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Properties of Contra (1,2)*-Sg-Continuous Maps

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Abstract

Ravi and Lellis Thivagar introduced a bitopological space in the year 2006. Many modern topologists have worked in the field of bitopological spaces using the idea of Ravi and Thivagar. In this paper, we study the bitopological properties of the new class of contra-(1,2)*-sg-continuous maps

Keywords: -bi topological Spaces, Contra-(1,2)*-sg-continuous maps .

Introduction

The concept of closedness is fundamental with respect to the investigation of general topological spaces. Levine [5] initiated the study of the so-called g-closed sets and by doing this; he generalized the concept of closedness. Following this, in 1987, Bhattacharyya and Lahiri [1] introduced the notion of semi-generalized closed sets in topological spaces by means of semi-open sets of Levine [6]. In continuation of this work, in 1991, Sundaram et al [13] studied and investigated semi-generalized continuous maps and semi-$T_{½}$-spaces. Recently, Dontchev and Noiri [3] have defined the concept of contra-sg-continuity between topological spaces. Ravi and Lellis Thivagar [8,9,10,11,12] introduced a bitopological space in the year 2006. In this paper, an attempt is carried out to study the bitopological properties of contra-(1,2)*-sg-continuous maps.

PRELIMINARIES

Throughout this paper, (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) (briefly, X, Y and Z) will denote bitopological spaces.

Definition 2.1

Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open [11] if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$.
The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed.
Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Definition 2.2 [11]

Let S be a subset of a bitopological space X. Then

[1] the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap$ {F : $S \subseteq F$ and F is $\tau_{1,2}$-closed}.

[2] the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup$ {F : F $\subseteq$ S and F is $\tau_{1,2}$-open}.

Definition 2.3 [8, 9, 10, 11, 12]

A subset A of a bitopological space X is called

(1) (1,2)*-semi-open if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A));
(2) (1,2)*-preopen if A $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A));
(3) $\tau_{1,2}$-clopen if A is $\tau_{1,2}$-closed and $\tau_{1,2}$-open;
(4) regular (1,2)*-open if A = $\tau_{1,2}$-int($\tau_{1,2}$-cl(A));
(5) (1,2)*- ß-open if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A));
(6) $\tau_{1,2}$-δ-open if it is a union of regular (1,2)*-open sets.

The complement of (1,2)*-semi-open (resp.(1,2)*-preopen, regular (1,2)*-open) set is said to be (1,2)*-semi-closed (resp.(1,2)*-preclosed, regular (1,2)*-closed).

The (1,2)*-semi-closure of a subset A of X is, denoted by (1,2)*-scl(A), defined to be the intersection of all (1,2)*-semi-closed sets of X containing A.

Definition 2.4

A subset A of a bitopological space X is called (1) (1,2)*-semi-generalized closed (briefly (1,2)*-sg-closed) set [9] if (1,2)*-scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-semi-open in X. (2) (1,2)*-generalized closed (briefly (1,2)*-g-closed) set [8] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.

The complement of (1,2)*-sg-closed (resp. (1,2)*-g-closed) set is
called (1,2)*-sg-open (resp. (1,2)*-g-open).
If a subset A is (1,2)*-sg-closed in a space X, then A = (1,2)*-sgcl(A). The converse of this implication is not true in general.
The family of all (1,2)*-sg-open (resp. (1,2)*-sg-closed, $\tau_{1,2}$-closed) sets of X is denoted by (1,2)*-SGO(X) (resp. (1,2)*-SGC(X), (1,2)*-C(X)). The family of all (1,2)*-sg-open (resp. (1,2)*-sg-closed, $\tau_{1,2}$-closed) sets of X containing a point x $\in$ X is denoted by (1,2)*-SGO(X, x) (resp. (1,2)*-SGC(X, x), (1,2)*-C(X, x)).

Remark 2.5

Let X be a bitopological space. Then
(i) Every (1,2)*-semi-closed set of X is (1,2)*-sg-closed in X, but not conversely. [2]
(ii) Every $\tau_{1,2}$-closed set of X is (1,2)*-sg-closed in X, but not conversely. [2]

CONTRA (1,2)*-sg-CONTINUOUS MAPS

Definition 3.1

A map f : X $\rightarrow$ Y is called :
(i) contra-(1,2)*-continuous if f^{-1}(V) is $\tau_{1,2}$-closed in X for each $\sigma_{1,2}$-open set V in Y;
(ii) contra-(1,2)*-semi-continuous if f^{-1}(V) is (1,2)*-semi-closed in X for each $\sigma_{1,2}$-open set V in Y;
(iii) contra-(1,2)*- sg-continous if f^{-1}(V) is (1,2)*-sg-closed in X for each $\sigma_{1,2}$-open set V in Y;
(iv) (1,2)*-sg-continous if f^{-1}(V) is (1,2)*-sg-closed in X for each $\sigma_{1,2}$-closed set V in Y;
(v) (1,2)*-sg-irresolute if f^{-1}(V) is (1,2)*-sg-closed in X for each (1,2)*-sg-closed set V in Y;
(vi) (1,2)*-preclosed if f(V) is (1,2)*-preclosed in Y for each $\tau_{1,2}$-closed set V in X;
(vii) (1,2)*-irresolute if f^{-1}(V) is (1,2)*-semi-closed in X for each (1,2)*-semi-closed set V in Y;
(viii) contra-(1,2)*- g-continous if f^{-1}(V) is (1,2)*-g-closed in X for each $\sigma_{1,2}$-open set V in Y;

