

Ö-J-Locally Closed Sets with Respect to an Ideal Topological Spaces

¹A. Saranya and ²O. Ravi

¹Assistant Professor, Department of Mathematics, Sri Adi Chunchanagiri Women's College,
Cumbum, Theni District, Tamil Nadu, India.

e-mail: saranyajey88@gmail.com, Research Scholar, Madurai Kamaraj University, Madurai - 21,
Tamilnadu, India.

²Principal, Pasumpon Muthuramalinga Thevar College, Usilampatti - 625 532,
Madurai District, Tamil Nadu, India. e-mail: siingam@yahoo.com.

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Abstract

In this paper, we introduce three forms of locally closed sets called Ö-J-locally closed sets, Ö-J- lc^* sets and Ö-J- lc^{**} sets.

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1. INTRODUCTION

M. Ganster and I. L. Reilly studied Locally closed sets and LC-continuous functions in the year 1989. Following this attempts, modern mathematics generalized this concept and are being found many generalization of locally closed sets. R. Vaidyanathaswamy studied the localization theory in set topology in 1945. D. Jankovic and T. R. Hamlett studied new topologies from old via ideals in 1990.

In this paper, we introduce three forms of locally closed sets called Ö-J-locally closed sets, Ö-J- lc^* sets and Ö-J- lc^{**} sets. Properties of these new concepts are studied as well as their relations to the other classes of locally closed sets are investigated.

2. PRELIMINARIES

Definition 2.1

A subset S of X is called locally closed (briefly, lc) if $S = U \cap F$, where U is open and F is closed in X .

Definition 2.2

A subset S of a space X is called:

- (i) generalized locally closed (briefly, glc) if $S = V \cap F$, where V is g-open and F is g-cld.

- (ii) semi-generalized locally closed (briefly, *sglc*) if $S = V \cap F$, where V is *sg-open* and F is *sg-cld*.
- (iii) regular-generalized locally closed (briefly, *rg-lc*) if $S = V \cap F$, where V is *rg-open* and F is *rg-cld*.
- (iv) generalized locally semi-closed (briefly, *glsc*) if $S = V \cap F$, where V is *g-open* and F is *semi-cld*.
- (v) locally semi-closed (briefly, *lsc*) if $S = V \cap F$, where V is *open* and F is *semi-cld*.
- (vi) α -locally closed (briefly, α -*lc*) if $S = V \cap F$, where V is α -*open* and F is α -*cld*.
- (vii) ω -locally closed (briefly, ω -*lc*) if $S = V \cap F$, where V is ω -*open* and F is ω -*cld*.

The class of all generalized locally closed (resp. generalized locally semi-closed, locally semi-closed, ω -locally closed) sets in X is denoted by $GLC(X)$ (resp. $GLSC(X)$, $LSC(X)$, ω - $LC(X)$).

An ideal on a topological space (X, τ) is a non-empty collection of subsets of X which satisfies the following properties:

- (i) $A \in I$ and $B \subset A$ implies $B \in I$
- (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$.

An ideal topological space (or An ideal space) is a topological space (X, τ) with an ideal I on X and is denoted by (X, τ, I) . For a subset $A \subset X$, $A^*(I, \tau) = \{x \in X : A \cap U \in I \text{ for every } U \in \tau(X, x)\}$ is called the local function of A with respect to I and τ . We simply write A^* incase there is no chance for confusion. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$ called the $*$ -topology, finer than τ is defined by $cl^*(A) = A \cup A^*$.

Definition 2.3

A subset S of X is called \ddot{O} - J -closed (briefly, \ddot{O} - J -cld) if $S^* \subseteq P$ whenever $S \subseteq P$ and P is *gs-open*. The complement of \ddot{O} - J -cld is called \ddot{O} - J -open.

The family of all \ddot{O} - J -cld in X is denoted by $\ddot{O}\text{-}J\text{-}C(X)$.

