Generalized Anti Fuzzy Implicative Ideals of Near – Rings

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Abstract: In this section, we introduce the new notion $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$ fuzzy implicative ideals of $\Re$ and discuss some of its properties. Also we define the different level set for the fuzzy set ξ .We bring semiprime and prime concept in $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$ fuzzy implicative ideals of $\Re$ .We investigate some theorems related to this concept.

Keywords: Near-ring, $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$ – fuzzy implicative ideals of $\Re$, prime $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$ – fuzzy implicative ideals of $\Re$, semiprime $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$ – fuzzy implicative ideals of $\Re$.

1. Introduction

Fuzzy concept was first introduced by Zadeh [7]. A new type of fuzzy subgroup, that is, the $(\in,\in\vee q)$ fuzzy sub group, was introduced by Bhakat and Das [1] using the combined notions of belongingness and quasicoincidence of fuzzy points and fuzzy sets. The idea of beside to and non quasi-coincident relation was given by Saeid and Jun [6]. Kim [3] studied the notion of anti fuzzy ideals in near rings. Zhan and Yin [8] introduced new type of fuzzy ideals of near rings. Recently, the authors defined Intuitionistic $\left(\in_{\gamma},\in_{\gamma},\vee q_{\delta}\right)$ - Fuzzy Prime Ideals of a near-rings [2].

In this paper, the concept of $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$ – fuzzy implicative ideals of a near-ring is given with its equivalent conditions. We give the relationship between $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$ - fuzzy implicative ideals and $\left(\Gamma_{\psi},\Gamma_{\psi}\right)$ fuzzy implicative ideals of $\Re$. We bring the definition for three level sets $\Gamma_{\psi},\Upsilon_{\phi}$ and $\Gamma_{\psi},\Upsilon_{\phi}$ of ξ. Moreover, we extend this $\left(\Gamma_{\psi},\Gamma_{\psi}\vee\Upsilon_{\phi}\right)$–fuzzy implicative ideals of near-rings to prime and semiprime concepts.

Definition 1.1. [6] A fuzzy set ξ of $\Re$ of the form

$$\xi(j)=\begin{cases} p\in[0,1) & if\ \ j=i, \\ 1 & if\ \ j\neq i, \end{cases}$$

is said to be an anti-fuzzy point with support i and value p and is denoted by i_p. An anti fuzzy point i_p is said to beside (respectively be non-quasi coincident with) a fuzzy set ξ, written as $i_p\ \Gamma\ \xi$ (respectively $i_p\ \Upsilon\,\xi$) if ξ (i) $\leq$ p (respectively ξ (i) + p<1).We say that Γ (respectively Υ) is a beside relation (respectively non-quasi coincident with) relation between anti fuzzy points and fuzzy sets.

If $i_p\ \Gamma\ \xi$ or $i_p\ \Upsilon\,\xi$, we say that $i_p\ \Gamma\vee\Upsilon\,\xi$ and $i_p\ \overline{\Gamma}\ \xi$ (respectively $i_p\ \overline{\Upsilon}\,\xi$, $i_p\ \overline{\Gamma\vee\Upsilon}\,\xi$) means that $i_p\ \Gamma\ \xi$ (respectively $i_p\ \Upsilon\,\xi$, $i_p\ \Gamma\vee\Upsilon\,\xi$) does not hold.

Result 1.2. Let $\phi,\psi\in[0,1]$ be such that $\phi<\psi$. For an anti-fuzzy point i_p and a fuzzy set ξ of $\Re$, we say that

1. $i_p\ \Gamma_\psi\xi$ if ξ (i) $\leq$ p < ψ
2. $i_p\ \Upsilon_\phi\xi$ if ξ (i) + p < 2ϕ
3. $i_p\ \Gamma_\psi\vee\Upsilon_\phi\xi$ if $i_p\ \Gamma_\psi\xi$ (or) $i_p\ \Upsilon_\phi\xi$.

Definition 1.3. [4] A non empty subset I of $\Re$ is called an implicative ideal of $\Re$ if it satisfies $\big((i(ji))k\big)\in$ I whenever i $\in$ I and k $\in$ I for all i, j, k $\in\Re$.

2. $(\Gamma_{\psi,}\Gamma_\psi\vee\Upsilon_\phi)$ fuzzy implicative ideal of $\Re$.