Definition 3.2

A bitopological space X is called

(i) (1,2)*- locally indiscrete if each $\tau_{1,2}$-open subset of X is $\tau_{1,2}$-closed in X;

(ii) (1,2)*-semi-$T_{½}$-space if each (1,2)*-sg-closed subset of X is (1,2)*-semi-closed in X;

(iii) (1,2)*-sg-connected if X cannot be written as a disjoint union of two non-empty (1,2)*-sg-open sets;

(iv) (1,2)*-ultra normal if each pair of non-empty disjoint $\tau_{1,2}$-closed sets can be separated by disjoint $\tau_{1,2}$-clopen sets;

(v) weakly (1,2)*-Hausdorff if each element of X is an intersection of regular (1,2)*-closed sets;

(vi) ultra (1,2)*-Hausdorff if for each pair of distinct points x and y in X, there exist $\tau_{1,2}$-clopen sets A and B containing x and y, respectively, such that $A \cap B = \phi$.

Let S be a subset of a bitopological space X. The set $\cap\{U \in (1,2)^*\text{-}O(X) : S \subseteq U\}$ is called the $\tau_{1,2}$-kernel of S and is denoted by $\tau_{1,2}$-ker(S). (1,2)*-O(X) denotes the family of all $\tau_{1,2}$-open sets of X.

Lemma 3.3

The following properties hold for the subsets U, V of a bitopological space X.

(i) $x \in \tau_{1,2}$-ker(U) if and only if $U \cap F \neq \phi$ for any $\tau_{1,2}$-closed set F containing x.

(ii) $U \subseteq \tau_{1,2}$-ker(U) and $U = \tau_{1,2}$-ker(U) if U is $\tau_{1,2}$-open in X.

(iii) $U \subseteq V$, then $\tau_{1,2}$-ker(U) $\subseteq \tau_{1,2}$-ker(V).

Remark 3.4

From the definitions we stated above, we observe that

(i) Every contra-(1,2)*-continuous map is contra-(1,2)*-sg-continuous.

(ii) Every contra-(1,2)*-semi-continuous map is contra-(1,2)*-sg-continuous.

However the separate converses of the above relations are not true from the following examples.

Example 3.5

Let X = Y = {a, b, c}, $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, Y, \{a\}\}$. Define $f : X \to Y$ as f(a) = c; f(b) = a; f(c) = b. Clearly f is contra-(1,2)*- sg-continuous map but it is not contra-(1,2)*-continuous.

Example 3.6

Let X = Y = {a, b, c}, $\tau_1 = \{\phi, X, \{a\}, \{b, c\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, Y, \{a\}\}$. Define $f : X \to Y$ as f(a) = b; f(b) = c; f(c) = a. Clearly f is contra-(1,2)*- sg-continuous map but it is not contra-(1,2)*-semi-continuous.

Theorem 3.7

Let $f : X \to Y$ be a map. The following statements are equivalent.

(i) f is contra-(1,2)*- sg-continuous.

(ii) The inverse image of each $\sigma_{1,2}$-closed set in Y is (1,2)*-sg-open in X.

Proof

Let G be a $\sigma_{1,2}$-closed set in Y. Then Y\G is an $\sigma_{1,2}$-open set in Y. By the assumption of (i), $f^{-1}(Y\backslash G) = X\backslash f^{-1}(G)$ is (1,2)*-sg-closed in X. It implies that $f^{-1}(G)$ is (1,2)*-sg-open in X. Converse is similar.

Theorem 3.8

Suppose that (1,2)*-SGC(X) is closed under arbitrary intersections. Then the following are equivalent for a map $f : X \to Y$.

(i) f is contra-(1,2)*-sg-continuous.

(ii) the inverse image of every $\sigma_{1,2}$-closed set of Y is (1,2)*-sg-open in X.

(iii) For each $x \in X$ and each $\sigma_{1,2}$-closed set B in Y with $f(x) \in B$, there exists an (1,2)*-sg-open set A in X such that $x \in A$ and $f(A) \subseteq B$.

(iv) $f((1,2)^*\text{-sgcl}(A)) \subseteq \sigma_{1,2}$-ker(f(A)) for every subset A of X.

(v) $(1,2)^*\text{-sgcl}(f^{-1}(B)) \subseteq f^{-1}(\sigma_{1,2}$-ker B) for every subset B of Y.

