Nonlinear Dynamics and Vibration Analysis of Rotating Machinery

Vaishally Dogra
Asst. Professor, Department of Mechanical Engineering, Graphic Era Hill University, Dehradun Uttarakhand India,

Abstract
In a variety of engineering disciplines, including the mechanical, aerospace, and energy industries, the study of nonlinear dynamics and vibration analysis of rotating machinery has become increasingly important. In many applications, including turbines, engines, compressors, and pumps, rotating machinery is essential for both the performance and safety of the entire system. Nonlinearities, on the other hand, add intricate dynamic behaviours and vibration phenomena into these systems, which can result in subpar performance, excessive wear, and even catastrophic failures. The goal of this research is to improve rotating machinery's design, operation, and maintenance by examining the nonlinear dynamics and vibration properties. In order to comprehend the complex interplay between the dynamics of the rotor, the dynamics of the bearing, and the nonlinear forces, the analysis makes use of sophisticated mathematical models and computer tools. To appropriately depict the system's nonlinear behaviour, important factors including rotor imbalance, shaft misalignment, bearing clearance, and non-circular journal shapes are taken into account. The objective of the project is to create reliable techniques for locating and forecasting important vibration modes, resonances, and instabilities in rotating machinery. Techniques like nonlinear time series analysis, bifurcation analysis, and chaos theory are used in this. The results of this study can help in the creation of efficient vibration mitigation plans and condition monitoring tools as well as useful insights into the underlying physics of rotating machinery dynamics. This paper findings may help to increase the performance, reliability, and lifetime of rotating machinery, which would increase operational effectiveness and lower maintenance costs across a range of industrial sectors.

Keywords: Vibrations, condition monitoring, rotary machines, predictive maintenance, diagnostics

Introduction
Power generation, transportation, and manufacturing are just a few of the industrial operations that depend on rotating machinery. Due to different elements such rotor unbalance, shaft misalignment, bearing dynamics, and nonlinear forces, these devices, such as turbines, engines, compressors, and pumps, are vulnerable to complicated dynamic behaviours and vibration phenomena. For rotating machinery to operate reliably and safely, it is crucial to comprehend and analyse these nonlinear dynamics and vibrations [2]. Recent years have seen a substantial increase in interest in the subject of nonlinear dynamics and vibration analysis of rotating machinery. Traditional linear analytic techniques
fall short of capturing these systems' complex interactions and nonlinear behaviour. In extreme circumstances, nonlinear effects can result in catastrophic failures, excessive wear, performance deterioration, and increased energy consumption. Therefore, it is imperative to create cutting-edge methods and procedures that can precisely describe and forecast the dynamic behaviour of rotating machinery. Nonlinear time series analysis, bifurcation analysis, chaos theory, and other cutting-edge methodologies will be used in the investigation of the intricate behaviour of spinning machinery [1]. The results of this study can help with the detection and forecasting of important vibrational modes, resonances, and instability. In the end, the findings will help develop effective methods for minimising vibrations, enhancing operating effectiveness, and lowering maintenance costs in rotating equipment systems. The goal of both online and offline monitoring methods is to distinguish between abnormal changes brought on by defects and typical variations brought on by operating circumstances. Maintenance staff can decide on maintenance measures, such as repairs, component replacements, or modifications, maintaining optimal performance and averting unforeseen breakdowns, by properly differentiating between these two sorts of alterations [5].

For rotating machinery to operate reliably and effectively, it is crucial to understand its dynamic behaviour and vibrational properties. Vibration analysis and nonlinear dynamics are effective methods for examining and analysing the intricate behaviour of these systems. The study of systems with nonlinear relationships between inputs and outputs is referred to as nonlinear dynamics. Nonlinearity can occur in rotating equipment for a variety of reasons, including material characteristics, friction, clearances, and the presence of nonlinear forces. The overall effectiveness and stability of spinning machinery can be considerably impacted by nonlinear influences. On the other hand, vibration analysis focuses on the investigation of the oscillations and vibrations displayed by rotating machinery. Unbalanced forces, misalignment, structural resonance, bearing flaws, and other mechanical irregularities are some of the causes of vibrations. The health and condition of the spinning machinery can be better understood by analysing and monitoring these vibrations.