Definition 2.1. A fuzzy set ξ of $\Re$ is called a $(\Gamma_{\psi,}\Gamma_\psi\vee\Upsilon_\phi)$ fuzzy implicative ideal of $\Re$ if for all i, j, k $\in\Re$ and p, n $\in$ [0, ψ),

(i_a) $i_p\ \Gamma_\psi\ \xi$ and $j_n\ \Gamma_\psi\ \xi\Rightarrow$ $(i+j)_{p\vee n}\Gamma_\psi\vee\Upsilon_\phi\xi$.

(i_b) $i_p\ \Gamma_\psi\ \xi\Rightarrow(-i)_p\Gamma_\psi\vee\Upsilon_\phi\xi$.

(ii) $i_p\ \Gamma_\psi\ \xi$ and $j_n\Gamma_\psi\ \xi\Rightarrow$ $(ij)_{p\vee n}\Gamma_\psi\vee\Upsilon_\phi\xi$.

(iii) $i_p\ \Gamma_\psi\ \xi\Rightarrow(j+i-j)_p\Gamma_\psi\vee\Upsilon_\phi\xi$.

(iv) $j_p\ \Gamma_\psi\ \xi$ and $i\in\Re\Rightarrow(ij)_p\Gamma_\psi\vee\Upsilon_\phi\xi$.

(v) $k_p\ \Gamma_\psi\ \xi\Rightarrow((i+k)j-ij)_p\Gamma_\psi\vee\Upsilon_\phi\xi$.

(vi) $i_p\ \Gamma_\psi\ \xi$ and $k_n\Gamma_\psi\ \xi\Rightarrow$ $((i(ji))k)_{p\vee n}\Gamma_\psi\vee\Upsilon_\phi\xi$.

Example 2.2. Let $\Re$ = {0, a, b, c} be a set. Consider the following klein's four group table. Define '+' and '.' as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	a	b	c
c	a	0	c	b

Then ($\Re$,+,.) is a near ring. Define the fuzzy set ξ of $\Re$ as ξ(0) = ξ(b) = 0.4, ξ(a) = 0.3, ξ(c) = 0.5. Then ξ is a $(\Gamma_{0.8}, \Gamma_{0.8} \vee \Upsilon_{0.2})$ fuzzy implicative ideals of $\Re$.

Theorem 2.3. For a fuzzy set ξ in R, the following conditions are equivalent.

a) $i_p\ \Gamma_\psi\ \xi$ and $k_n \Gamma_\psi\ \xi \Rightarrow$ ((i(ji))k) $_{p\vee n}\Gamma_\psi \vee \Upsilon_\phi \xi$.
b) ξ((i(ji))k) $\wedge \psi \le \xi$(i) $\vee \xi(k) \vee \phi$ for all i, j, k $\in \Re$.

Proof. Assume that $i_p\ \Gamma_\psi\ \xi$ and $k_n \Gamma_\psi\ \xi \Rightarrow$ ((i(ji))k) $_{p\vee n}\Gamma_\psi \vee \Upsilon_\phi \xi$. Let i, j $\in \Re$ Suppose that ξ((i(ji))k) $\wedge \psi > \xi$(i) $\vee \xi(k) \vee \phi$ Choose p such that ξ((i(ji))k)$\wedge \psi > p > \xi$(i) $\vee \xi(k) \vee \phi$ .This implies $i_p\ \Gamma_\psi\ \xi$, $k_p\ \Gamma_\psi\ \xi$ but ξ((i(ji))k) > p and ξ((i(ji))k)+p>2p$\ge 2\phi$. It follows that $((i(ji))k)_p\ \overline{\Gamma_\psi \vee \Upsilon_\phi} \xi$, which is a contradiction to our assumption. Therefore, ξ((i(ji))k) $\wedge \psi \le \xi$(i) $\vee \xi(k) \vee \phi$.

Assume that ξ((i(ji))k) $\wedge \psi \le \xi$(i) $\vee \xi(k) \vee \phi$ for all i, j, k $\in \Re$. Suppose there exists i, j, k $\in \Re$ such that $i_p\ \Gamma_\psi\ \xi$, $k_n \Gamma_\psi \xi$ but ((i(ji))k) $_{p\vee n}\overline{\Gamma_\psi \vee \Upsilon_\phi}\xi$. Then ξ(i) $\le$ p, ξ(k) $\le$ n but ξ((i(ji))k)>p$\vee$n and ξ((i(ji))k) + p$\vee$n $\ge 2\phi$.It follows that ξ((i(ji))k)>ϕ. So, ξ((i(ji))k) $\wedge \psi$ >p$\vee$n$\vee \phi \ge \xi$(i) $\vee \xi$(k)$\vee \phi$ which is a contradiction to our assumption. Therefore, ((i(ji))k) $_{p\vee n}\Gamma_\psi \vee \Upsilon_\phi \xi$.