Proof

(i) $\Rightarrow$ (iii): Let $x \in X$ and B be a $\sigma_{1,2}$-closed set in Y with $f(x) \in B$. By (i), it follows that $f^{-1}(Y\backslash B) = X\backslash f^{-1}(B)$ is (1,2)*-sg-closed and so $f^{-1}(B)$ is (1,2)*-sg-open. Take $A = f^{-1}(B)$. We obtain that $x \in A$ and $f(A) \subseteq B$.
(iii) $\Rightarrow$ (ii): Let B be a $\sigma_{1,2}$-closed set in Y with $x \in f^{-1}(B)$. Since $f(x) \in B$, by (iii) there exists an (1,2)*-sg-open set A in X containing x such that $f(A) \subseteq B$. It follows that $x \in A \subseteq f^{-1}(B)$. Hence $f^{-1}(B)$ is (1,2)*-sg-open.
(ii) $\Rightarrow$ (i): Follows from the previous theorem.
(ii) $\Rightarrow$ (iv): Let A be any subset of X. Let $y \notin \sigma_{1,2}$-ker(f(A)). Then there exists a $\sigma_{1,2}$-closed set F containing y such that $f(A) \cap F = \phi$. Hence, we have $A \cap f^{-1}(F) = \phi$ and (1,2)*-sgcl(A) $\cap f^{-1}(F) = \phi$. Hence we obtain f((1,2)*-sgcl(A)) $\cap F = \phi$ and $y \notin$ f((1,2)*-sgcl(A)). Thus, f((1,2)*-sgcl(A)) $\subseteq \sigma_{1,2}$-ker(f(A)).
(iv) $\Rightarrow$ (v): Let B be any subset of Y. By (iv), f((1,2)*-sgcl(f^{-1}(B))) $\subseteq \sigma_{1,2}$-ker(B) and (1,2)*-sgcl(f^{-1}(B)) $\subseteq f^{-1}(\sigma_{1,2}$-ker(B)).
(v) $\Rightarrow$ (i): Let B be any $\sigma_{1,2}$-open set of Y. By (v), (1,2)*-sgcl(f^{-1}(B)) $\subseteq f^{-1}(\sigma_{1,2}$-ker(B)) = f^{-1}(B) and (1,2)*-sgcl(f^{-1}(B)) = f^{-1}(B). We obtain that f^{-1}(B) is (1,2)*-sg-closed in X.

Theorem 3.9

Let $f : X \to Y$ be a map and $g : X \to X \times Y$ the graph function of f, defined by g(x) = (x, f(x)) for every $x \in X$. If g is contra-(1,2)*-sg-continuous, then f is contra-(1,2)*-sg-continuous.

Proof

Let U be an $\sigma_{1,2}$-open set in Y. Then $X \times U$ is an (1,2)*-open set in $X \times Y$. It follows that $f^{-1}(U) = g^{-1}(X \times U)$ is (1,2)*-sg-closed in X. Thus, f is contra-(1,2)*-sg-continuous.

For a map $f : X \to Y$, the subset $\{(x, f(x)) : x \in X\} \subseteq X \times Y$ is called the graph of f and is denoted by G(f).

Definition 3.10

The graph G(f) of a map $f : X \to Y$ is said to be contra-(1,2)*-sg-graph if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exist an (1,2)*-sg-open set U in X containing x and a $\sigma_{1,2}$-closed set V in Y containing y such that $(U \times V) \cap G(f) = \phi$.

Proposition 3.11

The following properties are equivalent for the graph G(f) of a map f:

(i) G(f) is contra-(1,2)*-sg-graph.

(ii) For each $(x, y) \in (X \times Y)\backslash G(f)$, there exist an (1,2)*-sg-open set U in X containing x and a $\sigma_{1,2}$-closed V in Y containing y such that $f(U) \cap V = \phi$

Theorem 3.12

If $f : X \to Y$ is contra-(1,2)*-sg-continuous and Y is (1,2)*-Urysohn, G(f) is contra-(1,2)*- sg-graph in $X \times Y$.

Proof

Let $(x, y) \in (X \times Y)\backslash G(f)$. It follows that $f(x) \neq y$. Since Y is (1,2)*-Urysohn, there exist $\sigma_{1,2}$-open sets B and C such that $f(x) \in B$, $y \in C$ and $\sigma_{1,2}$-cl(B) $\cap$ $\sigma_{1,2}$-cl(C) = ϕ. Since f is contra-(1,2)*-sg-continuous, there exists an (1,2)*-sg-open set A in X containing x such that $f(A) \subseteq \sigma_{1,2}$-cl(B). Therefore $f(A) \cap \sigma_{1,2}$-cl(C) = ϕ and G(f) is contra-(1,2)*-sg-graph in $X \times Y$.

Theorem 3.13

Let $\{Xi / i \in I\}$ be any family of bitopological spaces. If $f : X \to \Pi Xi$ is a contra-(1,2)*- sg-continuous map, then Pri o $f : X \to Xi$ is contra-(1,2)*-sg-continuous for each $i \in I$, where Pri is the projection of ΠX_i onto X_i.