Engineers and researchers can develop a comprehensive understanding of the behaviour of rotating machinery under various operating situations by combining the study of nonlinear dynamics and vibration analysis. It is possible to optimise the design, boost performance, increase reliability, and save maintenance costs by looking at how the system reacts to outside stimuli, identifying crucial factors, and foreseeing future failures or instabilities. The complex behaviours displayed by rotating machinery, such as limit cycles, chaos, bifurcations, and resonance phenomena, can be recognised and described using nonlinear dynamics. Designing, operating, and maintaining systems depend on an understanding of these behaviours [7].

Nonlinear dynamics [8] offers techniques for stability analysis, which aids in determining the stability limits of rotating machinery. Engineers can identify safe operating zones using stability analysis, avoiding potentially dangerous instabilities. Vibration analysis allows for the identification and diagnosis of defects in rotating machinery. It is feasible to pinpoint specific fault signatures linked to bearing wear, imbalance, misalignment, and other typical problems by analysing vibration
Tools for studying and enhancing the behaviour of rotating machinery include nonlinear dynamics and vibration analysis. Engineers can improve the performance, longevity, and dependability of spinning machinery in a variety of industrial applications by studying complicated dynamics and vibrations.

I. Related Work

Researchers and engineers have paid close attention to the study of nonlinear dynamics and vibration analysis of rotating machinery. Numerous research have investigated different facets of this subject, including as theoretical modelling, experimental studies, and real-world applications [9]. The analysis and diagnosis of torsional vibrations in rotating machinery has drawn more attention in recent studies. Torsional vibrations, in contrast to lateral vibrations, which have been thoroughly investigated and standardised, offer special insights on the behaviour of the shaft that is imparting torque. One [18] benefit of torsional vibrations is that time-varying transmission routes, which can cause additional amplitude modulation (AM) effects in lateral vibrations, are not an issue for them. This enables analysing torsional vibrations in terms of their frequency contents easier.

Concentrating on the examination of torsional vibrations in a centrifugal pump shaft in a nuclear research reactor. They found that the pump shaft had torsional vibrations that were similar to the rotor's initial torsional mode. These torsional vibrations provided crucial details about the pump's operational state that were not visible through lateral vibrations [20].

Torsional vibrations were used in laboratory experiments by Chen and Feng [12] to identify faults in spinning machinery. The local problems in parts like sun, planet, and ring gears were successfully identified despite the time-variability of the running conditions. Torsional vibrations were successful in identifying these problems and giving important diagnostic data. Researchers modelled induced defects in torsional vibrations inside the resonance zone using amplitude modulation and frequency modulation (AM-FM) methods. They were able to characterise and examine flaws in the torsional vibrations using this method, which revealed information about the system's behaviour.

Overall, there are various benefits to using torsional vibrations to analyse and diagnose defects in rotating machinery [15]. These vibrations offer a clearer understanding of the shaft's role in torque transfer, and analysis is made easier by their simpler frequency contents. Researchers and engineers can learn more important details about the health and operation of the machinery by including torsional vibration analysis that may not be obvious from lateral vibrations alone.

Some significant instances of similar work are provided below:

The study of a flexible rotor-bearing system's nonlinear oil film forces and nonlinear nonlinear dynamics. Under various operating situations, they looked at the system's vibrational properties, stability zones, and bifurcation behaviour. The study focused on how nonlinearities affect rotating machinery's dynamic reaction. The author analysed chaotic vibrations in rotating equipment. To characterise the chaotic behaviour of a rotor system with imbalance and nonlinear bearing forces, they
created a mathematical model. The study emphasised how crucial it is to take chaotic vibrations into account while designing and maintaining rotating machinery[16].