Theorem 2.4 A fuzzy set ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$ such that p, n $\in [\phi, \psi)$ for all i, j, k $\in \Re$ if and only if ξ is a $(\Gamma_\psi, \Gamma_\psi)$ fuzzy implicative ideals of $\Re$.

Proof: vi) Let ξ be a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$ such that p, n $\in [\phi, \psi)$ for all i, j, k $\in \Re$ (i.e) ξ((i(ji))k) $\wedge \psi \le \xi$(i) $\vee \xi$(k) $\vee \phi$.

Let i_p, $\Gamma_\psi \xi, k_n \Gamma_\psi \xi \Rightarrow \xi(i) \le p < \psi, \xi(k) \le n < \psi$. We have,

$\xi((i(ji)k) \wedge \psi \le \xi(i) \vee \xi(k) \vee \phi$

$\leq p \vee n \vee \phi$

$\leq p \vee n$ (since $p, n \in [\phi, \psi)$)

Therefore, ((i(ji))k) $_{p\vee n}\Gamma_{\psi}\ \xi.$ Similarly, we can prove other

Hence ξ is an $(\Gamma_{\psi}, \Gamma_{\psi})$ fuzzy implicative ideals of $\Re$.

Conversely, let ξ (i)= p, ξ (k)= n where p,n $\in$ [ϕ). Then ξ (i) $\leq$ p < ψ, ξ (k) $\leq$ n < $\psi \Rightarrow i_p \Gamma_{\psi} \xi, k_n \Gamma_{\psi} \xi.$ Since ξ is an $(\Gamma_{\psi}, \Gamma_{\psi})$ fuzzy implicative ideals of $\Re \Rightarrow ((i(ji))k)_{p\vee n}\Gamma_{\psi}\xi$ $(i.e)$ $\xi((i(ji))k) \leq p \vee n < \psi.$

Now,

$\xi((i(ji))k) \wedge \psi \leq p \vee n \wedge \psi$

= p $\vee$ n

= p $\vee$ n $\vee$ ϕ

= ξ (i) $\vee$ ξ (k) $\vee$ ϕ

Therefore, ξ ((i(ji))k) $\leq \xi$ (i) $\vee$ ξ (k) $\vee$ ϕ. Hence ξ is a $(\Gamma_{\psi}, \Gamma_{\psi} \vee \Upsilon_{\phi})$ fuzzy implicative ideals of $\Re$.

Theorem 2.5. The union of any family of $(\Gamma_{\psi}, \Gamma_{\psi} \vee \Upsilon_{\phi})$ fuzzy implicative ideals of $\Re$

is a $(\Gamma_{\psi}, \Gamma_{\psi} \vee \Upsilon_{\phi})$fuzzy implicative ideals of $\Re$.

Proof. Let $\{\xi_f\}_{f\in F}$ be any family of $(\Gamma_{\psi}, \Gamma_{\psi} \vee \Upsilon_{\phi})$fuzzy implicative ideals of $\Re$ and $\xi = \bigcup_{f\in F} \xi_f$. For any $i, j, k \in \Re$, we have

(vi) $\xi((i(ji))k \wedge \psi = \bigcup_{f\in F} \xi_f((i(ji))k) \wedge \psi$

$$= \bigcup_{f\in F} (\xi_f((i(ji))k) \wedge \psi)$$

$$\leq \bigcup_{f\in F} (\xi_f(i) \vee \xi_f(k) \vee \phi)$$

$$= (\bigcup_{f\in F} \xi_f)(i) \vee (\bigcup_{f\in F} \xi_f)(k) \vee \phi$$

$= \xi(i) \vee \xi(k) \vee \phi$

Therefore, $\xi((i(ji))k) \wedge \psi \leq \xi(i) \vee \xi(k) \vee \phi$

Definition 2.6. For any fuzzy set ξ in $\Re$ and $p \in [0,\psi)$ we define $\xi_p^\psi = \{i \in \Re / i_p \Gamma_\psi \xi\}, \xi_p^\phi = \{i \in \Re / i_p \gamma_\phi \xi\}$ and $|\xi|_p^\phi = \{i \in \Re / i_p \Gamma_\psi \vee \Upsilon_\phi \xi\}$. It is clear that $[\xi_p^\phi] = \xi_p^\psi \cup \xi_p^\phi$ where ξ_p^ψ, ξ_p^ϕ and $[\xi]_p^\phi$ are called Γ_ψ - level set, Υ_ϕ - level set and $\Gamma_\psi \vee \Upsilon_\phi$ – level set of ξ respectively.