Proof

We shall consider a fixed $i \in I$. Suppose U_i is an arbitrary $\tau_{1,2}$-open set of X_i. Since Pr_i is (1,2)*-continuous, $Pr_i^{-1}(U_i)$ is $\sigma_{1,2}$-open in ΠX_i. Since f is contra-(1,2)*-sg-continuous, we have by definition, $f^{-1}(Pr_i^{-1}(U_i)) = (Pr_i \circ f)^{-1}(U_i)$ is (1,2)*-sg-closed in X. Therefore Pr_i o f is contra-(1,2)*- sg-continuous.

Definition 3.14

A bitopological space X is said to be (1,2)*-sg-$T_{½}$ space if every (1,2)*-sg-closed set of X is $\tau_{1,2}$-closed in X.

Lemma 3.15

Let X be a bitopological space. Then (1,2)*-sg-τ = {U $\subseteq$ X : (1,2)*-sgcl(X\U) = X\U} is a topology for X.

Theorem 3.16

Let X be a bitopological space. Then every (1,2)*-sg-closed set is $\tau_{1,2}$-closed if and only if (1,2)*-sg-τ = τ.

Proof

Let A $\in$ (1,2)*-sg-τ. Then (1,2)*-sgcl(X\A) = X\A. By hypothesis, $\tau_{1,2}$-cl(X\A) = (1,2)*-sgcl(X\A) = X\A and A $\in$ τ. Conversely, let A be a (1,2)*-sg-closed set. Then (1,2)*-sgcl(A) = A and hence X\A $\in$ (1,2)*-sg-τ = τ. Hence, A is $\tau_{1,2}$-closed.

Theorem 3.17

Let f : X $\rightarrow$ Y be a map. Suppose that X is a (1,2)*-sg-$T_{½}$ space. Then the following are equivalent.

(i) f is contra-(1,2)*-sg-continuous .
(ii) f is contra-(1,2)*-semi-continuous.
(iii) f is contra-(1,2)*-continuous.

Proof

The proof is obvious.

Definition 3.18

A bitopological space X is said to be locally (1,2)*-sg-indiscrete if every (1,2)*-sg-open set of X is $\tau_{1,2}$-closed in X.

Theorem 3.19

If f : X $\rightarrow$ Y is contra-(1,2)*-sg-continuous with X as locally (1,2)*-sg-indiscrete, then f is (1,2)*-continuous.

Proof

Omitted.

Theorem 3.20

If f : X $\rightarrow$ Y is contra-(1,2)*-sg-continuous and X is (1,2)*-sg-$T_{½}$ space, then f is contra-(1,2)*-continuous.

Proof

Omitted.

Theorem 3.21

If f : X $\rightarrow$ Y is a surjective (1,2)*-preclosed contra-(1,2)*-sg-continuous with X as (1,2)*-sg-$T_{½}$ space, then Y is (1,2)*-locally indiscrete.

Proof

Suppose that V is $\sigma_{1,2}$-open in Y. Since f is contra-(1,2)*-sg-continuous, $f^{-1}(V)$ is (1,2)*-sg-closed in X. Since X is a (1,2)*-sg-$T_{½}$ space, $f^{-1}(V)$ is $\tau_{1,2}$-closed in X. Since f is (1,2)*-preclosed, then V is (1,2)*-preclosed in Y. Now we have $\sigma_{1,2}$-cl(V) = $\sigma_{1,2}$-cl($\sigma_{1,2}$-int(V)) $\subseteq$ V. This means V is $\sigma_{1,2}$-closed in Y and hence Y is (1,2)*-locally indiscrete.

Theorem 3.22

Suppose that X and Y are bitopological spaces and (1,2)*-SGO(X) is closed under arbitrary unions. If a map f : X $\rightarrow$ Y is contra-(1,2)*-sg-continuous and Y is (1,2)*-regular, then f is (1,2)*-sg-continuous.

Proof

Let x be an arbitrary point of X and V be an $\sigma_{1,2}$-open set of Y containing f(x). Since Y is (1,2)*-regular, there exists an $\sigma_{1,2}$-open set G in Y containing f(x) such that $\sigma_{1,2}$-cl(G) $\subseteq$ V. Since f is contra-(1,2)*-sg-continuous, there exists U $\in$ (1,2)*-SGO(X) containing x such that f(U) $\subseteq$ $\sigma_{1,2}$-cl(G). Then f(U) $\subseteq$ $\sigma_{1,2}$-cl(G) $\subseteq$ V. Hence f is (1,2)*-sg-continuous.

Theorem 3.23

A contra-(1,2)*-sg-continuous image of a (1,2)*-sg-connected space is (1,2)*-connected.