Nonlinear vibration analysis to explore the defect diagnosis of rolling element bearings in rotating equipment. They suggested a technique for identifying and categorising bearing failures based on the correlation dimension and the greatest Lyapunov exponent [11]. The study proved that nonlinear vibration analysis is useful for locating defective bearings. The nonlinear dynamics of fractured rotors in rotating equipment was researched, they created a mathematical model that took into account how the dynamic behaviour of the rotor would be affected by a transverse surface crack. The study emphasised the significance of crack detection in protecting the integrity of rotating machinery by highlighting the variations in vibration characteristics brought on by the presence of cracks. The enormous study on nonlinear dynamics and vibration analysis of rotating machinery is only partially represented by these papers[7]. They illustrate the variety of uses for these techniques that have been put to good use, such as defect diagnosis, condition monitoring, and design optimisation. The performance and dependability of rotating machinery may one day be better understood and improved with continued research in this area [9].

II. Design methodology

A systematic approach is used in design techniques for nonlinear dynamics and vibration analysis of rotating machinery in order to maximise the effectiveness, stability, and dependability of these systems. A comparison table can give an overview of the many techniques, each of which has advantages and factors to be taken into account. A comparison table showing various important design techniques is provided below:

Table 1: Comparison between Different Methodology used for nonlinear dynamics and vibration analysis of rotating machinery

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Description</th>
<th>Advantages</th>
<th>Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modal Analysis</td>
<td>Determines mode shapes and natural frequencies.</td>
<td>- Provides insights into system behavior</td>
<td>- Assumes linear behavior</td>
</tr>
<tr>
<td></td>
<td>means of experimental or numerical methods.</td>
<td>- Useful for initial design</td>
<td>- Limited to linear modes</td>
</tr>
<tr>
<td></td>
<td>Captures crucial speeds and resonances.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Aids in the analysis of dynamic response.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finite Element</td>
<td>Employs numerical discretization of the structure</td>
<td>- Accounts for geometric and material nonlinearities</td>
<td>- Requires accurate modeling</td>
</tr>
<tr>
<td>Analysis</td>
<td>and solves equations of motion</td>
<td>- Allows for detailed analysis of complex systems</td>
<td>- Computational resources and time-consuming</td>
</tr>
<tr>
<td></td>
<td>- Captures nonlinear</td>
<td></td>
<td></td>
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</tbody>
</table>
It's significant to remember that the selection of the design technique depends on the particular goals, restrictions, and resources available. To get a thorough understanding of the system's behaviour and improve its design, engineers and researchers frequently combine approaches.

The modal decomposition theorem states that vibration frequencies can be determined through the linear superposition of vibration modes, which together make up the space base of the structure's deformed configurations [9]. As a result, a displacement vector can be written as follows in modal coordinates:

\[ \mathbf{u}(t) = \sum \phi_i(t) \mathbf{x}_i + \sum \frac{\partial \phi_i}{\partial t} \mathbf{x}_i \]

where \( \mathbf{u}(t) \) is the displacement vector, \( \phi_i(t) \) are the vibration modes, \( \mathbf{x}_i \) are the modal coordinates, and \( \frac{\partial \phi_i}{\partial t} \) are the time derivatives of the vibration modes.

<table>
<thead>
<tr>
<th>Technique</th>
<th>Description</th>
<th>Benefits</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic Balance</td>
<td>Decomposes the dynamic response into harmonic components</td>
<td>- Captures nonlinear effects and behavior</td>
<td>- Limited to periodic excitations</td>
</tr>
<tr>
<td>Analysis</td>
<td>Components and solves for steady-state response</td>
<td>- Efficient for analyzing systems with harmonic excitation</td>
<td>- Requires accurate nonlinear models</td>
</tr>
<tr>
<td>Time-Domain</td>
<td>Simulates the system response in the time domain</td>
<td>- Captures transient behavior and dynamic interactions</td>
<td>- Computational resources and time-consuming</td>
</tr>
<tr>
<td>Simulation</td>
<td>By solving equations of motion numerically</td>
<td>- Allows for detailed analysis of complex systems</td>
<td>- Requires accurate modeling</td>
</tr>
<tr>
<td>Experimental Testing</td>
<td>Conducts physical tests and measurements on rotating machinery</td>
<td>- Captures real-world behavior and system response</td>
<td>- Limited control over operating conditions</td>
</tr>
<tr>
<td>- Provides insights into stability and response</td>
<td>- Validates numerical models and simulations</td>
<td>- Costly and time-consuming</td>
<td></td>
</tr>
<tr>
<td>- Can handle nonlinearities and system complexity</td>
<td>- Provides data for model calibration and validation</td>
<td>- May have limitations in capturing certain dynamic features</td>
<td></td>
</tr>
</tbody>
</table>
\[ x = XN_i = 1 \phi \Phi = \Phi q \] \hspace{1cm} (1)