3. \ddot{O} - J -LOCALLY CLOSED SETS

We introduce the following definition.

Definition 3.1

A subset S of X is called \ddot{O} - J -locally closed (briefly, \ddot{O} - J -lc) if $S = H \cap G$, where H is \ddot{O} - J -open and G is \ddot{O} - J -cld.

The class of all \ddot{O} - J -locally closed sets in X is denoted by $\ddot{O}\text{-}J\text{-}LC(X)$.

Proposition 3.2

Each \ddot{O} - J -cld (resp. \ddot{O} - J -open) is \ddot{O} - J -lc set but not reverse.

Proof

It follows from Definition 3.1(i).

Example 3.3

Let $X = \{p_1, q_1, r_1\}$ and $\tau = \{\phi, \{q_1\}, X\}$ with $\mathcal{J} = \{\phi\}$. Then the set $\{q_1\}$ is $\ddot{O}\text{-}\mathcal{J}$ -lc set but it is not $\ddot{O}\text{-}\mathcal{J}$ -cld and the set $\{p_1, r_1\}$ is $\ddot{O}\text{-}\mathcal{J}$ -lc set but it is not $\ddot{O}\text{-}\mathcal{J}$ -open in X .

Proposition 3.4

Each lc set is $\ddot{O}\text{-}\mathcal{J}$ -lc set but not reverse.

Proof

It follows from Proposition 3.2.

Example 3.5

Let $X = \{p_1, q_1, r_1\}$ and $\tau = \{\phi, \{q_1, r_1\}, X\}$ with $\mathcal{J} = \{\phi\}$. Then the set $\{q_1\}$ is $\ddot{O}\text{-}\mathcal{J}$ -lc set but it is not lc set in X .

Proposition 3.6

Each $\ddot{O}\text{-}\mathcal{J}$ -lc set is a (i) ω -lc set, (ii) glc set and (iii) sglc set. However the separate reverse is not true.

Proof

It is obviously.

Example 3.7

Let $X = \{p_1, q_1, r_1\}$ and $\tau = \{\phi, \{p_1\}, X\}$ with $\mathcal{J} = \{\phi\}$. Then the set $\{q_1\}$ is glc set but it is not $\ddot{O}\text{-}\mathcal{J}$ -lc set in X . Moreover, the set $\{r_1\}$ is sglc set but it is not $\ddot{O}\text{-}\mathcal{J}$ -lc set in X .

Example 3.8

Let $X = \{p_1, q_1, r_1\}$ and $\tau = \{\phi, \{q_1\}, \{p_1, r_1\}, X\}$ with $\mathcal{J} = \{\phi\}$. Then the set $\{p_1\}$ is ω -lc set but it is not $\ddot{O}\text{-}\mathcal{J}$ -lc set in X .

Remark 3.9

The concepts of α -lc sets and $\ddot{O}\text{-}\mathcal{J}$ -lc sets are independent of each other.

Example 3.10

The set $\{q_1, r_1\}$ in Example 3.3 is α -lc set but it is not a $\ddot{O}\text{-}\mathcal{J}$ -lc set in X and the set $\{p_1, q_1\}$ in Example 3.5 is $\ddot{O}\text{-}\mathcal{J}$ -lc set but it is not an α -lc set in X .

Remark 3.11

The concepts of lsc sets and $\ddot{O}\text{-}\mathcal{J}$ -lc sets are independent of each other.

Example 3.12

The set $\{p_1\}$ in Example 3.3 is lsc set but it is not a $\ddot{O}\text{-}\mathcal{J}$ -lc set in X and the set $\{p_1, q_1\}$ in Example 3.5 is $\ddot{O}\text{-}\mathcal{J}$ -lc set but it is not a lsc set in X .

Remark 3.13

The concepts of \ddot{O} - \mathcal{I} -lc sets and glsc sets are independent of each other.

