(1,2)*-$\hat{D}$-Closed Sets in Bitopological Spaces

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Abstract

In the paper, we introduce the notions of (1,2)*-$\hat{D}$-closed sets and (1,2)*-D-closed sets in bitopological spaces.

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1. INTRODUCTION

Levine introduced the concept of g-closed sets in topological spaces. Following this attempts, modern mathematics generalized this concept and are being found many generalization of g-closed sets. In the paper, we introduce the notion of (1,2)*-$\hat{D}$-closed sets and (1,2)*-D-closed sets in bitopological spaces.

2. PRELIMINARIES

Definition 2.1

A subset S of a TPS X is called:

(i) semi-open if S$\subseteq$ cl(int(S));

(ii) α-open if S$\subseteq$ int(cl(int(S)));

(iii) β-open (semi-pre-open) if S$\subseteq$ cl(int(cl(S)));

(vi) regular open if S= int(cl(S))

The complements of the above-mentioned open sets are called their respective closed sets.

The semi-closure (resp. α-closure, semi-pre-closure, regular-closure) of a subset S of X, scl(S) (resp. αcl(S), spcl(S), rcl(S)) is defined to be the intersection of all semi-closed (resp. α-closed, semi-pre-closed, regular-closed) of X containingS. It is known that scl(S) (resp. αcl(S), spcl(S), rcl(S)) is semi-closed (resp. α-closed, semi-pre-closed, regular-closed).

Definition 2.2
A subset S of a TPS X is called

(i) g-closed set (briefly, g-cld) if cl(S) $\subseteq$P whenever S$\subseteq$P and P is open.

(ii) αgs-closed (briefly, αgs -cld) if αcl(S) $\subseteq$Pwhenever S$\subseteq$P and P is semi-open.

(iii) semi-generalized closed (briefly, sg-cld) if scl(S) $\subseteq$P whenever S$\subseteq$P and P is semi-open.

(iv) ψ-closed (briefly, ψ-cld) if scl(S) $\subseteq$P whenever S$\subseteq$P and P is sg-open.

(v) generalized semi-closed (briefly, gs-cld) if scl(S)$\subseteq$P whenever S$\subseteq$P and P is open.

(vi) α -generalized closed (briefly, α g-cld) if α cl(S) $\subseteq$P whenever S$\subseteq$P and P is open.

(vii) generalized semi-pre-closed(briefly, gsp-cld) if spcl(S) $\subseteq$P whenever S$\subseteq$P and P is open .

The complements of the above-mentioned closed sets are called their respective open sets.

Definition 2.3

The intersection of all sg-open subsets of X containing S is called the sg-kernel of S and denoted by sg-ker(S).

Definition 2.4

A subset S of X is called locally closed (briefly, lc) if S = U $\cap$ F, where U is open and F is closed in X.

Definition 2.5

A subset S of a space X is called:

(i) ĝ-cld (=ω-cld) if cl(S) $\subseteq$P whenever S$\subseteq$P and P is semi-open in X. The complement of ĝ-cldis called ĝ-open set;

(ii) $\ddot{g}$ -cld if cl(S) $\subseteq$Pwhenever S$\subseteq$P and P is sg-open in X.

The complement of $\ddot{g}$ -cld is called $\ddot{g}$ -open.

Definition 2.6

A subset S of a space X is called a g*s-cld set if scl(S) $\subseteq$P whenever S$\subseteq$P and P is gs-open in X.
The complement of g*s-cld is called g*s-open.

Definition 2.7

A space X is called

(i) $T_{1/2}$-space if every g-cld is closed.

(ii) T_b-space if every gs-cldis closed.

(iii) αT_b-space if every α g-cld is closed.

(iv) T_ω-space if every ω-cldis closed.

(v) T_p*-space if every g*p-cldis closed.

(vi) $*_sT_p$-space if every gsp-cld is g*p-cld.

(vii) αT_d-space if every αg-cld is g-cld.

(viii) α-space if every α-cldis closed.

(ix) T_ω-space if every ω-cld is closed.

Definition 2.8

A topological space X is called:

(i) semi generalized -T_0 (briefly, sg-T_0) if and only if to each pair of distinct points x, y of X, there exists a sg-open set containing one but not the other.