Theorem 2.7. Let ξ be a fuzzy set in $\Re$.Then ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$ if and only if $[\xi]_p^\phi = \phi$ is an implicative ideals of $\Re$ for all $p \in [0,\psi)$.

Proof. Let $i,k \in [\xi]_p^\phi$ then $i_p \Gamma_\psi \vee \Upsilon_\phi \xi, k_p \Gamma_\psi \vee \Upsilon_\phi \xi$ .We can consider four cases.

(i) $\xi(i) \leq p$ and $\xi(k) \leq p$

(ii) $\xi(i) \leq p$ and $\xi(k) + p < 2\phi$

(iii) $\xi(i) + p < 2\phi$ and $\xi(k) \leq p$

(iv) $\xi(i) + p < 2\phi$ and $\xi(k) + p < 2\phi$

Since ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re, \xi((i(ji))k) \wedge \psi \leq \xi(i) \vee \xi(k) \vee \phi$. Let $p \in [o,\psi)$ .We consider the four cases for $\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi$.

Case (i) : $\xi(i) \leq p$ and $\xi(k) \leq p$

For $p \in [0,\phi)$ then $2\phi - p > \phi > p$

Now,

$\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi$

$\leq p \vee p \vee \phi$

$= \phi < 2\phi - p$

(or) $\xi((i(ji))k) \leq p \vee (2\phi - p) \vee \phi = 2\phi - p$ (or) $\xi((i(ji))k) \leq (2\phi - p) \vee (2\phi - p) \vee \phi = 2\phi - p$. Therefore, $\xi((i(ji))k) < 2\phi - p$ (i.e) $\xi((i(ji))k) + p < 2\phi$. Hence, $((i(ji))k)_p \Upsilon_\phi \xi$. Therefore, $((i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi$. For $p \in [\phi,\psi)$ then $2\phi - p < \phi \leq p$. Now, $\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi \leq p \vee p \vee \phi = p$ (or) $\xi((i(ji))k) \leq p \vee (2\phi - p) \vee \phi = p$ (or)

$\xi((i(ji))k) \leq (2\phi - p) \vee (2\phi - p) \vee \phi = \phi \leq p$. Therefore, $\xi((i(ji))k) \leq p$. Hence, $((i(ji))k)_p \Gamma_\psi \xi$. Therefore, $((i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi$.

Case (ii): $\xi(i) \leq p$ and $\xi(k) + p < 2\phi$

For $p \in [0,\phi)$, it is clear that $p < \phi$ then $2\phi - p > \phi$. Given, $\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi$. If $\xi(i) \vee \phi \geq \xi(k)$, then $\xi((i(ji))k) \leq \xi(i) \vee \phi = p \vee \phi = \phi$. Therefore, $\xi((i(ji))k) \leq \phi$. If $\xi(i) \vee \phi < \xi(k)$, then $\xi((i(ji))k) \leq \xi(k) < 2\phi - p \Rightarrow \xi((i(ji))k) + p < 2\phi$. Therefore, $((i(ji))k)_p \Upsilon_\phi \xi$. Hence, $((i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi$. For $p \in [\phi,\psi)$, it is clear that $p \geq \phi$ then $2\phi - p < \phi$. Given, $\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi$. If $\xi(i) \vee \phi \geq \xi(k)$, then $\xi((i(ji))k) \leq \xi(i) \vee \phi = p \vee \phi = p$. If $\xi(i) \vee \phi < \xi(k)$, then $\xi((i(ji))k) \leq \xi(k) < 2\phi - p \Rightarrow \xi((i(ji))k) + p < 2\phi$. Therefore, $((i(ji))k)_p \Upsilon_\phi \xi$. Thus, $((i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi$.