Example 3.14

The set $\{q_1, r_1\}$ in Example 3.3 is glsc set but it is not a \ddot{O} - \mathcal{I} -lc set in X and the set $\{p_1, q_1\}$ in Example 3.5 is \ddot{O} - \mathcal{I} -lc set but it is not a glsc set in X .

Remark 3.15

The concepts of \ddot{O} - \mathcal{I} -lc sets and $sglc^*$ sets are independent of each other.

Example 3.16

The set $\{q_1, r_1\}$ in Example 3.3 is $sglc^*$ set but it is not a \ddot{O} - \mathcal{I} -lc set in X and the set $\{p_1, q_1\}$ in Example 3.5 is \ddot{O} - \mathcal{I} -lc set but it is not a $sglc^*$ set in X .

Theorem 3.17

For a T \ddot{O} - \mathcal{I} -space X , the following properties hold:

- (i) $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) = LC(X)$.
- (ii) $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) \subseteq GLC(X)$.
- (iii) $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) \subseteq GLSC(X)$.
- (iv) $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) \subseteq \omega\text{-}LC(X)$.

Proof

(i) Since every \ddot{O} - \mathcal{I} -open set is open and every \ddot{O} - \mathcal{I} -cld is $*$ -closed, $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) \subseteq LC(X)$ and hence $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) = LC(X)$.

(ii), (iii) and (iv) follows from (i), since for any space X , $LC(X) \subseteq GLC(X)$, $LC(X) \subseteq GLSC(X)$ and $LC(X) \subseteq \omega\text{-}LC(X)$.

Corollary 3.18

If $GO(X) = \tau$, then $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) \subseteq GLSC(X) \subseteq LSC(X)$.

Proof

$GO(X) = \tau$ implies that X is a T \ddot{O} - \mathcal{I} -space and hence by Theorem 3.17, $\ddot{O}\text{-}\mathcal{I}\text{-LC}(X) \subseteq GLSC(X)$. Let $K \in GLSC(X)$. Then $K = V \cap F$, where V is g -open and F is semi-closed. By hypothesis, V is open and hence K is a lsc-set and so $K \in LSC(X)$.

Definition 3.19

A subset S of a space X is called:

- (i) $\ddot{O}\text{-}\mathcal{I}\text{-lc}^*$ set if $S = H \cap G$, where H is \ddot{O} - \mathcal{I} -open in X and G is closed in X .
- (ii) $\ddot{O}\text{-}\mathcal{I}\text{-lc}^{**}$ set if $S = H \cap G$, where H is open in X and G is \ddot{O} - \mathcal{I} -cld in X .

The class of all $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ (resp. $\check{\theta}\text{-}\mathcal{I}\text{-}lc^{**}$) sets in ideal topological space X is denoted by $\ddot{O}\text{-}\mathcal{I}\text{-}LC^*(X)$ (resp. $\ddot{O}\text{-}\mathcal{I}\text{-}LC^{**}(X)$).

Proposition 3.20

Each lc -set is $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ set but not reverse.

Proof

It follows from Definition 3.19 (i) and Definition of locally closed set.

Example 3.21

The set $\{q_1\}$ in Example 3.5 is $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ set but it is not a lc set in X .

Proposition 3.22

Each lc -set is $\ddot{O}\text{-}\mathcal{I}\text{-}lc^{**}$ set but not reverse.

Proof

It follows from Definition 3.19 (ii) and Definition of locally closed set.

Example 3.23

The set $\{p_1, r_1\}$ in Example 3.5 is $\ddot{O}\text{-}\mathcal{I}\text{-}lc^{**}$ set but it is not a lc set in X .

Proposition 3.24

Each $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ set is $\ddot{O}\text{-}\mathcal{I}\text{-}lc$ set but not reverse.

Proof

It follows from Definitions 3.1 and 3.19 (i).

Example 3.25

The set $\{p_1, q_1\}$ in Example 3.5 is $\ddot{O}\text{-}\mathcal{I}\text{-}lc$ set but it is not a $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ set in X .