Proof

Let f : X $\rightarrow$ Y be a contra-(1,2)*-sg-continuous map of a (1,2)*-sg-connected space X onto a bitopological space Y. If possible, let Y be (1,2)*-disconnected. Let A and B form a (1,2)*-disconnection of Y. Then A and B are $\sigma_{1,2}$-clopen and Y = A$\cup$B where A$\cap$B = ϕ. Since f is a contra-(1,2)*-sg-continuous map, X = $f^{-1}(Y)$ = $f^{-1}(A\cup B)$ = $f^{-1}(A) \cup f^{-1}(B)$, where $f^{-1}(A)$ and $f^{-1}(B)$ are non-empty (1,2)*-sg-open sets in X.

Also $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence X is not (1,2)*-sg-connected. This is a contradiction. Therefore Y is (1,2)*-connected.

Theorem 3.24

Let X be (1,2)*-sg-connected. Then each contra-(1,2)*-sg-continuous map of X into a (1,2)*-discrete space Y with atleast two points is a constant map.

Proof

Let $f : X \rightarrow Y$ be a contra-(1,2)*-sg-continuous map. Then X is covered by (1,2)*-sg-open and (1,2)*-sg-closed covering $\{f^{-1}(\{y\}) : y \in Y\}$. By assumption $f^{-1}(\{y\}) = \phi$ or X for each $y \in Y$. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, then f fails to be a map. Then there exists only one point $y \in Y$ such that $f^{-1}(\{y\}) \neq \phi$ and hence $f^{-1}(\{y\}) = X$ which shows that f is a constant map.

Theorem 3.25

If f is a contra-(1,2)*-sg-continuous map from a (1,2)*-sg-connected space X onto any bitopological space Y, then Y is not a (1,2)*-discrete space.

Proof

Suppose that Y is (1,2)*-discrete. Let A be a proper nonempty $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed subset of Y. Then $f^{-1}(A)$ is a proper nonempty (1,2)*-sg-open and (1,2)*-sg-closed subset of X, which is a contradiction to the fact that X is (1,2)*-sg-connected.

Definition 3.26

A bitopological space X is said to be (1,2)*-sg-normal if each pair of non-empty disjoint $\tau_{1,2}$-closed sets can be separated by disjoint (1,2)*-sg-open sets.

Theorem 3.27

If $f : X \rightarrow Y$ is a contra-(1,2)*-sg-continuous, (1,2)*-closed, injection and Y is (1,2)*-ultra normal, then X is (1,2)*-sg-normal.

Proof

Let F_1 and F_2 be disjoint $\tau_{1,2}$-closed subsets of X. Since f is (1,2)*-closed and injective, $f(F_1)$ and $f(F_2)$ are disjoint $\sigma_{1,2}$-closed subsets of Y. Since Y is (1,2)*-ultra normal, $f(F_1)$ and $f(F_2)$ are separated by disjoint $\sigma_{1,2}$-clopen sets V_1 and V_2 respectively. Hence $F_i \subseteq f^{-1}(V_i)$, $f^{-1}(V_i)$ is (1,2)*-sg-open in X for i = 1, 2 and $f^{-1}(V_1) \cap f^{-1}(V_2) = \phi$. Thus, X is (1,2)*-sg-normal.

Theorem 3.28

If $f : X \rightarrow Y$ is contra-(1,2)*-sg-continuous map and X is a (1,2)*-semi-$T_{½}$ space, then f is contra-(1,2)*-semi-continuous.

Proof

Omitted.

4 COMPOSITION OF MAPS

Theorem 4.1

The composition of two contra-(1,2)*-sg-continuous maps need not be contra-(1,2)*-sg-continuous.

The following example supports the above theorem.

Example 4.2

Let X = Y = Z = {a, b, c}, $\tau_1 = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$, $\tau_2 = \{\phi, X, \{a\}\}$, $\sigma_1 = \{\phi, Y\}$, $\sigma_2 = \{\phi, Y, \{a\}, \{b, c\}\}$, $\eta_1 = \{\phi, Z\}$ and $\eta_2 = \{\phi, Z, \{a, b\}\}$. Then the identity map $f : X \rightarrow Y$ is contra-(1,2)*-sg-continuous and the identity map $g : Y \rightarrow Z$ is contra-(1,2)*-sg-continuous. But their composition $g \circ f : X \rightarrow Z$ is not contra-(1,2)*-sg-continuous.

Theorem 4.3

Let X and Z be any bitopological spaces and Y be a (1,2)*-semi-$T_{½}$ space. Let $f : X \rightarrow Y$ be an (1,2)*-irresolute map and $g : Y \rightarrow Z$ be a contra-(1,2)*-sg-continuous map. Then $g \circ f : X \rightarrow Z$ is contra-(1,2)*-semi-continuous map.

Proof

Let F be any $\eta_{1,2}$-open set in Z. Since g is contra-sg-continuous, $g^{-1}(F)$ is (1,2)*-sg-closed in Y. But Y is (1,2)*-semi-$T_{½}$ space. Therefore $g^{-1}(F)$ is (1,2)*-semi-closed in Y. Since f is (1,2)*-irresolute, $f^{-1}(g^{-1}(F)) = (g \circ f)^{-1}(F)$ is (1,2)*-semi-closed in X. Thus, g o f is contra-(1,2)*-semi-continuous.