Where, \( \phi_i \) are the modes.

The displacement detected on the top floor is the system's sole output, as was previously mentioned. The complex conjugate poles, or eigenvalues, of matrix A can be used to define the system's behavior. You can describe these eigenvalues in terms of natural frequencies and damping coefficients. The poles of the system's transfer function in the setting of linear dynamic systems are represented by the eigenvalues of matrix A. The system's responsiveness and stability properties are determined by the poles. Each pole of a system with complex conjugate poles can be written as:

\[ y = \{0 \cdots 01\}1 \times n 01 \times n \{z\} C2 x(t) \] \hspace{1cm} (2)

where \( \beta \) denotes the imaginary portion of the pole and \( \alpha \) denotes the real portion of the pole. Complex conjugate poles have identical real and imaginary portions, although the imaginary parts have the opposite signs. The system's oscillation frequency (\( \omega \)) is represented by the natural frequency (\( \omega \)), and the decay rate of those oscillations over time is shown by the damping factor (\( \zeta \)). These variables shed light on the system's dynamic behavior and stability. Engineers and scientists can identify the system's natural frequencies and damping factors by analyzing the complicated conjugate poles. These elements are essential for comprehending and regulating the system's reaction. These variables are crucial for the system's performance design and optimization, as well as for evaluating the stability and reaction to various inputs.

III. Model For Rotor-Bearing System

Think about a six-pole Permanent Magnet Bearing (PMB) Homopolar Magnetic Bearing (HoMB) system, which consists of back irons, two stator lamination packs, 12 permanent magnets, and electric coils on each pole. Six permanent magnets are positioned around each stator lamination pack in an even distribution, as shown in Figure 1.

![Permanent magnet-biased homopolar bearing with two active planes](image.jpg)

**Figure 1: Permanent magnet-biased homopolar bearing with two active planes**

Providing an axially and radially flowing magnetic bias flux is the permanent magnets' main purpose. This bias flux produces a response force that balances the bearing's static load. It enhances the magnetic bearing's overall stability and capacity for handling loads. Contrarily, the control flux, which
is created by the electric coils, is limited to radial and circumferential flow within the lamination stacks of the rotor and stator. This flux element is used to accurately control where the shaft is located within the clearance circle or gap of the magnetic bearing. The system can maintain the shaft at the desired target point by adjusting the control flux. The control flux is also in charge of producing the desired stiffness. The system can precisely regulate the position of the shaft by adjusting the control flux through the electric coils, resulting in stability and dynamic performance. The ability to regulate stiffness, damping, and other forces thanks to the control flux leads to increased performance characteristics and disturbance rejection capacities.

Figure 2: Homopolar magnetic bearing magnetic circuit model with permanent magnet and electric coil sources

The magnetic field in the six-pole PMB HoMB system is represented simply in the magnetic circuit model in Figure 2. Six separate fluxes, six gap and six pole reluctances, six current sources, and a lumped permanent magnet source with its reluctance are all present. The model's simplicity is maintained while still accurately capturing the system's behaviour and attributes.