Proposition 3.26

Each $\ddot{O}\text{-}\mathcal{I}\text{-}lc^{**}$ set is $\ddot{O}\text{-}\mathcal{I}\text{-}lc$ set but not reverse.

Proof

It follows from Definitions 3.1 and 3.19 (ii).

Remark 3.27

The concepts of $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ sets and lsc sets are independent of each other.

Example 3.28

The set $\{r_1\}$ in Example 3.5 is $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ set but it is not a lsc set in X and the set $\{p_1\}$ in Example 2.3 is lsc set but it is not a $\ddot{O}\text{-}\mathcal{I}\text{-}lc^*$ set in X .

Remark 3.29

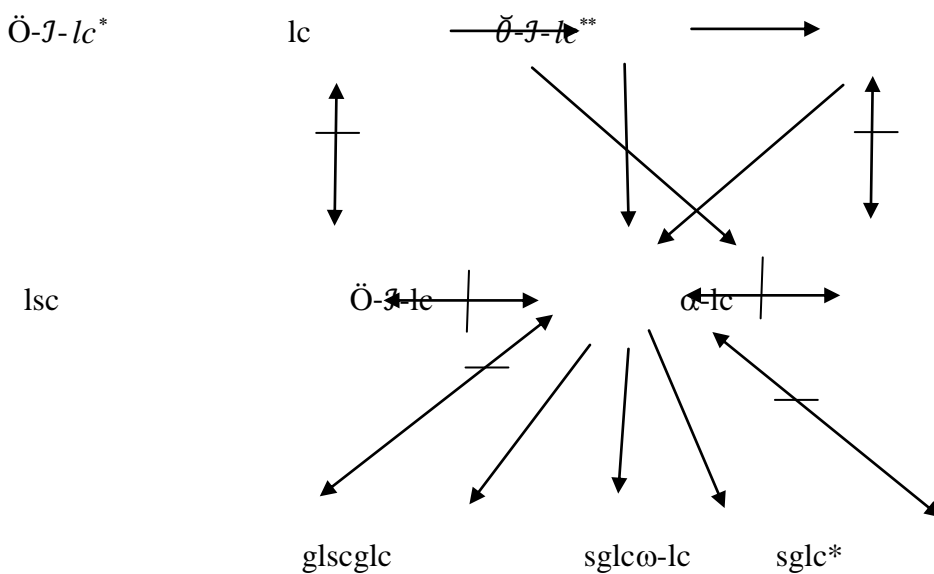
The concepts of $\ddot{O}\text{-}\mathcal{I}\text{-}lc^{**}$ sets and $\alpha\text{-}lc$ sets are independent of each other.

Example 3.30

The set $\{p_1, q_1\}$ in Example 3.5 is $\ddot{O}\text{-}\mathcal{I}\text{-}lc^{**}$ set but it is not a $\alpha\text{-}lc$ set in X and the set $\{p_1, q_1\}$ in Example 2.3 is $\alpha\text{-}lc$ set but it is not a $\ddot{O}\text{-}\mathcal{I}\text{-}lc^{**}$ set in X .

Remark 3.31

From the above discussions we have the following implications where $A \rightarrow B$ (resp. $A \nleftrightarrow B$) represents A implies B but not conversely (resp. A and B are independent of each other).

**Proposition 3.32**

If $G O(X) = \tau$, then $\ddot{O}\text{-}\mathcal{I}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{I}\text{-}LC^*(X) = \ddot{O}\text{-}\mathcal{I}\text{-}LC^{**}(X)$.

Proof

For any space X , $\tau \subseteq \ddot{O}\text{-}\mathcal{I}\text{-}O(X) \subseteq G O(X)$. Therefore by hypothesis, $\ddot{O}\text{-}\mathcal{I}\text{-}O(X) = \tau$. i.e., X is a $T\ddot{O}\text{-}\mathcal{I}$ space and hence $\ddot{O}\text{-}\mathcal{I}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{I}\text{-}LC^*(X) = \ddot{O}\text{-}\mathcal{I}\text{-}LC^{**}(X)$.