Theorem 4.4
Let f : X $\rightarrow$ Y be (1,2)*-sg-irresolute map and g : Y $\rightarrow$ Z be contra-(1,2)*-sg-continuous map. Then g o f : X $\rightarrow$ Z is contra-(1,2)*-sg-continuous.
Proof
Let F be an $\eta_{1,2}$-open set in Z. Then $g^{-1}(F)$ is (1,2)*-sg-closed in Y because g is contra-(1,2)*-sg-continuous. Since f is (1,2)*-sg-irresolute, $f^{-1}(g^{-1}(F)) = (g \text{ o } f)^{-1}(F)$ is (1,2)*-sg-closed in X. Therefore g o f is contra-(1,2)*-sg-continuous.
Corollary 4.5
Let f : X $\rightarrow$ Y be (1,2)*-sg-irresolute map and g : Y $\rightarrow$ Z be contra-(1,2)*-continuous map. Then g o f : X $\rightarrow$ Z is contra-(1,2)*-sg-continuous.
Definition 4.6
A map f : X $\rightarrow$ Y is said to be pre (1,2)*-sg-open if the image of every (1,2)*-sg-open subset of X is (1,2)*-sg-open in Y.
Theorem 4.7
Let f : X $\rightarrow$ Y be surjective (1,2)*-sg-irresolute pre (1,2)*-sg-open and g : Y $\rightarrow$ Z be any map. Then g o f : X $\rightarrow$ Z is contra-(1,2)*-sg-continuous if and only if g is contra-(1,2)*-sg-continuous.
Proof
The 'if' part is easy to prove. To prove the 'only if' part, let g o f : X $\rightarrow$ Z be contra-(1,2)*-sg-continuous and let F be a $\eta_{1,2}$-closed subset of Z. Then $(g \text{ o } f)^{-1}(F)$ is a (1,2)*-sg-open subset of X. That is $f^{-1}(g^{-1}(F))$ is (1,2)*-sg-open. Since f is pre (1,2)*-sg-open, $f(f^{-1}(g^{-1}(F)))$ is a (1,2)*-sg-open subset of Y. So, $g^{-1}(F)$ is (1,2)*-sg-open in Y. Hence g is contra-(1,2)*-sg-continuous.
Theorem 4.8
If f : X $\rightarrow$ Y is (1,2)*-sg-irresolute map with Y as locally (1,2)*-sg-indiscrete space and g : Y $\rightarrow$ Z is contra-(1,2)*-sg-continuous map, then g o f : X $\rightarrow$ Z is (1,2)*-sg-continuous.
Proof
Let F be any $\eta_{1,2}$-closed set in Z. Since g is contra-(1,2)*-sg-continuous, $g^{-1}(F)$ is (1,2)*-sg-open set in Y. But Y is locally (1,2)*-sg-indiscrete, $g^{-1}(F)$ is $\sigma_{1,2}$-closed in Y. Hence $g^{-1}(F)$ is (1,2)*-sg-closed set in Y. Since f is (1,2)*-sg-irresolute, $f^{-1}(g^{-1}(F)) = (g \text{ o } f)^{-1}(F)$ is (1,2)*-sg-closed in X. Therefore g o f is (1,2)*-sg-continuous.

SOME NEW SEPARATION AXIOMS

Definition 5.1
A bitopological space X is said to be:
(i) (1,2)*-sg-compact (strongly (1,2)*-S-closed) if every (1,2)*-sg-open ((1,2)*-closed) cover of X has a finite subcover;
(ii) countably (1,2)*-sg-compact (strongly countably (1,2)*-S-closed) if every countable cover of X by (1,2)*-sg-open (resp. $\tau_{1,2}$-closed) sets has a finite subcover;
(iii) (1,2)*-sg-Lindelöf (strongly (1,2)*-S-Lindelöf) if every (1,2)*-sg-open ((1,2)*-closed) cover of X has a countable subcover.
Theorem 5.2
The surjective contra-(1,2)*-sg-continuous images of (1,2)*-sg-compact (resp. (1,2)*-sg-Lindelöf, countably (1,2)*-sg-compact) spaces are strongly (1,2)*-S-closed (resp. strongly (1,2)*-S-Lindelöf, strongly countably (1,2)*-S-closed).
Proof
Suppose that f : X $\rightarrow$ Y is a contra-(1,2)*-sg-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any $\sigma_{1,2}$-closed cover of Y. Since f is contra-(1,2)*-sg-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is an (1,2)*-sg-open cover of X and hence there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and Y is strongly (1,2)*-S-closed.
The other proofs can be obtained similarly.

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Definition 5.3

A bitopological space X is said to be :

(i) (1,2)*-sg-closed-compact if every (1,2)*-sg-closed cover of X has a finite subcover;

(ii) countably (1,2)*-sg-closed-compact if every countable cover of X by (1,2)*-sg-closed sets has a finite subcover;

(iii) (1,2)*-sg-closed-Lindelöf if every (1,2)*-sg-closed cover of X has a countable subcover.

