New Topological Sets Close to ω-Open Sets

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ABSTRACT: This paper starts by introducing some topological sets that are very close to ω-open and ω-closed sets in the sense of Hdeib. Moreover the notions of regular ω*-open, semi- ω*-open, pre-ω*-open, α-ω*-open, β-ω*-open, b- ω*-open, *b-ω*-open , $b^{\#}$-ω*-open sets and the corresponding closed sets are introduced and studied in this paper.

1. Introduction

This paper starts by introducing some topological sets that are very close to ω-open and ω-closed sets in the sense of Hdeib. Moreover the notions of regular ω*-open, semi- ω*-open, pre-ω*-open, α-ω*-open, β-ω*-open, b- ω*-open, *b-ω*-open , $b^{\#}$-ω*-open sets and the corresponding closed sets are introduced and studied in this paper. Further, the concepts of pω-set, qω-set, Qω-set, pω*-set, qω*-set and Qω*-set are defined to investigate the properties of the above sets in conjunction with the recent related concepts that are available in the literature of point set topology.

2. Prelimeneries

Result 2.1

Let A and B be any two subsets of a topological space (X,τ). The following relations on the interior and closure operators will be useful.

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IntA ⊆ IntClIntA ⊆ ClIntA ⊆ Cl IntCl A ⊆ ClA.

IntA ⊆ IntClIntA ⊆ IntClA ⊆ Cl IntClA ⊆ ClA.

IntCl $(A\cap B)\subseteq$ (IntClA) $\cap$ (IntClB).

ClInt $(A\cap B)\subseteq$ (ClIntA) $\cap$ (ClIntB).

(IntClA) $\cup$ (IntClB) $\subseteq$ IntCl$(A\cup B)$.

(ClIntA) $\cup$ (ClIntB) $\subseteq$ ClInt $(A\cup B)$.

ClIntClIntA = ClIntA.

IntClInt ClA = IntClA.

Lemma 2.2

(i) If B is open then (ClA) $\cap$ B $\subseteq$ Cl$(A\cap B)$.
(ii) If B is closed then Int $(A\cup B)\subseteq$ (IntA) $\cup$ B.

Lemma 2.3

(i) ClInt Cl $(A\cup B)$ = ClIntClA $\cup$ ClIn ClB.

(ii) IntClInt $(A\cap B)$ = IntClIntA $\cap$ IntClIntB.

Definition 2.4 The set A is called

(i) regular open if A= Int ClA,
(ii) semi-open if A$\subseteq$ Cl IntA,
(iii) pre-open if A$\subseteq$IntClA,
(iv) b-open if A$\subseteq$Cl IntA$\cup$IntClA,
(v) *b-open if A$\subseteq$Cl IntA$\cap$IntClA,
(vi) $b^{\#}$-open if A=ClIntA$\cup$IntClA,

Definition 2.5 The set A is called

(i) a p-set if ClIntA$\subseteq$IntClA,
(ii) a q-set if IntClA$\subseteq$ ClIntA,
(iii) a Q-set if IntClA$\subseteq$ ClIntA,
(iv) a t-set if IntA = IntClA,
(v) a t*-set if ClA = ClIntA.

Definition 2.6 The set A is called

(i) α-open if A$\subseteq$IntClIntA.
(ii) β-open if A$\subseteq$ClIntClA.

Definition 2.7 The set A is called

(i) regular closed $\Leftrightarrow$ A= ClIntA,
(ii) semi-closed $\Leftrightarrow$IntClA $\subseteq$ A,

(iii) pre-closed $\Leftrightarrow$ ClIntA $\subseteq$ A,
(iv) b-closed $\Leftrightarrow$ Cl IntA $\cap$ IntClA $\subseteq$ A,
(v) *b-closed $\Leftrightarrow$ ClIntA $\cup$ IntClA $\subseteq$ A,
(vi) $b^{\#}$-closed $\Leftrightarrow$ Cl IntA $\cap$ IntClA $\subseteq$ A,
(vii) α-closed $\Leftrightarrow$ Cl IntClA $\subseteq$ A,
(viii) β-closed $\Leftrightarrow$ IntClIntA $\subseteq$ A.