Remark 3.33

The reverse of Proposition 3.32 need not be true.

For the ITPSX in Example 3.3. $\ddot{O}\text{-}\mathcal{I}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{I}\text{-}LC^*(X) = \ddot{O}\text{-}\mathcal{I}\text{-}LC^{**}(X)$. However, $G O(X) = \{\phi, \{p_1\}, \{q_1\}, \{r_1\}, \{p_1, q_1\}, \{q_1, r_1\}, X\} \neq \tau$.

Proposition 3.34

Let X be an ITPS. If $GO(X) \subseteq LC(X)$, then $\ddot{O}\text{-}\mathcal{J}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X)$.

Proof

Let $K \in \ddot{O}\text{-}\mathcal{J}\text{-}LC(X)$. Then $S = H \cap G$ where H is $\ddot{O}\text{-}\mathcal{J}$ -open and G is $\ddot{O}\text{-}\mathcal{J}$ -cld. Since $\ddot{O}\text{-}\mathcal{J}\text{-}O(X) \subseteq GO(X)$ and by hypothesis $GO(X) \subseteq LC(X)$, H is locally closed. Then $H = P \cap Q$, where P is open and Q is $*$ -closed. Therefore, $S = P \cap (Q \cap G)$. We have, $Q \cap G$ is $\ddot{O}\text{-}\mathcal{J}$ -cld and hence $S \in \ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X)$. i.e., $\ddot{O}\text{-}\mathcal{J}\text{-}LC(X) \subseteq \ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X)$. For any ITPS, $\ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X) \subseteq \ddot{O}\text{-}\mathcal{J}\text{-}LC(X)$ and so $\ddot{O}\text{-}\mathcal{J}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X)$.

Remark 3.35

The reverse of Proposition 3.34 need not be true in general.

For the ITPS X in Example 3.3, then $\ddot{O}\text{-}\mathcal{J}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X) = \{\emptyset, \{q_1\}, \{p_1, r_1\}, X\}$. But $GO(X) = \{\emptyset, \{p_1\}, \{q_1\}, \{r_1\}, \{p_1, q_1\}, \{q_1, r_1\}, X\} \not\subseteq LC(X) = \{\emptyset, \{q_1\}, \{p_1, r_1\}, X\}$.

Corollary 3.36

Let X be an ITPS. If $\omega O(X) \subseteq LC(X)$, then $\ddot{O}\text{-}\mathcal{J}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X)$.

Proof

It follows from the fact that $\omega O(X) \subseteq GO(X)$ and Proposition 3.34.

Remark 3.37

The reverse of Corollary 3.36 need not be true in general.

For the ITPSX in Example 2.8, then $\ddot{O}\text{-}\mathcal{J}\text{-}LC(X) = \ddot{O}\text{-}\mathcal{J}\text{-}LC^{**}(X) = \{\emptyset, \{q_1\}, \{p_1, r_1\}, X\}$. But $\omega O(X) = P(X) \not\subseteq LC(X) = \{\emptyset, \{q_1\}, \{p_1, r_1\}, X\}$.

The following results are characterizations of $\ddot{O}\text{-}\mathcal{J}$ -lc sets, $\ddot{O}\text{-}\mathcal{J}\text{-}lc^*$ sets and $\ddot{O}\text{-}\mathcal{J}\text{-}lc^{**}$ sets.