Theorem 5.4

The surjective contra-(1,2)*-sg-continuous images of (1,2)*-sg-closed-compact (resp. (1,2)*-sg-closed-Lindelöf, countably (1,2)*-sg-closed-compact) spaces are (1,2)*-compact (resp. (1,2)*-Lindelöf, countably (1,2)*-compact).

Proof

Suppose that $f : X \rightarrow Y$ is a contra-(1,2)*-sg-continuous surjection. Let $\{V_\alpha : \alpha \in I\}$ be any (1,2)*-open cover of Y. Since f is contra-(1,2)*-sg-continuous, then $\{f^{-1}(V_\alpha) : \alpha \in I\}$ is a (1,2)*-sg-closed cover of X. Since X is (1,2)*-sg-closed-compact, there exists a finite subset I_0 of I such that $X = \cup\{f^{-1}(V_\alpha) : \alpha \in I_0\}$. Therefore, we have $Y = \cup\{V_\alpha : \alpha \in I_0\}$ and Y is (1,2)*-compact.

The other proofs can be obtained similarly.

Definition 5.5

A bitopological space X is said to be (1,2)*-sg-T_1 iff for each pair of distinct points x and y in X, these exist (1,2)*-sg-open sets U and V containing x and y, respectively, such that $y \notin U$ and $x \notin V$.

Definition 5.6

A bitopological space X is said to be (1,2)*-sg-T_2 iff for each pair of distinct points x and y in X, there exist $U \in$ (1,2)*-SGO(X, x) and $V \in$ (1,2)*-SGO(X, y) such that $U \cap V = \phi$.

Theorem 5.7

Let X and Y be bitopological spaces. If for each pair of distinct points x and y in X, there exists a map f of X into Y such that $f(x) \neq f(y)$, Y is an (1,2)*-Urysohn space and f is contra-(1,2)*-sg-continuous at x and y, then X is (1,2)*-sg-T_2.

Proof

Let x and y be any distinct points in X. Then, there exists an (1,2)*-Urysohn space Y and a map $f : X \rightarrow Y$ such that $f(x) \neq f(y)$ and f is contra-(1,2)*-sg-continuous at x and y. Let $z = f(x)$ and $v = f(y)$. Then $z \neq v$. Since Y is (1,2)*-Urysohn, there exist $\sigma_{1,2}$-open sets V and W containing z and v, respectively, such that $\sigma_{1,2}\text{-cl}(V) \cap \sigma_{1,2}\text{-cl}(W) = \phi$. Since f is contra-(1,2)*-sg-continuous at x and y, then there exist (1,2)*-sg-open sets A and B containing x and y, respectively, such that $f(A) \subseteq \sigma_{1,2}\text{-cl}(V)$ and $f(B) \subseteq \sigma_{1,2}\text{-cl}(W)$. We have $A \cap B = \phi$ since $\sigma_{1,2}\text{-cl}(V) \cap \sigma_{1,2}\text{-cl}(W) = \phi$. Hence, X is (1,2)*-sg-$T_2$.

Corollary 5.8

Let $f : X \rightarrow Y$ is a contra-(1,2)*-sg-continuous injection. If Y is an (1,2)*-Urysohn space, then (1,2)*-sg-T_2.

Theorem 5.9

If $f : X \rightarrow Y$ is a contra-(1,2)*-sg-continuous injection and Y is weakly (1,2)*-Hausdorff, then X is (1,2)*-sg-T_1.

Proof

Suppose that Y is weakly (1,2)*-Hausdorff. For any distinct points x and y in X, there exist regular (1,2)*-closed sets A, B in Y such that $f(x) \in A$, $f(y) \notin A$, $f(x) \notin B$ and $f(y) \in B$. Since f is contra-(1,2)*-sg-continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are (1,2)*-sg-open subsets of X such that $x \in f^{-1}(A)$, $y \notin f^{-1}(A)$, $x \in f^{-1}(B)$ and $y \in f^{-1}(B)$. This shows that X is (1,2)*-sg-T_1.

Theorem 5.10

Let $f : X \rightarrow Y$ have a contra-(1,2)*-sg-graph. If f is injective, then X is (1,2)*-sg-T_1.

Proof

Let x and y be any two distinct points in X. Then, we have $(x, f(y)) \in (X \times Y)\backslash G(f)$. Then, there exist an (1,2)*-sg-open set U in X containing x and $F \in C(Y, f(y))$ such that $f(U) \cap F = \phi$; hence $U \cap f^{-1}(F) = \phi$. Therefore, we have $y \notin U$. This implies that X is (1,2)*-sg-T_1.

Theorem 5.11

Let f : X → Y be a contra-(1,2)*-sg-continuous injection. If Y is an ultra (1,2)*-Hausdorff space, then X is (1,2)*-sg-T_2.