Case (iii): $\xi(i) + p < 2\phi \, and \, \xi(k) \leq p$

For $p \in [0,\phi)$, it is clear that $p < \phi$ then $2\phi - p > \phi$. If $\xi(k) \vee \phi \geq \xi(i)$, then $\xi((i(ji))k) \leq \xi(k) \vee \phi \leq p \vee \phi$. If $\xi(k) \vee \phi < \xi(i)$, then $\xi((i(ji))k) \leq \xi(i) < 2\phi\, p$. Thus, $\xi((i(ji))k) + p < 2\phi$. Therefore, $(i(ji))k)_p \Upsilon_\phi \xi$. Hence, $(i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi$. For $p \in [\phi,\psi)$, assume that $p \geq \phi$ then $2\phi - p < \phi$. If $\xi(k) \vee \phi \geq \xi(i)$, then $\xi((i(ji))k) \leq \xi(k) \vee \phi \leq p \vee \phi = p$. Thus, $((i(ji))k)_p \Gamma_\psi \xi$. Therefore, $(i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi$. If $\xi(k) \vee \phi < \xi(i)$, then $((i(ji))k) \leq \xi(i) < 2\phi - p \Rightarrow \xi((i(ji))k) + p < 2\phi$. Therefore, $(i(ji))k)_p \Upsilon_\phi \xi$. Thus, $(i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi$.

Case (iv): $\xi(i) + p < 2\phi \, and \, \xi(k) + p < 2\phi$

For $p \in [0,\phi)$, it is clear that $p < \phi$ then $2_\phi - p > \phi$

$\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi$

$$= \begin{cases} \phi \leq 2\phi - p \, if \ \xi(i) \vee \xi(k) \leq \phi \\ \xi(i) \vee \xi(k) < 2\phi - p \, if \ \xi(i) \vee \xi(k) > \phi \end{cases}$$

$\xi((i(ji))k) \leq 2\phi - p \Rightarrow \xi((i(ji))k) + p < 2\phi$

Therefore, $((i(ji))k)_p \Upsilon_\phi \xi$. For $p \in [\phi,\psi)$, it is clear that $p \geq \phi$ then $2\phi - p < \phi$

Now,

$\xi((i(ji))k) \leq \xi(i) \vee \xi(k) \vee \phi$

$$= \begin{cases} \phi \leq p \, if \ \xi(i) \vee \xi(k) \leq \phi \\ \xi(i) \vee \xi(k) < 2\phi - p \, if \ \xi(i) \vee \xi(k) > \phi \end{cases}$$

$\Rightarrow \xi((i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi.$

Thus in all the four cases, $[\xi]_p^\phi$ is an implicative ideals of $\Re$.

Conversely, let i, j, k $\in \Re$ and i, j, k $\in [\xi]_p^\phi$. Since $[\xi]_p^\phi$ is an implicative ideals of $\Re \Rightarrow ((i(ji))k)_p \Gamma_\psi \vee \Upsilon_\phi \xi.$ Suppose that ξ is not a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$. If there exists p such that $\xi((i(ji))k) \wedge \psi > p > \xi(i) \vee \xi(k) \vee \phi \Rightarrow i_p \Gamma_\psi \xi, k_p \Gamma_\psi \xi$ but $((i(ji))k)_p \overline{\Gamma_\psi \vee \Upsilon_\phi \xi},$ which is a contradiction. Therefore, ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$.

Theorem 2.8. Let I be an implicative ideals of $\Re$ and ξ be a fuzzy set of $\Re$ such that

$$\xi(i) = \begin{cases} \leq \phi \ for\ i \in I \\ \psi\ otherwise \end{cases}$$

Then ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$.

Proof. (vi) Let i, j, k $\in \Re$ be such that $i_p \Gamma_\psi \xi, k_n \Gamma_\psi \xi.$ Then $\xi(i) \leq p, \xi(k) \leq n.$ Let $i, k \in I$ and so ((i(ji))k) $\in I$ (since I is an implicative ideals of $\Re$) $\Rightarrow \xi((i(ji))k) \leq \phi.$ If $p \vee n \geq \phi$ then $\xi((i(ji))k) \leq \phi \leq p \vee n.$ Hence $((i(ji))k)_{p \vee n} \Gamma_\psi \xi.$ If $p \vee n < \phi$ then $\xi((i(ji))k) + p \vee n < \phi + \phi = 2 \Rightarrow ((i(ji))k)_{p \vee n} \Gamma_\psi \vee \Upsilon_\phi \xi.$ Therefore, ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$.

