Mathematical Model of FYTHD Branching type with Hyphal Death

Tamadhir Jafaar Kadhim Al_Dahan[1], Prof. Dr. Ali Hussein Shuaa Al_Taie[2]

[1]College of Education for Pure Sciences || Wasit University || Wasit || Iraq || 52001 ||
[2]College of Education for Pure Sciences || Wasit University || Wasit || Iraq || 52001 ||

[1]Email: tamadirja1011@uowasit.edu.iq
[2]Email: alishuaa@uowasit.ed.iq

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Abstract

The teaching of models in biology is what gave rise to this study. to address challenges they are acquainted with a fresh perspective, to mentally generate a precise model, to translate it into mathematical words. This paper about mathematical model show behavior for growth of Lateral branching, Dichotomous branching, Tip death , Tip-hypha anastomosis with Haphal death . Despite the fact that there is an error ratio, mathematical modeling often decreases the amount of effort, time, and money needed to get the intended outcome. With the aid of a solution to a system of partial equations (PDEs), we will study the mathematical model of branching. The conclusions will show whether the tested fungus species grew successfully or not and in the numerical analysis, we applied a few symbols (pplane8, Pdepe).

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Introduction : For the development of fungus mycelia, we created new models. Table (1) presents a number of mathematically analyzed biological kinds along with an explanation of how they work and a list of their parameters. In this article, multiple fungus species will be combined [1].

Table (1): Illustrate branching, Biological type, Symbol of this type and version

Biological type	**Symbol**	**Version**	**Parameters description**
Lateral branching	**F**	$\delta=\alpha_2 p$	**α_2 is the number of branches produced per unit length hypha per unit time.**
Dichotomous branching	**Y**	$\delta=\alpha_1 n$	**α_1 is the number of tips produced per tip per unit time.**
Tip death	**T**	$\delta=-\alpha_3 n$	**α_3 is the loss rate of tips (constant for tip death).**
Tip-hypha anastomosis	**H**	$\delta=-\beta_1 n^2$	**β_1 is the rate of tip reconnections per unit time.**
Haphal death	**D**	$d=y_1 p$	**γ_1 is the loss rate of hyphal (constant for hyphal death).**

1-Mathematical Model

We'll investigate a different branching form Mathematical Model for fungal that includes hyphal death. Characterization of the fungus biologically: Mathematicians discovered that the fungi's branches may be transformed into letters, and that these letters rely on the species' behavior in terms of p= density of the hypha per unit area and filament length ,n = tip density. Our model includes a different type of fungus branching and with fungus death is Lateral branching with Dichotomous Branching Tip death with Tip-hypha anastomosis with Hyphal death (FYTHD) We can describe hyphal growth by the system below The model system of this type is [2,3,4]:

$$\frac{\partial p}{\partial t} = nv - dP$$

$$\frac{\partial n}{\partial t} = \frac{-\partial nv}{\partial x} + \sigma(p,n) \qquad (1)$$

Where $\sigma(p, n) = \alpha 2p + \alpha 1n - \alpha 3n - \beta 2np$

Then this system (1) becomes[5]:

$$\frac{\partial p}{\partial t} = n - p$$

$$\frac{\partial n}{\partial t} = \alpha p + \alpha n - \beta n + \beta np \qquad (2)$$

2-Non-dimensionlision and Stability

We will apply the non-dimensional form for the equations to aid in the analysis and numerical solution of the system (2). Ali H. Shuaa Al-Taie (2011) clear up how can put these parameters as dimensionlision less[1]

$$\frac{\partial p}{\partial t} = n - p$$

$$\frac{\partial n}{\partial t} = \alpha(p + n) - \beta n(1 + p) \qquad (3)$$

Where $\alpha = \frac{\alpha_1}{\gamma}, \beta = \frac{\alpha_3}{\gamma}$

We shall show or study the stability of the system to solve the above system as a stable solution.

$$\frac{\partial \rho}{\partial t} = n - \rho = 0 \rightarrow n = \rho \qquad (4)$$

$$\frac{\partial n}{\partial t} = \alpha(\rho + n) - \beta n(1 + \rho) = 0 \qquad (5)$$

$P = 0$ then $(\rho, n) = (0,0)$

and $\rho = \frac{2\alpha - \beta}{\beta}$ then $(p,n) = (\frac{2\alpha-\beta}{\beta}, \frac{2\alpha-\beta}{\beta})$

So that the steady state are $(\rho, n) = (0,0)$

and $(p, n) = (1,1)$

Therefor we take the Jacobin of these equations(4,5)

$$J(p,n) = \begin{bmatrix} -1 & 1 \\ \alpha - \beta n & \alpha - \beta(1+p) \end{bmatrix}$$

Now, determent the eigenvalues as λ_i ; i= 1,2

In This Situation , we'll Consider this point (0,0) is saddel point

And the point ($\frac{2\alpha-\beta}{\beta}$, $\frac{2\alpha-\beta}{\beta}$) stable spiral