Theorem 3.38

Assume that $\ddot{O}\text{-}\mathcal{J}\text{-}C(X)$ is closed under finite intersection. For a subset S of X , the following statements are equivalent:

- (i) $S \in \ddot{O}\text{-}\mathcal{J}\text{-}LC(X)$.
- (ii) $S = H \cap \ddot{O}\text{-}\mathcal{J}\text{-}cl(K)$ for some $\ddot{O}\text{-}\mathcal{J}$ -open set H .
- (iii) $\ddot{O}\text{-}\mathcal{J}\text{-}cl(S) - S$ is $\ddot{O}\text{-}\mathcal{J}$ -cld.
- (iv) $S \cup (\ddot{O}\text{-}\mathcal{J}\text{-}cl(S))^c$ is $\ddot{O}\text{-}\mathcal{J}$ -open.
- (v) $S \subseteq \ddot{O}\text{-}\mathcal{J}\text{-}int(S \cup (\ddot{O}\text{-}\mathcal{J}\text{-}cl(S))^c)$.

Proof

(i) \Rightarrow (ii). Let $K \in \tilde{O}\text{-}\mathcal{I}\text{-LC}(X)$. Then $S = H \cap G$ where H is $\tilde{O}\text{-}\mathcal{I}$ -open and G is $\tilde{O}\text{-}\mathcal{I}$ -cld. Since $S \subseteq G$, $\tilde{O}\text{-}\mathcal{I}\text{-cl}(S) \subseteq G$ and so $H \cap \tilde{O}\text{-}\mathcal{I}\text{-cl}(S) \subseteq S$. Also $S \subseteq H$ and $S \subseteq \tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$ implies $S \subseteq H \cap \tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$ and therefore $S = H \cap \tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$.

(ii) \Rightarrow (iii). $S = H \cap \tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$ implies $\tilde{O}\text{-}\mathcal{I}\text{-cl}(S) - S = \tilde{O}\text{-}\mathcal{I}\text{-cl}(S) \cap H^c$ which is $\tilde{O}\text{-}\mathcal{I}$ -cld since H^c is $\tilde{O}\text{-}\mathcal{I}$ -cld and $\tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$ is $\tilde{O}\text{-}\mathcal{I}$ -cld.

(iii) \Rightarrow (iv). $S \cup (\tilde{O}\text{-}\mathcal{I}\text{-cl}(S))^c = (\tilde{O}\text{-}\mathcal{I}\text{-cl}(S) - S)^c$ and by assumption, $(\tilde{O}\text{-}\mathcal{I}\text{-cl}(S) - S)^c$ is $\tilde{O}\text{-}\mathcal{I}$ -open and so is $S \cup (\tilde{O}\text{-}\mathcal{I}\text{-cl}(S))^c$.

(iv) \Rightarrow (v). By assumption, $S \cup (\tilde{O}\text{-}\mathcal{I}\text{-cl}(S))^c = \tilde{O}\text{-}\mathcal{I}\text{-int}(S \cup (\tilde{O}\text{-}\mathcal{I}\text{-cl}(S))^c)$ and hence $S \subseteq \tilde{O}\text{-}\mathcal{I}\text{-int}(S \cup (\tilde{O}\text{-}\mathcal{I}\text{-cl}(S))^c)$.

(v) \Rightarrow (i). By assumption and since $S \subseteq \tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$, $K = \tilde{O}\text{-}\mathcal{I}\text{-int}(S \cup (\tilde{O}\text{-}\mathcal{I}\text{-cl}(S))^c) \cap \tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$. Therefore, $S \in \tilde{O}\text{-}\mathcal{I}\text{-LC}(X)$.

Theorem 3.39

For a subset S of X , the following statements are equivalent:

- (i) $S \in \tilde{O}\text{-}\mathcal{I}\text{-LC}^*(X)$.
- (ii) $S = H \cap K^*$ for some $\tilde{O}\text{-}\mathcal{I}$ -open set H .
- (iii) $S^* - S$ is $\tilde{O}\text{-}\mathcal{I}$ -cld.
- (iv) $S \cup (S^*)^c$ is $\tilde{O}\text{-}\mathcal{I}$ -open.