3. Prime and Semiprime $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ Fuzzy Implicative Ideals of $\Re$

Definition 3.1. Let ξ be an $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$. A $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$ is called prime if for all $i, j \in \Re$ and $p \in [0, \psi)$ such that $(ij)_p \Gamma_\psi \xi$ implies that $i_p \Gamma_\psi \vee \Upsilon_\phi \xi$ (or) $j_p \Gamma_\Psi \vee \Upsilon_\phi \xi.$ A $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals ξ of $\Re$ is called semiprime if for all $i \in \Re$ and $p \in [0, \psi)$ such that $(i_p)^2$ $\Gamma_\psi \xi$ implies that $i_p \Gamma_\psi \vee \mathrm{Y}_\phi \xi$.

Theorem 3.2. Let P be a prime ideal of $\Re$ and ξ be a fuzzy set of $\Re$ such that

$$\xi(i) \begin{cases} \leq \phi \ for\ i \in P \\ \psi\ otherwise \end{cases}$$

Then ξ is a prime $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$

Proof. Let P be a prime ideal of $\Re$. By theorem (2.8), ξ is a $(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi)$ fuzzy implicative ideals of $\Re$. It is enough to prove that ξ is prime. Let $i, j \in \Re$ be such that

$(ij)_p\Gamma_\psi\xi \Rightarrow \xi(ij) \le p.$ Let $(ij) \in P,$ since P is prime ideal $\Rightarrow i \in P\,(or)\, j \in P.$ If $p \ge \phi$ then $\xi(i) \le \phi \le p$ (or) $\xi(j) \le \phi \le p$ (i.e) $\xi(i) \le p < \psi$ (or) $\xi(j) \le p < \psi \Rightarrow i_p\Gamma_\psi$ (or) $j_p\Gamma_\psi\xi.$ If $p < \phi$ then $\xi(i) + p < \phi + \phi = 2\phi$ (or) $\xi(j) + p < \phi + \phi = 2\phi \Rightarrow i_p\Upsilon_\phi\xi$ (or) $j_p\Upsilon_\phi\xi$. Hence, $i_p\Gamma_\psi \vee \Upsilon_\phi\xi$ (or) $j_p\Gamma_\psi \vee \Upsilon_\phi\xi.$ Therefore, ξ is a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$.

Corollary 3.3. Let P be a semiprime ideal of $\Re$ and ξ be a fuzzy set of $\Re$ such that

$$\xi(i) = \begin{cases} \le \phi \ for\ i \in P \\ \psi \ otherwise \end{cases}$$

Then ξ is a semiprime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$.

Proof. Straight Forward

Theorem 3.4. A $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of ξ of $\Re$ is prime if and only if it satisfies $\xi(i) \wedge \xi(j) \wedge \psi \le \xi(ij) \vee \phi$ for all $i, j \in \Re.$

Proof. Let ξ be a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$. Let $i, j \in \Re$. Suppose $\xi(i) \wedge \xi(j) \wedge \psi > \xi(ij) \vee \phi.$ Choose p such that $\xi(i) \wedge \xi(j) \wedge \psi > p > \xi(ij) \vee \phi$ for some $p \in [0, \psi),$ then $(ij)_p\Gamma_\psi\xi$ but $\xi(i) > p$ (or) $\xi(j) > p$ and $\xi(i) + p > 2p \ge 2\phi$ (or) $\xi(j) + p > 2p \ge 2\phi$ (i.e) $i_p\overline{\Gamma_\psi \vee \Upsilon_\phi}\xi$ (or) $j_p\overline{\Gamma_\psi \vee \Upsilon_\phi}\xi$, which is a contradiction.

Hence, $\xi(i) \wedge \xi(j) \wedge \psi \le \xi(ij) \vee \phi.$

Conversely, assume that for all $i, j \in \Re$ such that $\xi(i) \wedge \xi(j) \wedge \psi \le \xi(ij) \vee \phi.$ Let ξ be a $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$. It is enough to prove that ξ is prime. Let $(ij)_p\Gamma_\psi\xi \Rightarrow \xi(ij) \le p.$ Now, $\xi(i) \wedge \xi(j) \wedge \psi \le \xi(ij) \vee \phi \Rightarrow \xi(i) \wedge \xi(j) \wedge \psi \le p \vee \phi.$ If $p \ge \phi$ then either $\xi(i) \le p$ (or) $\xi(j) \le p \Rightarrow i_p\Gamma_\psi\xi$ (or) $j_p\Gamma_\psi\xi$. If $p < \phi$ then either $\xi(i) + p < p + p = 2p < 2\phi$ (or) $\xi(j) + p < p + p = 2p < 2\phi \Rightarrow i_p\gamma_\phi\xi$ (or) $j_p\Upsilon_\phi\xi.$

Hence, $i_p\Gamma_\psi \vee \Upsilon_\phi\xi$ (or) $j_p\Gamma_\psi \vee \Upsilon_\phi\xi$. Therefore, ξ is a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$.

