . Therefore f is contra-(1,2)*-continuous map.

Theorem 3.2

Let $f: X \rightarrow Y$ be contra-(1,2)*- α g-continuous map. If X is (1,2)*- α Tc space, then f is contra-(1,2)*-g*-continuous map.

Proof

Let V be an $\sigma_{1,2}$ -open set in Y . Since f is contra-(1,2)*- α g-continuous, $f^{-1}(V)$ is (1,2)*- α g-closed in X . But X is (1,2)*- α Tc space, $f^{-1}(V)$ is (1,2)*-g*-closed in X . Therefore f is contra-(1,2)*-g*-continuous map.

Theorem 3.3

Let $f: X \rightarrow Y$ be contra-(1,2)*-g-continuous map. If X is (1,2)*-T_{1/2}* space, then f is contra-(1,2)*-g*-continuous map.

Proof

Let V be an $\sigma_{1,2}$ -open set in Y . Since f is contra-(1,2)*-g-continuous, $f^{-1}(V)$ is (1,2)*-g-closed in X . But X is (1,2)*-T_{1/2}* space, $f^{-1}(V)$ is (1,2)*-g*-closed in X . Therefore f is contra-(1,2)*-g*-continuous map.

Theorem 3.4

Let $f: X \rightarrow Y$ be contra-(1,2)*-gs-continuous map. If X is (1,2)*-Tc space, then f is contra-(1,2)*-g*-continuous map.

Proof

Let V be an $\sigma_{1,2}$ -open set in Y . Since f is contra-(1,2)*-gs-continuous, $f^{-1}(V)$ is (1,2)*-gs-closed in X . But X is (1,2)*-Tc space, $f^{-1}(V)$ is (1,2)*-g*-closed in X . Therefore f is contra-(1,2)*-g*-continuous map.

Theorem 3.5

Let $f: X \rightarrow Y$ be a surjective, (1,2)*-preclosed, contra-(1,2)*-g*-continuous map and X be (1,2)*-T_{1/2}* space. Then Y is (1,2)*-locally indiscrete.

Proof

Suppose V is $\sigma_{1,2}$ -open set in Y . By hypothesis, f is contra-(1,2)*-g*-continuous, $f^{-1}(V)$ is (1,2)*-g*-closed in X . Since X is (1,2)*-T_{1/2}* space, $f^{-1}(V)$ is $\tau_{1,2}$ -closed in X . Since f is (1,2)*-preclosed, V is (1,2)*-preclosed in Y . Now we have $\sigma_{1,2}\text{-cl}(V) = \sigma_{1,2}\text{-cl}(\sigma_{1,2}\text{-int}(V)) \subseteq V$. This means that V is $\sigma_{1,2}$ -closed in Y . Thus, Y is (1,2)*-locally indiscrete.

Theorem 3.6

If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are contra-(1,2)*-g*-continuous maps with Y as a (1,2)*-T_{1/2}* space, then $g \circ f: X \rightarrow Z$ is (1,2)*-g*-continuous map.

Proof

Let G be an $\eta_{1,2}$ -open set in Z . Since g is contra-(1,2)*-g*-continuous, $g^{-1}(G)$ is (1,2)*-g*-closed in Y . Since Y is (1,2)*-T_{1/2}* space, $g^{-1}(G)$ is $\sigma_{1,2}$ -closed in Y . Since f is contra-(1,2)*-g*-continuous, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is (1,2)*-g*-open in X . Therefore $g \circ f$ is (1,2)*-g*-continuous map.

Theorem 3.7

Let X and Z be any bitopological spaces and Y be a (1,2)*-T_{1/2}* space. Then the composition $g \circ f: X \rightarrow Z$ is contra-(1,2)*-g*-continuous map if $f: X \rightarrow Y$ is (1,2)*-g*-continuous map and $g: Y \rightarrow Z$ is contra-(1,2)*-g*-continuous.

Proof

Let G be any $\eta_{1,2}$ -open set in Z . Since g is contra-(1,2)*-g*-continuous, $g^{-1}(G)$ is (1,2)*-g*-closed in Y . But Y is a (1,2)*-T_{1/2}* space, $g^{-1}(G)$ is $\sigma_{1,2}$ -closed in Y . Since f is (1,2)*-g*-continuous, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is (1,2)*-g*-closed in X . Therefore $g \circ f$ is contra (1,2)*-g*-continuous map.

Theorem 3.8

Let X and Z be any bitopological spaces and Y be a (1,2)*-T_{1/2}* space. Then $g \circ f: X \rightarrow Z$ is contra-(1,2)*-g*-continuous map if $f: X \rightarrow Y$ is (1,2)*-g*-irresolute map and $g: Y \rightarrow Z$ is contra-(1,2)*-g*-continuous map.