Lemma 2.8 The set A is

(i) regular open $\Leftrightarrow$ A= IntClInt A,

(ii) regular closed $\Leftrightarrow$ A= ClIntClA,

(iii) semi-open $\Leftrightarrow$ ClA= ClIntA,

(iv) semi-closed $\Leftrightarrow$ IntA= IntClA,

(v) $\boldsymbol{\beta}$-open $\Leftrightarrow$ ClA=ClIntClA,

(vi) $\boldsymbol{\beta}$-closed $\Leftrightarrow$ IntA= IntClIntA.

Lemma 2.9

(i) If A or B is semi-open then IntClA $\cap$ IntClB = IntCl(A $\cap$ B).

(ii) If A or B is semi-closed then ClInt(A $\cup$ B) =ClIntA $\cup$ ClIntB.

Definition 2. 10 Let A and B be any two subsets of a space (X,τ). We say that

(i) A is near to B in (X,τ) if IntA = IntB
(ii) A is closer to B in (X,τ) if ClA = ClB.
(iii) A is almost near to B in (X,τ) if IntClA = IntClB.
(iv) A is almost closer to B in (X,τ) if ClInt A = IntClB.

Definition 2.11 A function f : $(X,\tau)\rightarrow(Y,\sigma)$ is called

(i) regular continuous if $f^{-1}(V)$ is regular open in X for each V $\in\sigma$,
(ii) regular irresolute if $f^{-1}(V)$ is regular open in X for each V $\in RO(Y,\sigma)$.
Other types of continuity and irresoluteness can be analogously defined.

Definition 2.12 By a neighourhood (briefly nbd) of a point x in a space X we mean an open set containing x.

Definition 2.13 A space X is locally countable if the space has a base consisting of countable sets and is anti locally countable if every non-empty open set in X is uncountable.

Definition 2.14 For every open neighbourhood U of A,

(i) if ClA $\subseteq$ U then A is g-closed,

(ii) if Cl IntA⊆U then A is wg-closed,

(iii) if αClA⊆U then A is αg-closed,

(iv) if sClA⊆U then A is gs-closed,

(v) if pClA⊆U then A is gp-closed and

(vi) if βClA⊆U then A is gβ-closed.

Definition 2.15

(i) If A ⊆ V, V is regular open ⟹ClA ⊆ V then A is rg-closed.

(ii) If A ⊆ V, V is regular open ⟹pClA ⊆ V then A is gpr-closed.

(iii) If A ⊆ V, V is α-open ⟹αClA ⊆ V then A is gα-closed.

Definition 2.16 A point x of X is said to be a condensation point of A if for each U∈τ with x∈U, the set U∩A is uncountable.

Clearly every condensation point of A is its limit point. Let Cond(A)={x:x is a condensation point of A} and Limit(A) ={x:x is a limit point of A}. Obviously Limit(A)⊇Cond(A).

Definition 2.17 A subset B of X is said to be ω-closed in (X,τ) if B ⊇ Cond(B).

It is easy to see that every closed set is ω-closed. The complement of an ω-closed set is ω-open. Khalid Y.Al.Zoubi, Al.Nashef established that the collection of all ω-open sets in (X,τ) is a topology on X denoted by τ_ω which is finer than τ. Let $Cl_\omega(\ )$ and $Int_\omega(\ )$ denote the closure and interior operators in (X, τ_ω).

Lemma 2.18 A subset B of X is ω-open in (X,τ) if and only if for each x∈B there exists U∈τ such that U\B is countable. Equivalently x∈Int_ωBif and only if there exists U∈τ such that U\B is countable.

3. ρ-ω*- OPEN SETS where ρ∈{semi, pre, α, β, b}

The one level operators Int(.), Cl(.), Int_ω(.) and Cl_ω(.) in (X,τ) induce twelve two level operators in (X,τ) namely IntCl(.), IntInt_ω(.), IntCl_ω(.), ClInt(.), ClInt_ω(.), ClCl_ω(.), Int_ωInt(.), Int_ωCl(.), $Int_\omega Cl_\omega$ (.), $Cl_\omega Int_\omega$ (.), Cl_ωInt(.), Cl_ωCl(.). These two level operators have been linked as shown in the next lemma. It is interesting to note that the two level operators Int_ωInt(.), IntInt_ω(.) and Cl_ωCl(.), Cl Cl_ω(.) are reduced to the one level operators Int(.) and Cl(.) respectively. Throughout this paper, (X,τ) is a topological space, A and B are subsets of X.

Lemma 3.1 For any subset A of a space (X,τ), the following always hold.

