Properties of W$\tilde{\alpha}$-Closed Sets in Topological Spaces

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Abstract
In this paper, we introduce two new class of generalized closed sets called $\tilde{}$ -closed and weakly $\tilde{\alpha}$ -closed sets. Also, we investigate their relationships with others generalized closed sets.

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1. INTRODUCTION

Levine introduced generalized closed sets in general topology as a generalization of closed sets. Sheik John introduced a study on generalization of closed sets and continuous maps in topological space. P. Sundaram and N. Nagaveni introduced the concept of weakly generalized continuous maps, weakly generalized closed maps and weakly generalized irresolute maps in topological spaces.

In this paper, we introduce two new class of generalized closed sets called $\tilde{}$-closed and weakly $\tilde{\alpha}$ -closed sets. Also, we investigate their relationships with others generalized closed sets.

Throughout this paper (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent topological spaces (briefly **TPS**) on which no separation axioms are assumed unless otherwise mentioned. For a subset N of a space X, cl(N), int(N) and N^c or X$\setminus$ N or X – N denote the closure of N, the interior of N and the complement of N, respectively.

2. WEAKLY $\tilde{\alpha}$-CLOSED SETS IN TOPOLOGICAL SPACES

Definition 2.1

A subset *N* of a **TPS** is called

(i) a $\tilde{\alpha}$-closed (briefly $\tilde{\alpha}$-cld) if cl(N) $\subseteq$ **B** whenever N $\subseteq$ **B** and **B** is sg-open in X.

(ii) a weakly $\tilde{\alpha}$-closed (briefly w$\tilde{\alpha}$-cld) if cl(int(N)) $\subseteq$ **B** wheneverN $\subseteq$ **B** and **B** is sg-open in X.

The complements of the above mentioned closed sets are called their respective open sets.

Theorem 2.2
Any closed set is w$\tilde{\alpha}$-cld but the reverse is not true.

Proof
Let N be a closed. Then cl(N) = N. Let N $\subseteq$ **B** and **B** be sg-open. Since int(N) $\subseteq$ N, cl(int(N)) $\subseteq$ cl(N) = N. We have cl(int(N)) $\subseteq$
N $\subseteq$ **B** whenever N $\subseteq$ **B** and **B** is sg-open. Hence N is w$\tilde{\alpha}$-cld.

Example 2.3
Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2}, {w_1, w_2}, X}. Thenthe set {w_1, w_2} is w$\tilde{\alpha}$-cld but not closed in X.

Theorem 2.4
Any $\tilde{\alpha}$-cld set is w$\tilde{\alpha}$-cld but the reverse is not true.

Proof
The proof is straight forward.

Example 2.5
Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2}, {w_1, w_2}, X}. The set {w_1, w_2} is w$\tilde{\alpha}$-cld but not $\tilde{\alpha}$-cld in X.

Theorem 2.6
Any regular closed set is w$\tilde{\alpha}$-cld but the reverse is not true.

Proof
Let N be any regular closed and let **B** be sg-open containing N. Since N is regular closed, we have N = cl(int(N)) $\subseteq$ **B**. Thus, N is w$\tilde{\alpha}$- cld.

Example 2.7
Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2}, {w_1, w_2}, X}. The set{w_1} is w$\tilde{\alpha}$-cld but not regular cld in X.

Theorem 2.8
Any w$\tilde{\alpha}$-cld set is gsp-cld but the reverse is not true.

Proof
Let N be any w$\tilde{\alpha}$-cld and **B** be open containing N. Then **B** is a sg- open containing N and cl(int(N)) $\subseteq$ **B**. Since **B** is open, we get

int(cl(int(N))) $\subseteq$ **B** which implies spcl(N) = N $\cup$ int(cl(int(N))) $\subseteq$ **B**.Thus, N is gsp-cld.

Example 2.9
Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2}, {w_1, w_2}, X}. Thenthe set {w_1} is gsp-cld but not

w$\tilde{\alpha}$-cld.

Theorem 2.10
If a subset N of a **TPS** X is both closed and α g-cld, then it is w$\tilde{\alpha}$- cld in X.

Proof
Let N be an α g-cld in X and **B** be an open containing N. Then **B** $\supseteq \alpha$ cl(N) = N $\cup$ cl(int(cl(N))). Since N is closed, **B** $\supseteq$ cl(int(N)) andhence N is w$\tilde{\alpha}$-cld in X.

Theorem 2.11
If a subset N of a **TPS** X is both open and w$\tilde{\alpha}$-cld, then it is closed.

Proof

Since N is both open and w$\tilde{\alpha}$-cld, N $\supseteq$ cl(int(N)) = cl(N) and hence N is closed in X.

Corollary 2.12
If a subset N of a **TPS** X is both open and w$\tilde{\alpha}$-cld, then it is both regular open and regular cld in X.

Theorem 2.13
Suppose that B $\subseteq$ N $\subseteq$ X, B is a gs-cld relative to N and that N isboth open and sg-cld subset of X. Then B is gs-cld relative to X.