Proof

Let x and y be any two distinct point in X. Then, f(x) ≠ f(y) and there exist $\sigma_{1,2}$-clopen sets A and B containing f(x) and f(y), respectively, such that A ∩ B = ϕ. Since f is contra-(1,2)*-sg-continuous, then $f^{-1}(A) \in$ (1,2)*-SGO(X) and $f^{-1}(B) \in$ (1,2)*-SGO(X) such that $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence, X is (1,2)*-sg-$T_2$.

Definition 5.12

A map f : X → Y is said to be :

(i) perfectly (1,2)*-continuous if $f^{-1}(V)$ is $\tau_{1,2}$-clopen in X for every $\sigma_{1,2}$-open set V of Y;

(ii) RC-(1,2)*-continuous if $f^{-1}(V)$ is regular (1,2)*-closed in X for each $\sigma_{1,2}$-open set V of Y;

(iii) Strongly (1,2)*-continuous if the inverse image of every set in Y is $\tau_{1,2}$-clopen in X;

(iv) Contra-(1,2)*-R-map if $f^{-1}(V)$ is regular (1,2)*-closed in X for every regular (1,2)*-open set V of Y;

(v) Contra-(1,2)*-super-continuous if for each x ∈ X and each F ∈ C(Y, f(x)), there exists a regular (1,2)*-open set U in X containing x such that f(U) ⊆ F.

(vi) Almost contra-(1,2)*-super-continuous if $f^{-1}(V)$ is $\tau_{1,2}$-δ-closed in X for every regular (1,2)*-open set V of Y;

(vii) Regular (1,2)*-set-connected if $f^{-1}(V)$ is $\tau_{1,2}$-clopen in X for every regular (1,2)*-open set V in Y;

(viii) Almost (1,2)*-s-continuous if for each x ∈ X and each V ∈ (1,2)*-SO(Y, f(x)), there exists an $\tau_{1,2}$-open set U in X containing x such that f(U) ⊆ (1,2)*-scl(V);

(ix) (Θ, s)-(1,2)*-continuous if for each x ∈ X and each V ∈ (1,2)*-SO(Y, f(x)), there exists an $\tau_{1,2}$-open set U in X containing x such that f(U) ⊆ $\sigma_{1,2}$-cl(V).

Remark 5.13

The following diagram holds for a map f : X → Y :

Strongly (1,2)*-continuous	⇒	almost (1,2)*-s-continuous
Perfectly (1,2)*-continuous	⇒	regular (1,2)*-set-connected
RC-(1,2)*-continuous	⇒	contra-(1,2)*-R-map
Contra-(1,2)*-super-continuous	⇒	almost contra-(1,2)*-super-continuous
Contra-(1,2)*-continuous	⇒	(Θ, s)- (1,2)*-continuous
Contra-(1,2)*-semi-continuous	⇒	contra-(1,2)*-sg-continuous

Remark 5.14

None of these implications is reversible.

Remark 5.15

(Θ,s)-(1,2)*-continuity and contra-(1,2)*-sg-continuity are independent of each other. It may be seen by the following examples.

Example 5.16

Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {a, b}, {a, c}}, τ_2 = {ϕ, X, {a}}, σ_1 ={ϕ, X} and σ_2 = {ϕ, X, {a}, {a, b}}. Then the identity map f : (X, τ) → (X, σ) is (Θ, s)- (1,2)*-continuous map which is not contra-(1,2)*-sg-continuous.

Example 5.17

Let X = {a, b, c}, τ_1 = {ϕ, X, {b}, {c}, {b, c}}, τ_2 = {ϕ, X, {b}}, σ_1 ={ϕ, X} and σ_2 = {ϕ, X, {a}, {b}, {a, b}, {a, c}}. Then the identity map f : (X, τ) → (X, σ) is contra-(1,2)*-sg-continuous map which is not (Θ, s)-(1,2)*-continuous.

Definition 5.18

A map f : X → Y is called ß-(1,2)*-continuous if $f^{-1}(V)$ is (1,2)*-ß-open in X for every $\sigma_{1,2}$-open set V of Y.

Theorem 5.19

If X is (1,2)*-sg-$T_{½}$, then the following equivalent for a map f : X → Y :

(i) f is RC-(1,2)*-continuous.

(ii) f is ß-(1,2)*-continuous and contra-(1,2)*-sg-continuous.
(iii) f is ß-(1,2)*-continuous and contra-(1,2)*-g-continuous.
(iv) f is ß-(1,2)*-continuous and contra-(1,2)*-continuous.

Proof

Omitted.

Conclusion

General Topology has shown its fruitfulness in both the pure and applied directions. In data mining , computational topology for geometric design and molecular design, computer – aided geometric design and engineering design (briefly CAGD), digital topology, information systems, non-commutative geometry and its application to particle physics, one can observe the influence made in these realms of applied research by general topological spaces, properties and structures. Rosen and Peters have used topology as a body of mathematics that could unify diverse areas of CAGD and engineering design research. They have presented several examples of the application of topology to CAGD and design.

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