(i) Int_ωIntA= IntA = IntInt_ωA.

(ii) Cl_ωClA =Cl Cl_ωA = ClA.

(iii) $IntCl_\omega A \subseteq IntClA \subseteq Int_\omega ClA$.
(iv) $IntCl_\omega A \subseteq Int_\omega Cl_\omega A \subseteq Int_\omega ClA$.
(v) $Cl_\omega IntA \subseteq ClIntA \subseteq ClInt_\omega A$.
(vi) $Cl_\omega IntA \subseteq Cl_\omega Int_\omega A \subseteq ClInt_\omega A$.

Proof. We have $IntA \subseteq Int_\omega A \subseteq A \subseteq Cl_\omega A \subseteq ClA$. By applying the interior operator on $IntA \subseteq Int_\omega A \subseteq A$, we get $IntA \subseteq Int\ Int_\omega A \subseteq IntA$ so that $Int\ Int_\omega A = Int\ A$. Since IntA is ω-open we have $Int_\omega IntA = IntA$.This proves the assertion (i). The assertion (ii) can be analogously established.

By replacing A by ClA and A by Cl_ω A in $IntA \subseteq Int_\omega A$, we have $IntClA \subseteq Int_\omega ClA$ and

$Int\ Cl_\omega A \subseteq Int_\omega Cl_\omega A$. By replacing A by IntA and A by Int_ω A in $Cl_\omega A \subseteq ClA$, we have $Cl_\omega IntA \subseteq ClIntA$ and $Cl_\omega Int_\omega A \subseteq Cl\ Int_\omega A$.

Taking closure and ω-closure operation on either side of $IntA \subseteq Int_\omega A$ we get

$ClIntA \subseteq ClInt_\omega A$ and $Cl_\omega IntA \subseteq Cl_\omega Int_\omega A$.

Taking interior and ω-interior operation on either side of $Cl_\omega A \subseteq ClA$ we get

$IntCl_\omega A \subseteq IntClA$ and $Int_\omega Cl_\omega A \subseteq Int_\omega ClA$.

We have $IntCl_\omega A \subseteq IntClA \subseteq Int_\omega ClA$;

$IntCl_\omega A \subseteq Int_\omega Cl_\omega A \subseteq Int_\omega ClA$;

$Cl_\omega IntA \subseteq ClIntA \subseteq ClInt_\omega A$ and

$Cl_\omega IntA \subseteq Cl_\omega Int_\omega A \subseteq ClInt_\omega A$.

Remark 3.2

(i) $IntClA \neq IntCl_\omega A$ and $ClIntB \neq ClInt_\omega B$,
(ii) $IntClA \neq Int_\omega Cl_\omega A$ and $ClIntB \neq Cl_\omega Int_\omega B$,
(iii) $IntClA \neq Int_\omega ClA$ and $ClIntB \neq Cl_\omega IntB$,
(iv) $IntCl_\omega A \neq Int_\omega ClA$ and $ClInt_\omega B \neq Cl_\omega IntB$,
(v) $IntCl_\omega A \neq Int_\omega Cl_\omega A$ and $ClInt_\omega B \neq Cl_\omega Int_\omega B$,
(vi) $Int_\omega ClA \neq Int_\omega Cl_\omega A$ and $Cl_\omega IntB \neq Cl_\omega Int_\omega B$.

Example 3.3 Let E^1 be the real line. Then the set Q of all rational numbers is ω-closed but not ω-open and Q^c, the set of all irrational numbers is ω-open but not ω-closed. It is easy to see that $IntClQ = E^1$, $Int_\omega ClQ = E^1$,

$IntCl_\omega Q = \emptyset$, $Int_\omega Cl_\omega Q = \emptyset$,

$ClIntQ^c = \emptyset$, $Cl_\omega IntQ^c = \emptyset$,

$ClInt_\omega Q^c = E^1$, $Cl_\omega Int_\omega Q^c = E^1$.

Let A=Q and B=Q^c. Then from the above we see that

IntClA = $E^1 \neq \emptyset$= $IntCl_\omega A$ and ClIntB = $\emptyset \neq E^1$ = $ClInt_\omega B$.

IntClA = $E^1 \neq \emptyset$= $Int_\omega Cl_\omega A$ and ClIntB = $\emptyset \neq E^1$ =Cl_ω Int_ω B.