The magnetic circuit model is shown on the left side of figure 2, and the right side is considered to be symmetrical and to function the same way with the same sources, parameter values, and fluxes. However, for the sake of simplicity, only a portion of the right side is depicted. The 3-dimensional magnetic field that actually exists in the real system is quite accurately modelled by the magnetic circuit model. It enables examination and comprehension of the system's behaviour and performance despite being greatly simplified. The fluxes are frequently de-rated in the model by a factor (normally between 0.75 and 0.9) to take leakage and fringing effects at air gaps into consideration. The proposed model, however, makes the assumption of unity de-rate components for demonstration reasons. In order to simplify the representation while keeping important aspects of the magnetic field distribution, the model focuses the material routes in the pole reluctance terms.

The following is a representation of the nonlinear algebraic equations for the six unknown fluxes in the magnetic circuit model of the six-pole PMB HoMB system:

\[
\Phi_1 = (R_{gap1} + R_{pole1}) \times I_1 + R_{gap2} \times I_2 + R_{gap3} \times I_3 + R_{gap4} \times I_4 \\
+ R_{gap5} \times I_5 + R_{gap6} \times I_6
\]
\[
\Phi_2 = R_{gap1} \times I_1 + (R_{gap2} + R_{pole2}) \times I_2 + R_{gap3} \times I_3 + R_{gap4} \times I_4 \\
+ R_{gap5} \times I_5 + R_{gap6} \times I_6
\]
\[
\Phi_3 = R_{gap1} \times I_1 + R_{gap2} \times I_2 + (R_{gap3} + R_{pole3}) \times I_3 + R_{gap4} \times I_4 \\
+ R_{gap5} \times I_5 + R_{gap6} \times I_6
\]
\[
\Phi_4 = R_{gap1} \times I_1 + R_{gap2} \times I_2 + R_{gap3} \times I_3 + (R_{gap4} + R_{pole4}) \times I_4 \\
+ R_{gap5} \times I_5 + R_{gap6} \times I_6
\]
\[
\Phi_5 = R_{gap1} \times I_1 + R_{gap2} \times I_2 + R_{gap3} \times I_3 + R_{gap4} \times I_4 + (R_{gap5} \\
+ R_{pole5}) \times I_5 + R_{gap6} \times I_6
\]
\[
\Phi_6 = R_{gap1} \times I_1 + R_{gap2} \times I_2 + R_{gap3} \times I_3 + R_{gap4} \times I_4 + R_{gap5} \times I_5 \\
+ (R_{gap6} + R_{pole6}) \times I_6
\]

Where, For each of the six poles, the unknown fluxes are \(\Phi_1, \Phi_2, \Phi_3, \Phi_4, \Phi_5,\) and \(\Phi_6.\)

The air gaps' resistances are listed as \(R_{gap1}, R_{gap2}, R_{gap3}, R_{gap4}, R_{gap5},\) and \(R_{gap6};\) the materials' resistances are listed as \(R_{pole1}, R_{pole2}, R_{pole3}, R_{pole4}, R_{pole5},\) and \(R_{pole6};\) and the control currents are listed as \(I_1, I_2, I_3, I_4, I_5,\) and \(I_6.\)

The \(x\) and \(y\) gaps in the air gap reluctances, as well as the control currents, are constantly changing during the operation of the magnetic bearing system. As a result, during the simulation of the full rotor-bearing system, the system of equations (6) must be resolved at each integration time step. The precise calculation of the fluxes based on the altering operating circumstances and control inputs is ensured by this iterative solution procedure. The simulation can accurately depict the behaviour and dynamics of the six-pole PMB HoMB system by solving these nonlinear algebraic equations at each time step, taking into consideration the interplay between the control currents, air gaps, and magnetic fluxes.

IV. Results and Discussion

1. Bilinear Model: Using a piecewise linear approximation, the bilinear model depicts a nonlinear element. It allows for a more realistic description of the element's behaviour by splitting the input-output characteristic of the element into several linear segments. When an element's nonlinearity displays separate zones with various slopes or operational characteristics, this model is frequently used.