Proof
Let B $\subseteq$ O and suppose that O is open in X. Then B $\subseteq$ N $\cap$ O and scl_A(B) $\subseteq$ N $\cap$ O. It follows then that N $\cap$ scl(B) $\subseteq$ N $\cap$ O and N $\subseteq$ O $\cup$ $(scl(B))^c$. Since N is sg-cld in X, we have scl(N) $\subseteq$ O $\cup$ $(scl(B))^c$ since the union of open and semi-open is semi-open. Therefore scl(B) $\subseteq$ scl(N) $\subseteq$ O $\cup$ $(scl(B))^c$ and consequently, scl(B) $\subseteq$ O. Then B is gs-cld relative to X.

Corollary 2.14
Let N be both open and sg-cld and suppose that F is closed. ThenN $\cap$ F is gs-cld.

Proof
N $\cap$ F is closed in N and hence gs-cld in N (Apply Theorem2.13).

Theorem 2.15
A set N is w$\tilde{\alpha}$-cld if and only if cl(int(N)) – N contains no non- empty gs-cld.

Proof
Necessity: Let F be a gs-cld such that F $\subseteq$ cl(int(N)) – N. Since $F^c$is gs-open and N $\subseteq$ F^c, from the definition of w$\tilde{\alpha}$-cld it follows that cl(int(N)) $\subseteq$ F^c. ie. F $\subseteq$ $(cl(int(N)))^c$. This implies that F $\subseteq$ (cl(int(N)))
$\cap$ $(cl(int(N)))^c$ = ϕ.

Sufficiency: Let N $\subseteq$ G, where G is both closed and sg-open in X. If cl(int(N)) is not contained in G, then cl(int(N)) $\cap$ G^c is a non-empty gs-closed subset of cl(int(N)) – N , we obtain a contradiction. This proves the sufficiency and hence the theorem.

Theorem 2.16

Let X be a **TPS** and N ⊆ Y ⊆ X. If N is open and w$\tilde{\alpha}$-closed in X, then N is w$\tilde{\alpha}$-cld relative to Y.

Proof

Let N ⊆ Y ∩ G where G is gs-open in X. Since N is w$\tilde{\alpha}$-cld in X,

N ⊆ G implies cl(int(N)) ⊆ G. That is Y ∩ (cl(int(N))) ⊆ Y ∩ G where Y ∩ cl(int(N)) is closure of interior of N in Y. Thus, is w$\tilde{\alpha}$-cld relative to Y.

Theorem 2.17

If a subset N of a **TPS** X is nowhere dense, then it is w$\tilde{\alpha}$-cld.

Proof

Since int(N) ⊆ int(cl(N)) and N is nowhere dense, int(N) = ϕ.
Therefore, cl(int(N)) = ϕ and hence, N is w$\tilde{\alpha}$-cld in X.

The converse of Theorem 2.17 need not be true as seen in thefollowing example.

Example 2.18

Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2, w_3}, X}. Then the set {w_1} is w$\tilde{\alpha}$-cld but not nowhere dense in X.

Remark 2.19

The following examples show that w$\tilde{\alpha}$-closedness and semi-closedness are independent.

Example 2.20

Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2, w_3}, X}. The set {w_2}is w$\tilde{\alpha}$-cld but not semi-cld in X.

Example 2.21

Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2}, {w_1, w_2}, X}. Thenthe set {w_2} is semi-closed but not w$\tilde{\alpha}$-cld in X.

Definition 2.22

A subset N of a **TPS** X is called w$\tilde{\alpha}$-open if N^c is w$\tilde{\alpha}$-cld in X.

Theorem 2.23

Any open set is w$\tilde{\alpha}$-open.

Poof

Let N be an open in a **TPS** X. Then N^c is closed in X. By Theorem 2.2 it follows that N^c is w$\tilde{\alpha}$-cld in X. Hence N is w$\tilde{\alpha}$-open in X.

The converse of Theorem 2.23 need not be true as seen in the following example.

Example 2.24

Let X = {w_1, w_2, w_3} and τ = {ϕ, {w_1}, {w_2}, {w_1, w_2}, X}. The set{w_3} is w$\tilde{\alpha}$-open set but it is not open in X.

Proposition 2.25

(i) Any w$\tilde{\alpha}$-open is w$\tilde{\alpha}$-open but reverse is not true.

(ii) Any regular open set is w$\tilde{\alpha}$-open but the reverse is not true.

(iii) Any g-open set is w$\tilde{\alpha}$-open but the reverse is not true.

(iv) Any w$\tilde{\alpha}$-open set is gsp-open but the reverse is not true.

It can be shown that the converse of (i), (ii), (iii) and (iv) need not be true.

Theorem 2.26

A subset N of a **TPS** X is w$\tilde{\alpha}$-open if G $\subseteq$ int(cl(N)) whenever G$\subseteq$ N and G is gs-cld.

Proof

Let be any w$\tilde{\alpha}$-open. Then N^c is w$\tilde{\alpha}$-cld. Let G be a sg-cld contained in N. Then G^c is a sg-open containing N^c. Since N^c is w$\tilde{\alpha}$-cld, we have cl(int(N^c)) $\subseteq$ G^c. Therefore G $\subseteq$ int(cl(N)).

Conversely, we suppose that G $\subseteq$ int(cl(N)) whenever G $\subseteq$ N and G is sg-cld. Then G^c is a sg-open containing N^c and $G^c \supseteq$ (int(cl(N)))^c. It follows that $G^c \supseteq$ cl(int(N^c)). Hence N^c is w$\tilde{\alpha}$-cld and so N is w$\tilde{\alpha}$-open.

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