$Int_\omega ClA$ = $E^1 \neq \emptyset$= $Int_\omega Cl_\omega A$ and $Cl_\omega IntB$ = $\emptyset \neq E^1$ = Cl_ω $Int_\omega B$.

Thus (i), (ii) and (vi) in the above remark are verified.

Example 3.4 Let R denote the set of all real numbers and τ={∅, {r}, R} where r is an arbitrary rational number. Then it is easy to see that

$IntClQ^c$ = Int(R\{r}) =∅ and ClIntQ = Cl({r}) = R ,

$Int_\omega ClQ^c$ = Int_ω(R\{r}) = R\{r} and $Cl_\omega IntQ$ = Cl_ω(r}) = {r},

$IntCl_\omega Q^c$ = Int(R\{r})=∅ and $ClInt_\omega Q$ = Cl(r})= R,

$Int_\omega Cl_\omega Q^c$ = Int_ω(R\{r}) = R\{r} and $Cl_\omega Int_\omega Q$ = Cl_ω(r}) = {r},

Let A = Q^c and B= Q. Then from the above we see that

IntClA=∅≠R\{r}= $Int_\omega ClA$ and ClIntB =R≠{r}= $Cl_\omega IntB$,

$IntCl_\omega A$ =∅≠R\{r}=$Int_\omega ClA$ and $ClInt_\omega B$ =R≠{r}=$Cl_\omega IntB$,

$IntCl_\omega A$ =∅≠R\{r}= $Int_\omega Cl_\omega A$ and $ClInt_\omega B$ =R≠{r}= Cl_ω $Int_\omega B$,

Thus (iii) , (iv) and (v) in the above remark are verified.

The above discussion motivates to define different types of b-open sets. Andrijevic had chosen the pair (IntCl, ClInt) to define the notion of b-open sets. Noiri et.al. took the pair ($Int_\omega Cl$, $ClInt_\omega$) to introduce the notion of b-ω-open sets. The other pairs can also be chosen to define some topological sets that are very close to an open set or ω-open set. The two level operators $IntCl_\omega$ and $Cl_\omega Int$ are used to define certain topological sets as given in the next definition.

Definition 3.5 The set A is called

(i) regular ω*-open if A= $IntCl_\omega A$,
(ii) semi-ω*-open if A⊆$Cl_\omega IntA$,
(iii) pre-ω*-open if A⊆ $IntCl_\omega A$,
(iv) b-ω*-open if A⊆ $IntCl_\omega A$∪ $Cl_\omega IntA$.

The three level operators $IntCl_\omega$ Int and Cl_ωInt Cl_ω are used to define some topological sets as given in the next definition.

Definition 3.6 The set A is called

(i) α-ω*-open if A⊆ $IntCl_\omega$ IntA.

(ii) β-ω*-open if $A \subseteq Cl_{\omega}$ Int $Cl_{\omega}A$.

Remark 3.7 It is worthwhile to see that regular ω*-open, semi-ω*-open, pre-ω*-open, b-ω*-open , α-ω*-open and β-ω*-open sets are defined by replacing "Cl " by "Cl_{ω} " in the definitions of regular open, semi-open, pre-open, b-open , α-open and β-open sets respectively.

The complements of the regular ω*-open , semi-ω*-open, pre-ω*-open, b-ω*-open, α-ω*-open , β-ω*-open are called regular ω*-closed, semi-ω*-closed, pre-ω*-closed, b-ω*-closed, α-ω*-closed, β-ω*-closed respectively. Such weak forms of ω*-closed sets are characterized in the next proposition.

Proposition 3.8 The set A is

(i) regular ω*-closed $\Leftrightarrow A = ClInt_{\omega}A$,
(ii) semi-ω*-closed $\Leftrightarrow Int_{\omega}ClA \subseteq A$,
(iii) pre-ω*-closed in $(X,\tau) \Leftrightarrow ClInt_{\omega}A \subseteq A$,
(iv) b-ω*-closed in $(X,\tau) \Leftrightarrow Int_{\omega} ClA \cap ClInt_{\omega}A \subseteq A$.
(v) α-ω*-closed in $(X,\tau) \Leftrightarrow ClInt_{\omega} ClA \subseteq A$.
(vi) β-ω*-closed in $(X,\tau) \Leftrightarrow Int_{\omega} Cl\ Int_{\omega}A \subseteq A$.

Proof. The set A is regular ω*-closed$\Leftrightarrow$ X\A is regular ω*-open.

