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# Implementing a Multi-Objective Linear Fractional Inventory Model Using a Fuzzy Approach

**Naresh Kumar**
Associate Professor, Department of Mathematics
Aggarwal College Ballabgarh, Faridabad, Haryana, India
Email id: nareshkamra10@gmail.com

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**Abstract**

This paper presents a fuzzy programming model as a means of addressing multiple management objectives concurrently within the context of a linear fractional inventory system. The objective of this study is to present a developing model and illustrate how it can be implemented through the use of numerous numerical examples. In this work, the methodology is applied to a problem with two distinct objectives. The generated fuzzy model is then analysed using a linear programming format. For convenience, the researchers have assumed that the cycle period only consists of a single phase. The optimal modelling formulation takes into account both the total amount of available funds and the total amount of available storage space.

**Keywords**: - MOLFPP (Multi-Objective Linear Fractional Programming Problem) model; Inventory model; Fuzzy approaches; Min operator

## Introduction

Fractional programming is essential for modeling & optimisation in the domains of business, engineering, economics, finance, & science, since it involves optimizing a fraction of two functions according to certain present restrictions. Significant changes have currently occurred in this region. Typically, FPP (Fractional Programming Problem) - decision-making system that seeks to maximize the ratio while taking certain restrictions into account. The decision maker (DM) might occasionally have to calculate inventory & sales ratio, production & labor force, & more, when both the denominator & the numerator are linear. The issue is known as a linear FPP when a ratio is taken into account as an objective function subject to linear restrictions. For application, FPP is utilized in a variety of areas, including traffic planning. Meanwhile, **[6]** discussed several FPP applications & strategies for these kinds of issues & established some fuzzy methods for multi-objective linear FPP.

The traditional Economic Order Quantity (EOQ) problem has been resolved for a long time under a variety of formats & presumptions **[1].** The traditional inventory model makes the assumption that demand will always be deterministic. However, a lot of real-world case studies deal with stochastic issues **[2].** On the other hand, the idea of multi-objective programming (MOP) has gained popularity among scholars recently because numerous single goal optimisation techniques cannot assist users in arriving at suitable solutions. A fascinating area of research that has several applications for production planning is the notion of MOP paired with fractional programming **[3].**

Investigators formulate a multi-objective issue having fractional components all objective functions in this study. After that, the generated model is converted; multi-objective LP. The researchers evaluate each goal function independently in order to offer a lower limit for each. The multi-objective function is then converted into a standard LP algorithm using Zimmermann's min values notion **[4].** MOLFPP is solvable utilising a variety of techniques.

An algorithm that selects one of several effective solutions from a pool of effective solutions. Additionally, they offer a generic approach for handling goal programming problems involving linear fractional objectives. Other persons have also investigated MOLFPP **[5].** However, in terms of computational complexity, their methods are often difficult. Numerous attempts have been made to include the idea of fuzzy programming with MOLFPP in order to address the complexity that is apparent in the solution. Implementation makes use of language characteristics to reveal the decision-maker's level of aspiration **[6].** A strategy for solving the same issue that resulted in a very non-linear non-convex issue. Solving such an issue may frequently be quite challenging, especially when there are many variables involved. Such as utilizing an iterative strategy to solve MOLFPP, which after a number of iterations obtains the global solution **[7].**

By translating MOLFPP into an Linear Programming (LP) problem that might be readily solved, the researchers in given study try to provide a novel strategy that would result in a linear optimisation problem & would achieve the universal answer in 2 steps. This would be accomplished by transforming LP into MOLFPP. The following is the structure of this document. The researchers start by providing some of the essential definitions **[8]**. The resultant model's problem formulation is then given inside the framework of an inventory issue. In the next part, the implementation of the suggested approach presented in this work is illustrated using a numerical example. This section comes later in the text. At last, some concluding thoughts are presented at the end of this study in order to summarize its contribution **[9]**.

**Problem formulation**

In this part of the article, we provided a synopsis of some of the essential terminology, as well as the transformation that has to be put into place for suggested research. In this part, we also provide the transformation that users must do in order to use the approach developed by to turn a multi-objective issue into a standard LP problem **[10]**.

The programming using the linear fractional algebra. The following is an example of a generic form that may be used to formulate linear fractional programming (LFP) **[11]**,

$$max\left\{\frac{c'x+\alpha}{d'x+\beta} \mid Ax=b, x\geq 0\right\} \dots (1.1)$$

where, $c, d \in R^{n\times 1}, A \in R^{m\times n}, b \in R^{m\times 1}$ & $\alpha, \beta \in R$. $d'x+\beta$ might be zero for particular $x$ values. Therefore, an extra assumption is required.

**Assumption 1.** So, if $Ax=b$ & $x\geq 0$; $d'x+\beta>0$ or $d'x+\beta<0$

Problem with linear fractional programming that has several objectives. The following is an example of a definition for a generic MOLFPP, which stands for MOLF programming **[12]**,

$$maxz(x)=\{z_1(x),\cdots,z_n(x) \mid Ax\leq b\} \dots (1.2)$$

where $x \in R^{n\times 1}, A \in R^{m\times n}, b \in R^{m\times 1}$ and

$$z_i(x)=\frac{c_i'x+\alpha_i}{d_i'x+\beta_i}, c_i, d_i \in R^{n\times 1} \ \& \ \alpha_i, \beta_i \in R$$

**Transformation**

The steps necessary to convert MOLFPP into a LP issue are outlined in the following **[13]**,

**Step 1.** Let

$$z_i(x) = \frac{c_i' x + \alpha_i}{d_i' x + \beta_i} = \frac{N_i(x)}{D_i(x)}.$$

Let $I$ be the index set; $I = \{i \mid N_i(x) \geq 0$, for some $x\}$ & $I^c = \{i \mid N_i(x) < 0$, for some $x\}$.
**Step 2.** Let $y = tx$ with $t \geq 0$, thus, utilising elaborated procedure, **[8]** we have,

$$max\left\{\left(tN_i\left(\frac{y}{t}\right)\right)\ i \in I, \left(tD_i\left(\frac{y}{t}\right)\right)\ i \in I^c\right\}$$

Constraints:

$$tD_i\left(\frac{y}{t}\right) \leq 1, i \in I$$

$$-tN_i\left(\frac{y}{t}\right) \leq 1, i \in I^c \ldots.(1.3)$$

$$A\left(\frac{y}{t}\right) - b \leq 0, \qquad t \geq 0, y \geq 0.$$

**Step 3** Deterministic multi-objective LP problem gives,
$Max\lambda$ Subject to:

$$\mu_i\left(t, N_i\left(\frac{y}{t}\right)\right) \geq \lambda \text{ for } i \in I$$

$$\mu_i\left(t. D_i\left(\frac{y}{t}\right)\right) \geq \lambda \text{ for } i \in I^c$$

$$t \cdot D_i\left(\frac{y}{t}\right) \leq 1 \text{ for } i \in I \ldots(1.4)$$

$$-t \cdot D_i\left(\frac{y}{t}\right) \leq 1 \text{ for } i \in I^c$$

$$A\left(\frac{y}{t}\right) - b \leq 0\ t \geq 0, y \geq 0,$$

Here,

$$\mu_i\left(t. N_i\left(\frac{y}{t}\right)\right) = \{0\ \ if\ \left(t. N_i\left(\frac{y}{t}\right)\right) \leq 0\ \frac{t. N_i\left(\frac{y}{t}\right) - 0}{\tilde{z}_i - 0}\ \ if\ \ 0 < \left(t. N_i\left(\frac{y}{t}\right)\right)$$

$$\leq \tilde{z}_i \ldots.(1.5)\ 1\ \ if\ \left(t. N_i\left(\frac{y}{t}\right)\right) > \tilde{z}_i$$

and

$$\mu_i\left(t. D_i\left(\frac{y}{t}\right)\right) = \{0\ \ if\ \left(t \cdot D_i\left(\frac{y}{t}\right)\right) \leq 0\ \frac{t \cdot D_i\left(\frac{y}{t}\right) - 0}{\tilde{z}_i - 0}\ \ if\ \ 0 < \left(t \cdot D_i\left(\frac{y}{t}\right)\right)$$

$$\leq \tilde{z}_i \ldots.(1.6)\ 1\ \ if\ \left(t \cdot D_i\left(\frac{y}{t}\right)\right) > \tilde{z}_i$$

To determine an attribute’s intersection in a statistical population, the reason for choosing the min operator as opposed to two other operators termed "*and* & *product*" is that the min operator is more precise **[14]**.
Let's assume that $I, I^c, i$ & $N_i(x)$ are known for $i = 1, \ldots, k$. But let's assume all we know about the denominators is that they're all positive in the tractable range. To find the index sets & the maximum aspiration levels$\tilde{z}_i$,
We must perform the given actions,

1 Maximize all objective functions $z_i(x)$ per the mentioned limit-set. Assume $z_i^*$ is $z_i(x)$ optimal value for $i = 1, \dots k$.