Proof

Let G be an $\eta_{1,2}$ -open set in Z . Since g is contra- $(1,2)^*$ - g^* -continuous, $g^{-1}(G)$ is $(1,2)^*$ - g^* -closed in Y . But Y is $(1,2)^*$ - $T_{1/2}$ space, $g^{-1}(G)$ is $(1,2)^*$ - g^* -closed in Y . Since f is $(1,2)^*$ - g^* -irresolute, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is $(1,2)^*$ - g^* -closed in X . Therefore $g \circ f$ is contra- $(1,2)^*$ - g^* -continuous map.

Theorem 3.9

Let X and Z be any bitopological spaces and Y be a $(1,2)^*$ - $T_{1/2}$ space. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous map if $f: X \rightarrow Y$ is contra- $(1,2)^*$ - g^* -continuous map and $g: Y \rightarrow Z$ is $(1,2)^*$ - g^* -irresolute map.

Proof

Let G be any $\eta_{1,2}$ -closed set in Z . Then G is $(1,2)^*$ - g^* -closed in Z . Since g is $(1,2)^*$ - g^* -irresolute, $g^{-1}(G)$ is $(1,2)^*$ - g^* -closed in Y . But Y is a $(1,2)^*$ - $T_{1/2}$ space, $g^{-1}(G)$ is $\sigma_{1,2}$ -closed in Y . Since f is contra- $(1,2)^*$ - g^* -continuous, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is $(1,2)^*$ - g^* -open in X . Therefore $g \circ f$ is contra- $(1,2)^*$ - g^* -continuous map.

RELATIONS WITH OTHER MAPS

Theorem 4.1

Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two maps. Then $g \circ f$ is contra- $(1,2)^*$ - g^* -continuous map if g is contra- $(1,2)^*$ -continuous map and f is $(1,2)^*$ - g^* -continuous map.

Proof

Let V be an $\eta_{1,2}$ -open set in Z . Since g is contra- $(1,2)^*$ -continuous, $g^{-1}(V)$ is $\sigma_{1,2}$ -closed in Y . Since f is $(1,2)^*$ - g^* -continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $(1,2)^*$ - g^* -closed in X . Therefore $g \circ f$ is contra- $(1,2)^*$ - g^* -continuous map.

Theorem 4.2

Let $f: X \rightarrow Y$ be surjective, $(1,2)^*$ - g^* -irresolute and pre $(1,2)^*$ - g^* -closed and $g: Y \rightarrow Z$ be any map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous if and only if g is contra- $(1,2)^*$ - g^* -continuous.

Proof

Let $g \circ f: X \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Let F be an $\eta_{1,2}$ -open subset of Z . Then $(g \circ f)^{-1}(F) = f^{-1}(g^{-1}(F))$ is a $(1,2)^*$ - g^* -closed subset of X . Since f is pre $(1,2)^*$ - g^* -closed, $f(f^{-1}(g^{-1}(F))) = g^{-1}(F)$ is $(1,2)^*$ - g^* -closed in Y . Thus g is contra- $(1,2)^*$ - g^* -continuous map.

Conversely, let $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous. Let G be an $\eta_{1,2}$ -open subset of Z . Since g is contra- $(1,2)^*$ - g^* -continuous, $g^{-1}(G)$ is $(1,2)^*$ - g^* -closed in Y . Since f is $(1,2)^*$ - g^* -irresolute, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is $(1,2)^*$ - g^* -closed in X . Hence $g \circ f$ is contra- $(1,2)^*$ - g^* -continuous map.

Theorem 4.3

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous map.

Proof

Let F be an $\eta_{1,2}$ -open set in Z . Since g is contra- $(1,2)^*$ - g^* -continuous, $g^{-1}(F)$ is $(1,2)^*$ - g^* -closed in Y . Since f is $(1,2)^*$ - g^* -irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is $(1,2)^*$ - g^* -closed in X . Thus, $g \circ f$ is contra- $(1,2)^*$ - g^* -continuous map.

Corollary 4.4

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous map.

Corollary 4.5

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - αg^* -continuous map.

Corollary 4.6

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous map.

Corollary 4.7

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous map.

Corollary 4.8

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous map.

Corollary 4.9

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - g^* -continuous map.

Corollary 4.10

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - α^* - g^* -continuous map.

Corollary 4.11

Let $f: X \rightarrow Y$ be $(1,2)^*$ - g^* -irresolute map and $g: Y \rightarrow Z$ be contra- $(1,2)^*$ - g^* -continuous map. Then $g \circ f: X \rightarrow Z$ is contra- $(1,2)^*$ - rg^* -continuous map.

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