$\Leftrightarrow X\backslash A = IntCl_{\omega}(X\backslash A)$.

$\Leftrightarrow X\backslash A = X\backslash ClInt_{\omega}A$.

$\Leftrightarrow A = ClInt_{\omega}A$.

This proves (i). The set A is α-ω*-closed $\Leftrightarrow$ X\A is α-ω*-open.

$\Leftrightarrow X\backslash A \subseteq IntCl_{\omega}Int\ (X\backslash A)$.

$\Leftrightarrow X\backslash A \subseteq X\backslash ClInt_{\omega}ClA$.

$\Leftrightarrow A \supseteq ClInt_{\omega}ClA$.

$\Leftrightarrow ClInt_{\omega} ClA \subseteq A$.

This proves (v) and the other assertions can be analogously proved.

Remark 3.9 The concepts of regular ω*-closed, semi-ω*-closed, pre-ω*-closed,

b-ω*-closed, α-ω*-closed and β-ω*-closed sets can also be defined replacing "Int " by "Int_{ω}" in the definitions of regular closed, semi-closed, pre-closed, b-closed , α-closed and β-closed sets respectively.

The next proposition gives specific formulas to verify whether a given set A is regular- ω*-open, regular ω*-closed, semi-ω*-open, semi-ω*-closed, β-ω*-open, β-ω*-closed.

Proposition 3.10 The set A is

(i) regular ω*-open $\Leftrightarrow$ $A = IntCl_{\omega}IntA$,
(ii) regular ω*-closed $\Leftrightarrow$ $A = ClInt_{\omega}ClA$,
(iii) semi-ω*-open $\Leftrightarrow$ $Cl_{\omega}A = Cl_{\omega} IntA$,
(iv) semi-ω*-closed $\Leftrightarrow$ $Int_{\omega}ClA = Int_{\omega} A$,
(v) β-ω*-open $\Leftrightarrow$ $Cl_{\omega}A = Cl_{\omega} Int Cl_{\omega}A$,
(vi) β-ω*-closed $\Leftrightarrow$ $Int_{\omega} Cl Int_{\omega}A = Int_{\omega}A$.

Proof. The set A isregular ω*-open $\Rightarrow$ $A = IntCl_{\omega}A$ and A is open. $\Rightarrow$ $A = IntCl_{\omega}IntA$. Now $A = IntCl_{\omega}IntA$ $\Rightarrow IntA = IntCl_{\omega}IntA$ and A is open $\Rightarrow$ $A = IntCl_{\omega}A$.

This proves (i) and the proof for (ii) is analogous.

The set A is semi-ω*-open $\Rightarrow A \subseteq Cl_{\omega}IntA$

$\Rightarrow A \subseteq Cl_{\omega}IntA \subseteq Cl_{\omega}A$.

$\Rightarrow Cl_{\omega}A \subseteq Cl_{\omega}Cl_{\omega}IntA \subseteq Cl_{\omega}A$.

$\Rightarrow Cl_{\omega}A \subseteq Cl_{\omega}IntA \subseteq Cl_{\omega}A$.

$\Rightarrow Cl_{\omega}A = Cl_{\omega}IntA$.

Conversely, $Cl_{\omega}A = Cl_{\omega}IntA \Rightarrow A \subseteq Cl_{\omega}A = Cl_{\omega}IntA$

$\Rightarrow A \subseteq Cl_{\omega}IntA$

$\Rightarrow$ A is semi- ω*-open

This proves (iii) and the proof for (iv) is analogous.

The set A is β-ω*-open $\Rightarrow A \subseteq Cl_{\omega}Int\, Cl_{\omega}A$

$\Rightarrow A \subseteq Cl_{\omega}Int\, Cl_{\omega}A \subseteq Cl_{\omega}A$.

$\Rightarrow Cl_{\omega}A \subseteq Cl_{\omega}Cl_{\omega}Int\, Cl_{\omega}A \subseteq Cl_{\omega}A$.

$\Rightarrow Cl_{\omega}A \subseteq Cl_{\omega}Int\, Cl_{\omega}A \subseteq Cl_{\omega}A$.

$\Rightarrow Cl_{\omega}A = Cl_{\omega}Int\, Cl_{\omega}A$.