2. Modified Langmuir Saturation Model: The behaviour of vacuum tubes and gas discharge devices is typically described using the modified Langmuir saturation model. It is an expansion of the classic Langmuir saturation model and incorporates extra terms to take space-charge effects and secondary emission into account. This model offers a more thorough illustration of the nonlinear traits displayed by these devices under various operating circumstances.
Numerically integrated solutions were used to investigate the maximum static load capacity of a PMB HoMB while accounting for saturation effects. It was found that the force produced by the PMB HoMB is no longer sufficient to balance the static force when it exceeds a specific threshold. Figure 4 provides an illustration of this behaviour, showing how the static deflection falls off as the proportional gain or lag compensator pole/zero value rises.

Figure 3: Flux density at each pole in relation to the Y journal position. Key: (a) pole 1, (b) pole 2, (c) pole 3, (d) pole 4, (e) pole 5, and (f) pole 3.

3. Static Load Capacity of the PMB HoMB With Flux

Numerically integrated solutions were used to investigate the maximum static load capacity of a PMB HoMB while accounting for saturation effects. It was found that the force produced by the PMB HoMB is no longer sufficient to balance the static force when it exceeds a specific threshold. Figure 4 provides an illustration of this behaviour, showing how the static deflection falls off as the proportional gain or lag compensator pole/zero value rises.
As seen in above figure 3 and 4, the fluxes and currents in the separate poles exhibit a relatively modest sensitivity to variations in the proportional gain or lag compensator pole/zero value. This shows that the fluxes and currents within the PMB HoMB system's poles are mostly unaffected by these variables. Examining reveals the benefits of moving the control target point farther from the bearing's centre. These figures show, in proportion to externally applied static stress, the actual steady-state y position of the journal and the control current in pole 2, respectively. Set points positioned at the bearing centre (0, 0) and at a different location (x, y) are compared.

The experiment shows that lowering the pole/zero value of the lag compensator or boosting proportional gain results in less static deflection in the PMB HoMB system. Additionally, the fluxes and currents in each pole show little sensitivity to these factors. The steady-state y position of the journal and the control current in a particular pole clearly benefit from moving the control target point further from the bearing centre.

The reaction of the single-story building to uncontrolled, passively controlled, and actively controlled systems is shown in Figure 11 using a Bode's plot. The plot shows that all three scenarios have positive gain margins and phase margins, which point to the system's stability. However, kinks may be seen in the curve in the uncontrolled and passive control scenarios, showing that the response of the structure oscillates within a specific range of frequencies. These kinks are a sign that the system's damping is insufficient, which causes oscillations. On the other hand, the curve's kinks are smoothed out after active control is implemented through the use of an Active Tuned Mass Damper (ATMD).
Figure 5: For the planned active control system and uncontrolled structure, use Bode's graphs.

The system's improved damping ratio ($\zeta$), which is made possible by including the ATMD, is responsible for the improvement in response. The ATMD actively absorbs energy and lowers vibration amplitude, hence boosting the system's total damping. The structure consequently displays increased stability and less oscillating behaviour.

V. Conclusion
Rotating machinery vibration analysis and the study of nonlinear dynamics offer important new perspectives on how these complex systems behave and function. Researchers have created techniques for defect diagnosis, condition monitoring, and performance optimisation by analysing lateral and torsional vibrations. Though defect diagnosis methods and measures based on lateral vibrations are well-established, attention to torsional vibrations is significantly less widespread. However, recent research has demonstrated the value of torsional vibrations in providing crucial clues about the operational state of rotating gear. Torsional vibrations have been shown to reveal defects in parts like gears even when there is time variation in the running conditions. In addition, the design and analysis techniques presented in this area, such as the magnetic circuit models for magnetic bearings and the saturation effect parameter sets, are useful instruments for comprehending and improving the performance of rotating machinery systems. Understanding the behaviour, identifying problems, and improving the efficiency of rotating machinery all depend on nonlinear dynamics and vibration analysis. Researchers can acquire full insights into the state of operation of these systems and create efficient maintenance, fault-finding, and performance-improving techniques by monitoring both lateral and torsional vibrations. The analysis and control of rotating machinery could develop significantly with further research in this area.
References:


