Heptagonal Graceful Labeling on Simple Graphs

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Abstract

Heptagonal numbers are numbers that have the form 5n2-2n2 for all n≥1.Consider the graph *G* with *p* vertices and *q* edges. Assume that f : V (G)→{0,1,2...Nq} is an injective function with Nqbeing the qth heptagonal number. Define f* :E(G) → {N1,N2,N3,....,Nq} such that f*(uv) = |f(u) - f(v)| for all edges (uv) ∈ E(G).If f*(E(G)) is a sequence of distinct sequential numbers {N1,N2,N3,....,Nq} then the function f is said to have heptagonal graceful labeling and the graph admitting such a labeling is termed as heptagonal graceful graph. This work investigates heptagonal graceful labeling of various graphs.

Keywords- Graceful Labeling, Heptagonal graceful number, Heptagonal graceful labeling, Heptagonal graceful graphs,

1.Introduction

In this research, we look at basic, undirected, finite graphs. Let $G = (V,E)$ denotes a graph having p vertices and q. Labeling in a graph is the process of assigning numbers to its vertices, edges, or both. If the domain of the mapping is the set of vertices (edge/both), the labelling is referred to as a vertex (edge/both) labelling. Rosa [1] proposed the concept of β-valuation of a graph. According to Golomb [10 it was graceful labeling. Assume *G* be a (p,q) graph. A graceful labeling of *Gis* defined as a one to one function $f: V(G) \to \{0,1,2\ldots,q\}$ if the induced edge labeling defined by $f^*(e) = |f(u) - f(v)|$ for each edges $e = uv$ of G is also one to one. The graph G with graceful labeling is called graceful graph. In [2], certain families of graceful graphs were constructed. There are several types of graceful labeling and a detailed survey is found in [3]. Labeled graphs are becoming more valuable family of mathematical models for a wide variety of applications such as designing X-Ray crystallography, formulating a communication network addressing system, determining an optimal circuit layouts, problems in additive number theory and so on [5]. Heptagonal graceful labeling of various graphs is investigated in this study.

2.Preliminary

Definition 2.1. Heptagonal numbers are numbers that have the form $\frac{5n^2-2n}{2}$ for all n ≥1. $1,7,18,34,55,81,112,\ldots$ are the first few heptagonal numbers.

3. Main Results

Theorem 3.1. *Let G be the graph formed by connecting the leaves of $K_{1,n}$ to the central vertex of $K_{1,2}$ [6]. Then for any $n \geq 1$, G is heptagonal graceful.*

Proof.

Let *G* be the graph obtained by identifying the leaves of $K_{1,n}$ with the central vertex of $K_{1,2}$.

Let $V(G) = \{v, v_i, v_{ij};\ 1 \leq i \leq n, 1 \leq j \leq 2\}$ and

$E(G) = \{v\, v_i, v_i\, v_{ij};\ 1 \leq i \leq n, 1 \leq j \leq 2\}$

.

G has $3n + 1$ vertices and $3n$ edges.

Let $q = 3n$.

Let $f : V(G) \to \{0, 1, 2, \ldots, Nq\}$ be defined as follows:

$f(v) = 0$

$f(v_i) = N_{3(n-(i-1))};\ 1 \leq i \leq n$

$f(v_{ij}) = f(v_i) - N_{q-(i-1)n-j};\ 1 \leq i \leq n, 1 \leq j \leq 2$

Let f^* be the induced edge labeling of f.

Then

$f^*(v\, v_i) = N_{3(n-(i-1))};\ 1 \leq i \leq n$

$f^*(v_i\, v_{ij}) = N_{q-(i-1)n-j};\ 1 \leq i \leq n, 1 \leq j \leq 2$

The induced edge labels $N_1, N_2, N_3, \ldots, N_q$ are distinct and has consecutive heptagonal numbers. Hence G is heptagonal graceful for all $n \geq 1$.

Example 3.2. Below graph depicts a heptagonal graceful labeling of $K_{1,3} \odot K_{1,2}$[6].

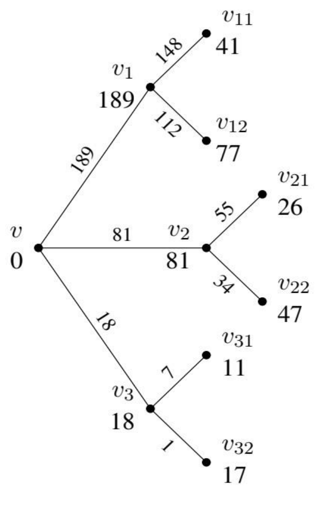

Figure 1. $K_{1,3} \odot K_{1,2}$

Theorem 3.3. *The F-tree* FP_n, $n \geq 3$ [6]*is heptagonal graceful.*

Proof.

Let G be FP_n, $n \geq 3$.

Let $V(G) = \{u, v, v_i : 1 \leq i \leq n\}$ and

$E(G) = \{v_i v_{i+1} : 1 \leq i \leq n - 1\} \cup \{u v_{n-1}, v v_n\}$

G has $n + 2$ vertices and $n + 1$ edges.

Let $q = n + 1$

Let $f: V(G) \to \{0, 1, 2, \dots, N_q\}$ be defined as follows.

$f(v_1) = 0$

$$f(v_i) = \begin{cases} f(v_{i-1}) - N_{q-i+2}, & if\ i\ is\ odd, 2 \leq i \leq n \\ f(v_{i-1}) + N_{q-i+2}, & if\ i\ is\ even, 2 \leq i \leq n \end{cases}$$

$f(v) = f(v_n) - 1$

$f(v) = f(v_{n-1}) - 7$

Let f^* be the induced edge labeling of f.

Then

$f^*(v_i v_{i+1}) = N_{q-i+1}, \quad 1 \leq i \leq n - 1$,

$f^*(u v_{n-1}) = N_2$

$f^*(v v_n) = N_1$

The induced edge labels $N_1, N_2, N_3, \dots, N_q$ are distinct and has consecutive heptagonal numbers. Hence *F-tree* FP_n, $n \geq 3$ is heptagonal graceful.

Example 3.4. Heptagonal graceful labeling of FP_5 [6]is given below.

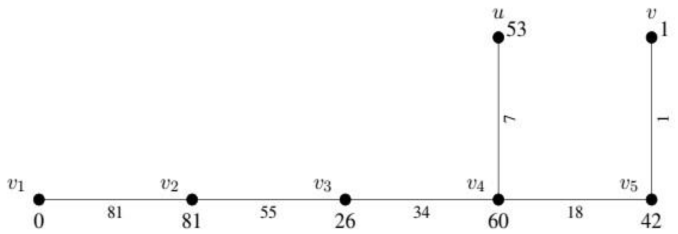

Figure 2. FP_5

Theorem 3.5. *Let G be the graph formed by connecting a pendant vertex of* P_m *to a leaf of* $K_{1,n}$ [6]. *Then, for any* $m \geq 2$ *and* $n \geq 1$, *G is heptagonal graceful.*

Proof.

Let G be the graph formed by connecting a pendant vertex v_1 of P_m with a leaf u_n of $K_{1,n}$.

Let $V(G) = \{u, u_i, v_j : 1 \leq i \leq n - 1; 1 \leq j \leq m\}$ and

$E(G) = \{u u_i, u v_1, v_j v_{j+1}; 1 \leq i \leq n - 1, 1 \leq j \leq m - 1\}$

.

G has $m + n$ vertices and $m + n - 1$ edges.

Let $q = m + n - 1$

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Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, N_q\}$ be defined as follows:

$f(u) = 0$

$f(u_i) = N_{q-(i-1)}, 1 \le i \le n$

$f(v_1) = N_m$

$$f(v_j) = \begin{cases} f(v_{j-1}) + N_{n-(j-2)}, if\ j\ is\ odd, 2 \le j \le m \\ f(v_{j-1}) - N_{n-(j-2)}, if\ j\ is\ even, 2 \le i \le m \end{cases}$$

Let f^* be the induced edge labeling of f.

Then $f^*(u\,u_i) = N_{q-(i-1)};\ 1 \le i \le n - 1$

$f^*(u\,v_1) = N_m$

$f^*(v_j\,v_{j+1}) = N_{m-j};\ 1 \le j \le m - 1$

The induced edge labels $N_1, N_2, N_3, \ldots, N_q$ are distinct and has consecutive heptagonal numbers. Hence G is heptagonal graceful for all $m \ge 2$ *and* $n \ge 1$.

Example 3.6. The heptagonal graceful labeling graph created by identifying a pendant vertex of P_5 with a leaf of $K_{1,4}$ [6] is shown below.

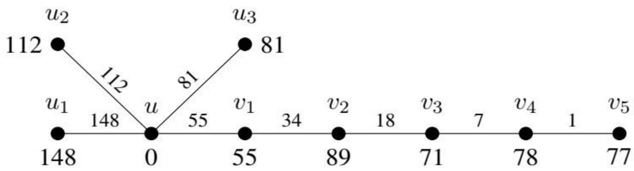

Figure 3. *G*raph created by identifying a pendant vertex of $P_5$with a leaf of $K_{1,4}$

Theorem 3.7. *For any $n \ge 1$, the graph formed by subdividing the edges of the star$K_{1,n}$ [6]is heptagonal graceful.*

Proof.

Let G be the graph obtained by subdividing the edges of the star $K_{1,n}$ for all $n \ge 1$.

Let $V(G) = \{u, v_i, u_i : 1 \le i \le n\}$ and

$E(G) = \{uv_i, v_i\,u_i : 1 \le i \le n\}$

G has $2n + 1$ vertices and$2n$ edges.

Let $q = 2n$

Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, N_q\}$ be defined as follows:

$f(u) = 0$

$f(v_i) = N_{q-(i-1)};\ 1 \le i \le n$

$f(u_i) = f(v_i) - N_{n-(i-1)};\ 1 \le i \le n$

Let f^* be the induced edge labeling of f.

Then $f^*(uv_i) = N_{q-(i-1)};\ 1 \le i \le n$

$f^*(v_iu_i) = N_{n-(i-1)};\ 1 \le i \le n$

The induced edge labels $N_1, N_2, N_3, \ldots, N_q$ are distinct and has consecutive heptagonal numbers. Hence the graph G is heptagonal graceful for all $n \ge 1$.

Example 3.8. The graph of heptagonal graceful labeling created by subdividing the edges of the star $K_{1,6}$ [6] is shown below.

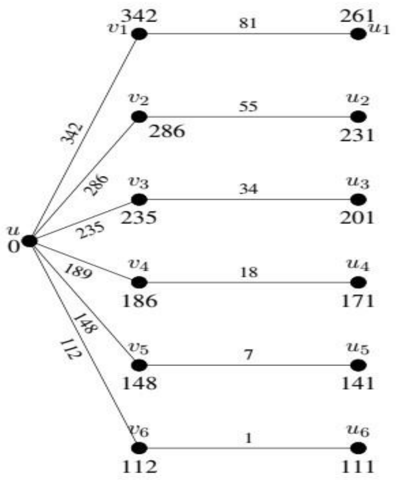

Figure 4. Subdivision of the edges of the star$K_{1,6}$.

Theorem 3.9. *For any $n \geq 2$, the graph generated from$P_n \odot K_1$ by subdividing the edges of the path P_n[6] is heptagonal graceful.*

Proof.

Let G be the graph generated from $P_n \odot K_1$ by subdividing the edges of the path P_n

Let $V(G) = \{v_i, u_i, w_j;\ 1 \leq i \leq n;\ 1 \leq j \leq n-1\}$ and

$E(G) = \{v_i w_i, v_j u_j, w_k w_{k+1};\ 1 \leq i \leq n-1, 1 \leq j \leq n, 1 \leq k \leq n-1\}$

G has $3n - 1$ vertices and $3n - 2$edges.

Let $q = 3n - 2$

Let $f : V(G) \rightarrow \{0, 1, 2, \ldots, N_q\}$ be defined as follows:

$$f(v_1) = 0$$
$$f(v_i) = f(w_{i-1}) - N_{q-1-(2(i-2))};\ 2 \leq i \leq n$$
$$f(w_j) = f(v_j) + N_{q-2(j-1)};\ 1 \leq j \leq n-1$$
$$f(u_i) = f(v_i) + N_{n-i+1};\ 1 \leq i \leq n$$

Letf^* be the induced edge labeling of f.

Then $f^*(v_i w_i) = N_{q-2(i-1)};\ 1 \leq i \leq n-1$

$f^*(v_j u_j) = N_{n-j+1};\ 1 \leq j \leq n$

$f^*(w_k v_{k+1}) = N_{q-2(k-1)};\ 1 \leq k \leq n-1$

The induced edge labels $N_1, N_2, N_3, \ldots, N_q$ are distinct and has consecutive heptagonal numbers. Hence the graph G is heptagonal graceful for all $n \geq 2$.

Example 3.10. The graph depicts heptagonal graceful labeling of $P_3 \odot K_1$[6] by subdividing the edges of the path P_3.

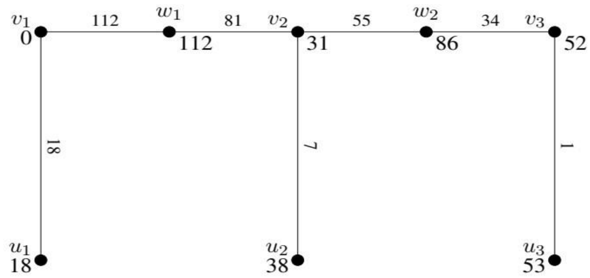

Figure 5. $P_3 \odot K_1$by subdividing the edges of the path P_3.

Conclusions

The authors investigated the heptagonal graceful labeling of several graphs in this study. A similar investigation might be conducted for other graphs. In addition to this various other labeling related to heptagonal graceful labeling have also been investigated and proved by the authors.

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