Conversely, $Cl_{\omega}A = Cl_{\omega}Int\, Cl_{\omega}A \Rightarrow A \subseteq Cl_{\omega}A = Cl_{\omega}Int\, Cl_{\omega}A$

$\Rightarrow A \subseteq Cl_{\omega}Int\, Cl_{\omega}A$

$\Rightarrow$ A is β- ω*-open

This proves (iv) and the proof for (v) is analogous.

Proposition 3.11 Let A be regular ω*-open.

(i) A is open, pre-open, pre-ω-open in (X,τ) and

(ii) A is pre-open in (X,τ_ω)

Proof. The set A is regular ω*-open $\Rightarrow A= IntCl_\omega A \subseteq IntClA \subseteq Int_\omega ClA$.

$\Rightarrow$ A is open, pre-open, pre-ω-open in (X,τ).

This proves (i).

A is regular ω*-open $\Rightarrow A= IntCl_\omega A \subseteq Int_\omega Cl_\omega A$.

$\Rightarrow$ A is pre-open in (X,τ_ω).

This proves (ii).

Proposition 3.12 Let A be semi-ω*-open.

(i) A is semi-open, semi-ω-open in (X,τ) and

(ii) A is semi-open in (X,τ_ω).

Proof. The set A is semi-ω*-open in $(X,\tau) \Rightarrow A\subseteq Cl_\omega\, IntA\subseteq ClIntA \subseteq ClInt_\omega A$

$\Rightarrow$ A is semi-open , semi-ω-open in (X,τ).

This proves (i).

The set A is semi-ω*-open in $(X,\tau) \Rightarrow A\subseteq Cl_\omega IntA\subseteq Cl_\omega Int_\omega A$.

$\Rightarrow$ A is semi-open in (X,τ_ω).

This proves (ii).

Proposition 3.13 Let A be pre-ω*-open.

(i) A is pre-open, pre-ω-open in (X,τ) and

(ii) A is pre-open in (X,τ_ω).

Proof. The set A is pre-ω*-open in $(X,\tau) \Rightarrow A\subseteq IntCl_\omega A\subseteq IntClA \subseteq Int_\omega ClA$.

$\Rightarrow$is pre-open, pre-ω-open in (X,τ).

This proves (i).

The set A is pre-ω*-open in $(X,\tau) \Rightarrow A\subseteq Cl_\omega IntA\subseteq Cl_\omega Int_\omega A$.

$\Rightarrow$ A is semi-open in (X,τ_ω).

This proves (ii).

Proposition 3.14 Let A be b-ω*-open.

(i) A is b-open, b-ω-open in (X,τ) and

(ii) A is b-open in (X,τ_ω).

Proof: The set A is b-ω*-open in $(X,\tau) \Rightarrow A \subseteq Cl_\omega IntA \cup IntCl_\omega A$.

$\subseteq ClIntA \cup IntClA \subseteq ClInt_\omega A \cup Int_\omega ClA$.

$\Rightarrow$ A is b-open, b-ω-open in (X,τ) .

This proves (i).

The set A is b-ω*-open in $(X,\tau) \Rightarrow A \subseteq Cl_\omega IntA \cup IntCl_\omega A \subseteq Cl_\omega Int_\omega A \cup Int_\omega Cl_\omega A$.

$\Rightarrow$ A is b-open in (X,τ_ω).

This proves (ii).

Proposition 3.15 Let A be α-ω*-open in (X,τ).

(i) A is pre-ω*-open, pre-open, semi-ω*-open, semi-open,α-open, β-ω*-open in (X,τ) and

(ii) A is pre-open, semi-open, α-open in (X,τ_ω).

Proof. The set A is α-ω*-open in $(X,\tau) \Rightarrow A \subseteq IntCl_\omega IntA \subseteq IntCl_\omega A \subseteq IntClA$,

$A \subseteq IntCl_\omega IntA \subseteq Cl_\omega IntA \subseteq Cl\, IntA$,

$A \subseteq IntCl\, IntA$ and

$A \subseteq IntCl_\omega IntA \subseteq Cl_\omega IntCl_\omega IntA \subseteq Cl_\omega IntCl_\omega A$

$\Rightarrow$ A is pre-ω*-open, pre-open, semi-ω*-open, semi-open ,α-open, β-ω*-open in (X,τ)

This proves (i).

The set A is α-ω*-open in $(X,\tau) \Rightarrow A \subseteq IntCl_\omega IntA$

$\Rightarrow A \subseteq Int_\omega Cl_\omega A$, $A \subseteq Cl_\omega Int_\omega A$,

$A \subseteq Int_\omega Cl_\omega Int_\omega A$.