(ii) semi generalized -T_1 (briefly, sg-T_1) if and only if to each pair of distinct points x, y of X, there exists a pair of sg-open sets, one containing x but not y, and the other containing y but not x.

(iii) semi generalized -R_0 (briefly, sg-R_0) if and only if for each sg-open set G and $x \in G$ implies sg-cl({x}) $\subseteq$ G.

Remark 2.9

The collection of all rg-closed sets in X is denoted by RG C(X).

The collection of all rg-open sets in X is denoted by RG O(X).

Definition 2.10

A subset S of a space X is called:

(i) generalized locally closed (briefly, glc) if $S = V\cap F$, where V is g-open and F is g-cld.

(ii) semi-generalized locally closed (briefly, sglc) if $S = V\cap F$, where V is sg-open and F is sg-cld.

(iii) regular-generalized locally closed (briefly, rg-lc) if $S = V\cap F$, where V is rg-open and F is rg-cld.

(iv) generalized locally semi-closed (briefly, glsc) if $S = V\cap F$, where V is g-open and F is semi-cld.

(v) locally semi-closed (briefly, lsc) if $S = V\cap F$, where V is open and F is semi-cld.

(vi) α-locally closed (briefly, α-lc) if $S = V\cap F$, where V is α-open and F is α-cld.

(vii) ω-locally closed (briefly, ω-lc) if $S = V\cap F$, where V is ω-open and F is ω-cld.

The class of all generalized locally closed (resp. generalized locally semi-closed, locally semi-closed, ω-locally closed) sets in X is denoted by GLC (X) (resp. $GLSC$ (X), LSC (X), ω-LC(X)).

Throughout this paper (X, τ_1, τ_2) or X will always denote bitopological spaces when A is a subset of $\tau_{1,2}$-cl(A) and $\tau_{1,2}$-int(A) denote the $\tau_{1,2}$-closure set of A and $\tau_{1,2}$-interior set of A respectively.

3. (1,2)*-$\widehat{D}$-CLOSED SETSIN BITOPOLOGICAL SPACES

Definition 3.1

A subset A of X is called

(i) (1,2)*-D-closed (briefly, (1,2)*-D-cld) if (1,2)*-scl(A) $\subseteq \tau_{1,2}$-int U whenever A$\subseteq$U and U is (1,2)*-ω-open. The complement of (1,2)*-D-closed set is called (1,2)*-D-open.

(ii) (1,2)*-$\widehat{D}$-closed (briefly, (1,2)*-$\widehat{D}$-cld) if (1,2)*-spcl(A) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-D-open. The complement of (1,2)*-$\widehat{D}$-closed set is called (1,2)*-$\widehat{D}$-open.

The class of all (1,2)*-$\widehat{D}$-cld in X is denoted by (1,2)*-$\widehat{D}$C.

Proposition 3.2

Each $\tau_{1,2}$-closed (resp. (1,2)*-α-cld, (1,2)*-pre-cld, (1,2)*-semi-cld) is (1,2)*-$\widehat{D}$-cld.

Proof

Let A be any $\tau_{1,2}$-closed set. Let A $\subseteq$ U and U is (1,2)*-D-open set in X. Then $\tau_{1,2}$-cl(A) $\subseteq$U. But (1,2)*-spcl(A) $\subseteq \tau_{1,2}$-cl(A) $\subseteq$U. Thus A is (1,2)*-$\widehat{D}$-cld. The proof follows from the facts that(1,2)*-spcl(A) $\subseteq$(1,2)*-scl(A) $\subseteq \tau_{1,2}$-cl(A) and (1,2)*-spcl(A) $\subseteq$(1,2)*-pcl(A) $\subseteq$(1,2)*-αcl(A) $\subseteq \tau_{1,2}$-cl(A).

Remark 3.3

The reverse of the above proposition need not be true.

Example 3.4

Let X={m,n,o,p,q} with τ_1={ϕ,{m},{m,n},X} and τ_2={ϕ,{o,p},X}. Then $\tau_{1,2}$={ϕ,{m},{m,n},{o,p},{m,o,p},{m,n,o,p},X}. Here, J={m,n,p} is (1,2)*-$\widehat{D}$-cld (resp. not (1,2)*-pre-cld, not (1,2)*-α-cld, not (1,2)*-semi-cld).