Proof

(i) \Rightarrow (ii). Let $S \in \tilde{O}\text{-}\mathcal{I}\text{-LC}^*(X)$. There exist an $\tilde{O}\text{-}\mathcal{I}$ -open set S and a \star -closed set G such that $S = H \cap G$. Since $S \subseteq H$ and $S \subseteq S^*$, $S \subseteq H \cap S^*$. Also, since $S^* \subseteq G$, $H \cap S^* \subseteq H \cap G = S$. Therefore $S = H \cap S^*$.

(ii) \Rightarrow (i). Since H is $\tilde{O}\text{-}\mathcal{I}$ -open and S^* is a \star -closed set, $S = H \cap S^* \in \tilde{O}\text{-}\mathcal{I}\text{-LC}^*(X)$.

(ii) \Rightarrow (iii). Since $S^* - S = S^* \cap H^c$, $S^* - S$ is $\tilde{O}\text{-}\mathcal{I}$ -cld.

(iii) \Rightarrow (ii). Let $H = (S^* - S)^c$. Then by assumption H is $\tilde{O}\text{-}\mathcal{I}$ -open in X and $S = H \cap S^*$.

(iii) \Rightarrow (iv). Let $G = S^* - S$. Then $G^c = S \cup (S^*)^c$ and $S \cup (S^*)^c$ is $\tilde{O}\text{-}\mathcal{I}$ -open.

(iv) \Rightarrow (iii). Let $H = S \cup (S^*)^c$. Then H^c is $\tilde{O}\text{-}\mathcal{I}$ -cld and $H^c = S^* - S$ and so $S^* - S$ is $\tilde{O}\text{-}\mathcal{I}$ -cld.

Theorem 3.40

Let S be a subset of X . Then $S \in \tilde{O}\text{-}\mathcal{I}\text{-LC}^{**}(X)$ if and only if $S = H \cap \tilde{O}\text{-}\mathcal{I}\text{-cl}(S)$ for some open set H .

Proof

Let $S \in \tilde{O}\text{-}\mathcal{J}\text{-LC}^{**}(X)$. Then $S = H \cap G$ where H is open and G is $\tilde{O}\text{-}\mathcal{J}\text{-cld}$. Since $S \subseteq G$, $\tilde{O}\text{-}\mathcal{J}\text{-cl}(S) \subseteq G$. We obtain $S = S \cap \tilde{O}\text{-}\mathcal{J}\text{-cl}(S) = H \cap G \cap \tilde{O}\text{-}\mathcal{J}\text{-cl}(S) = H \cap \tilde{O}\text{-}\mathcal{J}\text{-cl}(S)$.

Converse part is trivial.

Corollary 3.41

Let S be a subset of X . If $S \in \tilde{O}\text{-}\mathcal{J}\text{-LC}^{**}(X)$, then $\tilde{O}\text{-}\mathcal{J}\text{-cl}(S) - S$ is $\tilde{O}\text{-}\mathcal{J}\text{-cld}$ and $S \cup (\tilde{O}\text{-}\mathcal{J}\text{-cl}(S))^c$ is $\tilde{O}\text{-}\mathcal{J}\text{-open}$.

Proof

Let $S \in \tilde{O}\text{-}\mathcal{J}\text{-LC}^{**}(X)$. Then by Theorem 3.40, $S = H \cap \tilde{O}\text{-}\mathcal{J}\text{-cl}(S)$ for some open set H and $\tilde{O}\text{-}\mathcal{J}\text{-cl}(S) - S = \tilde{O}\text{-}\mathcal{J}\text{-cl}(S) \cap H^c$ is $\tilde{O}\text{-}\mathcal{J}\text{-cld}$ in X . If $G = \tilde{O}\text{-}\mathcal{J}\text{-cl}(S) - S$, then $G^c = S \cup (\tilde{O}\text{-}\mathcal{J}\text{-cl}(S))^c$ and G^c is $\tilde{O}\text{-}\mathcal{J}\text{-open}$ and so is $S \cup (\tilde{O}\text{-}\mathcal{J}\text{-cl}(S))^c$.

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