$\Rightarrow$ A is pre-open, semi-open, α-open in (X,τ_ω). This proves (ii).

Proposition 3.16 If A is β-ω*-open in (X,τ), then A is β-open in (X,τ) and in (X,τ_ω).

Proof. The set A is β-ω*-open in $(X,\tau) \Rightarrow A \subseteq Cl_\omega\, Int\, Cl_\omega A$

$\Rightarrow A \subseteq Cl\, Int\, ClA$ and $A \subseteq Cl_\omega\, Int_\omega\, Cl_\omega A$

$\Rightarrow$ A is β-open in (X,τ) as well as in (X,τ_ω).

Proposition 3.17 If A is regular ω*-closed then it is closed, pre-closed, pre-ω-closed in (X,τ) and is pre-closed in (X,τ_ω).

Proof. The set A is regular ω*-closed $\Rightarrow$ X\A is regular ω*-open.

$\Rightarrow$ X\A is open, pre-open, pre-ω-open in (X,τ) and pre-open in (X,τ_ω).

$\Rightarrow$ A is closed, pre-closed , pre-ω-closed in (X,τ) and pre-closed in (X,τ_ω).

Proposition 3.18 If A is semi-ω*-closed, then it is semi-closed, semi-ω- closed in (X,τ) and semi-closed in (X,τ_ω) .

Proof. The set A is semi-ω*-closed in (X,τ) $\Rightarrow$ X\A is semi-ω*-open.

$\Rightarrow$X\A is semi-open, semi-ω-open in (X,τ) and semi-open in (X,τ_ω)

$\Rightarrow$A is semi-closed, semi-ω-closed in (X,τ) and semi-closed in (X,τ_ω).

Proposition 3.19 If A is pre-ω*-closed, then it is pre-closed, pre-ω-closed in (X,τ) and pre-closed in (X,τ_ω).

Proof. The set A is pre- ω*- closed in (X,τ) $\Rightarrow$ X\A is pre- ω*- open.

$\Rightarrow$X\A is pre-open, pre-ω-open in (X,τ) and semi-open in (X,τ_ω).

$\Rightarrow$A is pre-closed, pre-ω-closed in (X,τ) and semi-closed in (X,τ_ω).

Proposition 3.20 If A is b-ω*-closed then it is b-closed, b-ω-closed in (X,τ) and b-closed in (X,τ_ω).

Proof. The set A is b-ω*-closed in (X,τ) $\Rightarrow$ X\A is b-ω*-open.

$\Rightarrow$X\A is b-open, b-ω-open in (X,τ) and b-open in (X,τ_ω).

$\Rightarrow$A is b-closed, b-ω-closed in (X,τ) and b-closed in (X,τ_ω).

Proposition 3.21 If A is α-ω*-closed in (X,τ), then it is closed, pre-ω*- closed, pre-closed, semi-ω*-closed, semi-closed, α-closed, β-ω*-closed in (X,τ) and is pre-closed, semi-closed, α-closed in (X,τ_ω).

Proof. The set A is α-ω*-closed in (X,τ) $\Rightarrow$ X\A is α-ω*-open.

$\Rightarrow$X\A is pre-ω*-open, pre-open, semi-ω*-open, semi-open,α-open, β-ω*-open in (X,τ) and pre-open, semi-open, α-open in (X,τ_ω)

$\Rightarrow$A is pre-ω*-closed, pre-closed, semi-ω*-closed, semi-closed, α-closed,

β-ω*-closed in (X,τ) and pre-closed, semi-closed, α-closed in (X,τ_ω).

Proposition 3.22 If A is β-ω^*-closed in (X,τ) then A is β-closed in (X,τ) and also β-closed in (X,τ_ω).

Proof. The set A is β- ω^*-closed in $(X,\tau) \Rightarrow X\backslash A$ is β-ω^*-open.

$\Rightarrow X\backslash A$ is β-open in (X,τ) and in (X,τ_ω).

$\Rightarrow$A is β-closed in (X,τ) and in (X,τ_ω).

Proposition 3.23 Let A and B be any two subsets of a space X.