2 Study $z_i^*$'s nature for all values of $i$, (e.g. if $z_i^* \geq 0$ ; $i \in I$ & if $z_i^* < 0$ ; $i \in I^c$).

3 Similarly we use $\tilde{z}_i = z_i^*$ if $i \in I$ & $\tilde{z}_i = -\frac{1}{z_i^*}$ when $i \in I^c$.

**Method**

The majority of genuine inventory issues involve more than one objective function, some of which may be in direct opposition to one another. In this part of the report, the researchers analyze a model of an inventory problem in which they want to maximize ratio of profit to cost of holding while concurrently decreasing ratio of back orders total requested amounts **[15]**. The researchers have made the assumption that the cycle time only contains a single phase for the purpose of simplicity. In the end, they take into account two restrictions in our modeling formulation: the available budget & the amount of space in the warehouse. Because of this, we have,

$$maxZ_1 = \frac{\sum_{i=1}^{n} (S_i - P_i)Q_i}{\sum_{i=1}^{n} \frac{h_i Q_i}{2}},$$

$$minZ_2 = \frac{\sum_{i=1}^{n}(D_i - Q_i)}{\sum_{i=1}^{n} Q_i} \dots (1.7)$$

Subject to:

$$\sum_{i=1}^{n} P_i Q_i \leq B \quad \sum_{i=1}^{n} f_i Q_i \leq F$$

where:

$Q_i$ : Product $i$'s ordering quantity

$P_i$ : $i$'s purchasing price

$S_i$ : $i$'s selling price

$h_i$ : $i$'s inventory holding cost

$D_i$ : $i$'s demand

$B$ : Total spending limit

$f_i$: $i$'s space requirement

$F$ : Maximum potential storage

**Result**s

Results section includes a few numerical examples that illustrate how the planned application was carried out with Take into consideration the material that follows.,

**Table 1.1. Numerical example's input information.**

| Item | $h_i$ | $p_i$ | $S_i$ | $D_i$ | $f_i$ | $B$ | $F$ |
|---|---|---|---|---|---|---|---|
| $i = 1$ | 20 | 600 | 650 | 8000 | 1 | 400000 | 240 |
| $i = 2$ | 24 | 720 | 750 | 4000 | 2 | 400000 | 240 |

A summary of MOLF inventory issue having 2 goal functions:

$$max\ z_1 \ = \frac{50Q_1 + 30Q_2}{10Q_1 + 12Q_2}\ min\ z_2 \ = \frac{8000 - Q_1 + 4000 - Q_2}{Q_1 + Q_2}$$

Subject to:

$$600Q_1 + 720Q_2 \leq 400000\ \ Q_1 + 2Q_2 \leq 240\ \ Q_1, Q_2 \geq 0\ \ldots (1.8)$$

$$10y_1 + 12y_2 \leq 1$$

$$-y_1 - y_2 - 12000 \leq 1\ \ 600y_1 + 720y_2 - 400000 \leq 0\ \ y_1 + 2y_2 - 240t \leq 0\ \ y_1, y_2, t \geq 0$$

Let, $f_1(y,t) \geq 5$ & $f_2(y,t) \geq 0.1$.

The resulting fuzzy model & its membership function are expressed in the given crisp model & equation, respectively (1.9)

$$max\lambda$$

Subject to:

$$50y_1 + 30y_2 - 5\lambda \geq 0\ \ y_1 + y_2 - 0.1\lambda \geq 0\ \ 10y_1 + 12y_2 \leq 1\ \ \ -y_1 - y_2 + 12000t \leq 1\ \ 600y_1 + 720y_2 - 400000 \leq 0\ \ y_1 + 2y_2 - 240t \leq 0\ \ldots (1.9)$$

(10)’s optimal solution is given,

$$\lambda^* = 0.2041, t^* = 0.0001, y_1^* = 0.0204, y_2^* = 0.$$

Therefore we have,

$$Q_1^* = 204, Q_2^* = 0, z_1^* = 5, z_2^* = 57.82$$

**Conclusions**

Based on a set theoretic approach & fuzzy programming, the researchers have provided a way to address MOLF inventory problems. A MOLF inventory model has been transformed into a LP issue via a transformation. A numerical example has been used to demonstrate how the researchers' technique was put into practice. The inventory model described in this work may be applied to a variety of more challenging inventory issues, including models that incorporate deteriorations, discounts, dynamic demand, replenishment, etc.

However, there are a number of unresolved issues that need to be investigated in the multilevel quadratic fractional optimization field in the future, including:

1. An interactive method is essential for multi – level & objective quadratic fractional programming with fuzzy parameters.
2. With fuzzy parameters, multi-level integer multi-objective quadratic fractional problems need a fuzzy goal programming technique.
3. Fuzzy goal programming is essential to deal with multi-level integer multi-objective quadratic fractional problems within complex settings.

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