Proposition 3.5

Each (1,2)*-$\widehat{D}$-cld is (1,2)*-gspr-cld

Proof

Let A be any (1,2)*-$\widehat{D}$-cld set. Let A $\subseteq$ U and U is regular (1,2)*-open in X. Since each regular (1,2)*-open set is $\tau_{1,2}$-open and each $\tau_{1,2}$-open is (1,2)*-D-open, we get (1,2)*-spcl(A) $\subseteq$U. Hence, A is (1,2)*-gspr-cld.

Remark 3.6

The reverse of the above proposition need not be true.

Example 3.7

Let X={m,n,o,p} with τ_1={ϕ,{m},{n},{m,n},X} and τ_2={ϕ,{p}, {n,p},X}. The $\tau_{1,2}$={ϕ,{m},{n},{p},{m,n},{n,p},{m,p},{m,n,p},X}.Then, J={m,n,p} is (1,2)*-gspr-cld but not (1,2)*-$\widehat{D}$-cld.

Theorem 3.8

Each (1,2)*-ω-cld is (1,2)*-$\widehat{D}$-cld.

Proof

Let A be (1,2)*-ω-cld in X. Let $A \subseteq U$ and U is (1,2)*-D-open. Then $\tau_{1,2}$-cl(A) $\subseteq$U. Since each (1,2)*-ω-cld set is (1,2)*-pre-cld and each (1,2)*-pre-cld set is (1,2)*-semi-pre-cld, A is (1,2)*-semi-pre-cld.

Then A $\subseteq$(1,2)*-pcl(A) $\subseteq$(1,2)*-ωcl(A), Since each$\tau_{1,2}$-closed is (1,2)*-ω-cld, (1,2)*-ω-cl(A) $\subseteq\tau_{1,2}$-cl(A).Therefore, (1,2)*-spcl(A) $\subseteq$(1,2)*-pcl(A) $\subseteq\tau_{1,2}$-cl(A) $\subseteq$U. Hence, A is (1,2)*-$\widehat{D}$-cld.

Remark 3.9

The reverse of the above proposition need not be true.

Example 3.10

Let X={m,n,o} with τ_1={ϕ,{m},X} and τ_2={ϕ,{n},X}. Then $\tau_{1,2}$={ϕ,{m},{n},{m,n},X}. Then, J ={m} is (1,2)*-$\widehat{D}$-cld but not (1,2)*-ω-cld.

Proposition 3.11

Each (1,2)*-$\widehat{D}$-cld is (1,2)*-gsp-cld.

Proof

Let A be any (1,2)*-$\widehat{D}$-cld in X. Let $A \subseteq U$ and U is $\tau_{1,2}$-open set in X. Since every $\tau_{1,2}$-open is (1,2)*-D-open, we get (1,2)*-spcl(A) $\subseteq$U. Hence A is (1,2)*-gsp-cld.

Remark 3.12

The reverse of the above proposition need not be true.

Example 3.13

Let X={m,n,o} with τ_1={ϕ,X} and τ_2={ϕ,{m},X}. Then $\tau_{1,2}$={ϕ,{m},X}. Then, J={m,n} is (1,2)*-gsp-cld but not (1,2)*-$\widehat{D}$-cld.

Proposition 3.14

Each (1,2)*-$\widehat{D}$-cld is (1,2)*-pre-semi-cld

Proof

Let A be any (1,2)*-$\widehat{D}$-cld in X. Let $A \subseteq U$ and U is (1,2)*-g-open in X. Since each (1,2)*-g-open is (1,2)*-D-open, we get (1,2)*-spcl(A) $\subseteq$U. Hence, A is (1,2)*-pre-semi-cld.

Remark 3.15

The reverse of the above proposition need not be true.

Example 3.16

Let X={m,n,o,p} with τ_1={ϕ,{m},X} and τ_2={ϕ,{m,n,o},X}. Then $\tau_{1,2}$={ϕ,{m},{m,n,o},X}. Then, J={m,n,o,p} is (1,2)*-pre-semi-cld but not (1,2)*-$\widehat{D}$-cld.

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