(i) If A and B are regular ω^*-open in (X,τ) then $A\cap B$ is regular ω^*-open.
(ii) If A and B are ρ-ω^*-open in (X,τ) then $A\cup B$ is ρ-ω^*-open.
(iii) If Aand B are regular ω^*-closed in (X,τ) then $A\cup B$ is regular ω^*-closed.
(iv) If A and B are ρ-ω^*-closed in (X,τ) then $A\cap B$ is ρ-ω^*-closed.

Proof. Let A, B be regular ω^*-open in (X,τ). Then $IntCl_\omega A=A$ and $IntCl_\omega B=B$ that implies $IntCl_\omega(A\cap B) \subseteq Int(Cl_\omega A\cap Cl_\omega B) = Int\ Cl_\omega A\cap Int\ Cl_\omega B = A\cap B\subseteq Cl_\omega(A\cap B)$ so that $IntCl_\omega(A\cap B) \subseteq Int(A\cap B) = A\cap B \subseteq IntCl_\omega(A\cap B)$ which further implies that $IntCl_\omega(A\cap B) = A\cap B$. Therefore $A\cap B$ is regular ω^*-open. This proves (i) and the assertion (iii) follows from (i).

Now, let A, B be semi- ω^*-open in (X,τ). Then $Cl_\omega IntA = Cl_\omega A$ and

$Cl_\omega IntB = Cl_\omega B$.

$Cl_\omega Int(A\cup B)\supseteq Cl_\omega(Int\ A\cup Int\ B) = Cl_\omega Int\ A\cup Cl_\omega Int\ B = Cl_\omega A\cup Cl_\omega B \supseteq A\cup B$ that implies $A\cup B$ is semi- ω^*-open.

If A and B are pre-ω^*-open in (X,τ) then $A\subseteq IntCl_\omega A$ and $B\subseteq IntCl_\omega B$ so that

$IntCl_\omega(A\cup B) = Int(Cl_\omega A\cup Cl_\omega B) \supseteq Int\ Cl_\omega A\cup Int\ Cl_\omega B \supseteq A\cup B$ that implies

$A\cup B$ is pre- ω^*-open.

If A and B are α-ω^*-open in (X,τ) then $A\subseteq IntCl_\omega\ Int\ A$ and $B\subseteq IntCl_\omega\ Int\ B$ so that

$IntCl_\omega Int\ (A\cup B) \supseteq IntCl_\omega\ (IntA\cup IntB) = Int(Cl_\omega IntA\cup Cl_\omega IntB)$

$\supseteq Int\ Cl_\omega IntA\cup Int\ Cl_\omega IntB \supseteq A\cup B$ that implies $A\cup B$ is α-ω^*-open.

If A and B are β-ω^*-open in (X,τ) then $A\subseteq Cl_\omega\ IntCl_\omega A$ and $B\subseteq Cl_\omega\ IntCl_\omega B$ so that

$Cl_\omega IntCl_\omega\ (A\cup B) = Cl_\omega Int\ (Cl_\omega A\cup Cl_\omega B) \supseteq Cl_\omega(Int\ Cl_\omega A\cup Int\ Cl_\omega B)$

$= Cl_\omega Int\ Cl_\omega A\cup Cl_\omega Int\ Cl_\omega B$

$\supseteq A\cup B$ that implies $A\cup B$ is β-ω^*-open.

If A and B are b-ω*-open in (X,τ) then $A \subseteq IntCl_{\omega}A \cup Cl_{\omega}IntA$ and

$B \subseteq IntCl_{\omega}B \cup Cl_{\omega}IntB$ so that $IntCl_{\omega}(A \cup B) \cup Cl_{\omega}Int(A \cup B)$

$= Int(Cl_{\omega}A \cup Cl_{\omega}B) \cup Cl_{\omega}Int(A \cup B)$

$\supseteq Int(Cl_{\omega}A \cup Cl_{\omega}B) \cup Cl_{\omega}(Int A \cup B)$

$\supseteq (Int Cl_{\omega}A \cup IntCl_{\omega}B) \cup Cl_{\omega}(Int A \cup Int B)$

$= (Int Cl_{\omega}A \cup IntCl_{\omega}B) \cup (Cl_{\omega}Int A \cup Cl_{\omega}Int B)$

$= (Int Cl_{\omega}A \cup Cl_{\omega}Int A) \cup (IntCl_{\omega}B \cup Cl_{\omega}Int B)$

$\supseteq A \cup B$ that implies $A \cup B$ is b- ω*-open. This proves (ii) and the assertion (iv) can be analogously proved.

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