Corollary 3.5. A $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals ξ of $\Re$ is semiprime if and only if $\xi(i) \wedge \psi \le \xi(i^2) \vee \phi$ for all $i \in \Re.$

Proof. Straight Forward

Theorem 3.6. The union of any family of prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$ is a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$.

Proof. By theorem (2.5), $\xi = \bigcup_{f\in F} \xi_f$ is a $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$. Let $i, j \in \Re$. Now,

$$\xi(i) \wedge \xi(j) \wedge \psi = \bigcup_{f\in F} \xi_f(i) \wedge \bigcup_{f\in F} \xi_f(j) \wedge \psi$$

$$= \bigcup_{f\in F} (\xi_f(i)) \wedge \bigcup_{f\in F} (\xi_f(j)) \wedge \psi$$

$$\leq \bigcup_{f\in F} (\xi_f(ij)) \vee \phi)$$

$$= \xi(ij) \vee \phi$$

Therefore $\xi(i) \wedge \xi(j) \wedge \psi \leq \xi(ij) \vee \phi$. Hence, ξ is a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$.

Corollary 3.7. The union of any family of semiprime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$ is a semiprime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$fuzzy implicative ideals of $\Re$.

Proof. Straight forward.

Theorem 3.8. A $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$fuzzy implicative ideals ξ of $\Re$ is prime if and only if for $p \in [0,\psi), [\xi_\rho]^\phi \neq \phi$ is a prime implicative ideals of $\Re$.

Proof. Let ξ be a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$. Let $p \in [0,\psi)$.

Then by theorem (2.7), $[\xi]_p^\phi \neq \phi$ is an implicative ideals of $\Re$, it is enough to prove that $[\xi]_p^\phi$ is prime. Let $i, j \in \Re$ be such that $(ij) \in [\xi]_p^\phi \Rightarrow (ij) \in \xi_p^\psi \cup \xi_p^\phi$ (i.e) $(ij)_p\ \Gamma_\psi \xi$ (or) $(ij)_p\ \Upsilon_\phi \xi$. Since ξ is a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$,we have $i_p \Gamma_\psi \vee \Upsilon_\phi \xi (or)\, j_p \Gamma_\psi \vee \Upsilon_\phi \xi (i.e) i \in [\xi]_p^\phi (or)\, j \in [\xi]_p^\phi$.

Hence, $[\xi]_p^\phi$ is a prime implicative ideals of $\Re$.

Conversely, let $i, j \in \Re$ be such that $(ij)_p \Gamma_\psi \xi$. For $p \in [0,\psi)$, let $(ij) \in [\xi]_p^\phi$. Since $[\xi]_p^\phi$ is a prime, $i \in [\xi]_p^\phi$ (or) $j \in [\xi]_p^\phi \Rightarrow$ $i_p \Gamma_\psi \vee \Upsilon_\phi \xi (or)\, j_p \Gamma_\psi \vee \Upsilon_\phi \xi$. Hence, ξ is a prime $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals of $\Re$.

Corollary 3.9. A $\left(\Gamma_\psi, \Gamma_\psi \vee \Upsilon_\phi\right)$ fuzzy implicative ideals ξ of $\Re$ is semiprime if and only if for $p \in [0,\psi)$,$[\xi]_p^\phi \neq \phi$ is a semiprime implicative ideals of $\Re$.

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Proof. Straight Forward

4 Conclusions

In this research paper, We provided some conditions for being an $(\Gamma_{\psi}, \Gamma_{\psi} \vee \Upsilon_{\phi})$ Fuzzy Implicative Ideal of $\Re$ and Prime $(\Gamma_{\psi}, \Gamma_{\psi} \vee \Upsilon_{\phi})$ Fuzzy Implicative Ideal of $\Re$ and discussed some of its properties.

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