For all α,β > 0 such that α > β , see fig (1) using **MATLAP** pplane 8

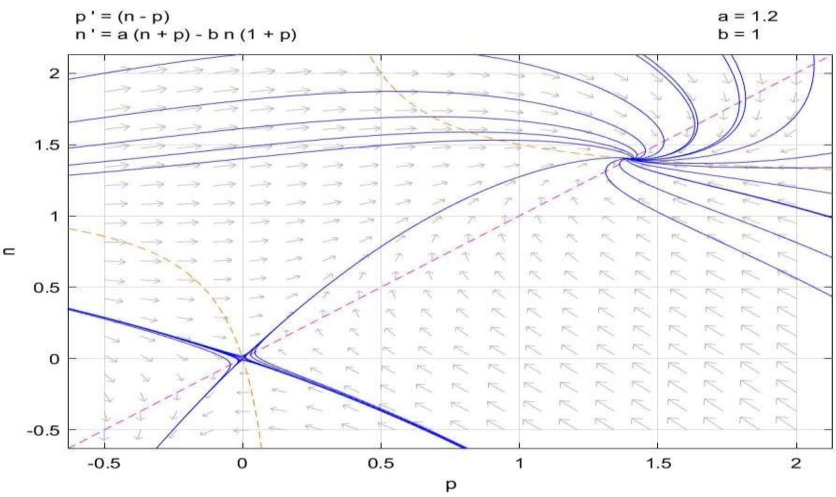

Figure 1: the (n,p) – plane , note that atrajectory connects the Saddle point at (0,0) and stable spiral at ($\frac{2\alpha-\beta}{\beta}$, $\frac{2\alpha-\beta}{\beta}$) For all α = 1.2 and β = 1

3-Traveling wave solution

We shall discuss the travelling wave solution in this section let z = x – ct, and we impose

$n(x,t) = N(z)$

$p(x,t) = P(z)$ (6)

where P(z) indicate to density profiles, and (c) rate of propagation of colony. N(z) and P(z) positive and are moves at constant speed wave c in function for (z) The function N(x,t), p(x,t) are traveling positive x – direction where c > 1 and α = β = 1 We appearance the traveling wave solution of the system in t and t in the form(4,5)

$$\frac{\partial p}{\partial t} = -c\frac{\partial P}{\partial z}, \qquad \frac{\partial n}{\partial t} = -c\frac{\partial N}{\partial z}, \qquad \frac{\partial n}{\partial t} = \frac{\partial N}{\partial z}$$

Therefore becomes the system

$\frac{\partial P}{\partial z} = \frac{-1}{c}$ [N-P] (7)

$\frac{\partial N}{\partial Z} = \frac{1}{1-C}$ [α (P + N) – βN (1 + P)] , $c \neq 1 , -\infty < z < \infty$

We study the steady states of the system (7), we get the saddle point (p,n)=(0,0) and **(** $\frac{2\alpha-\beta}{\beta}$ **,** $\frac{2\alpha-\beta}{\beta}$ **) ,** Unstable node for c = 1.5 , α = 1.2 and β = 1 unsing **MATLAB** pplane 8

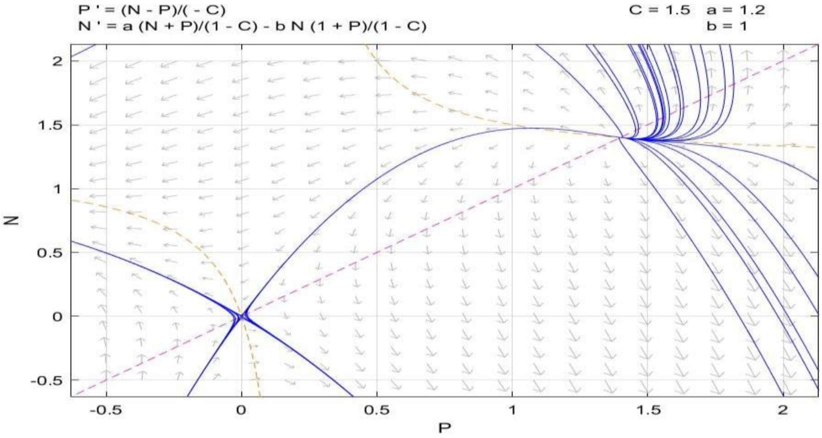

Figure 2 . The (N,P) plane not that atrajestory connects Unstable spiral ($\frac{2\alpha-\beta}{\beta}$ **,** $\frac{2\alpha-\beta}{\beta}$ **) to a saddle point at (0,0) for all c > 1 such that α = 1.2 and β = 1**

4-Numerica Solution

We turn to numerical solutions because the system cannot be solved, and we employ **MATLAB** in this case. pdepe code will be used to solve the system (3)

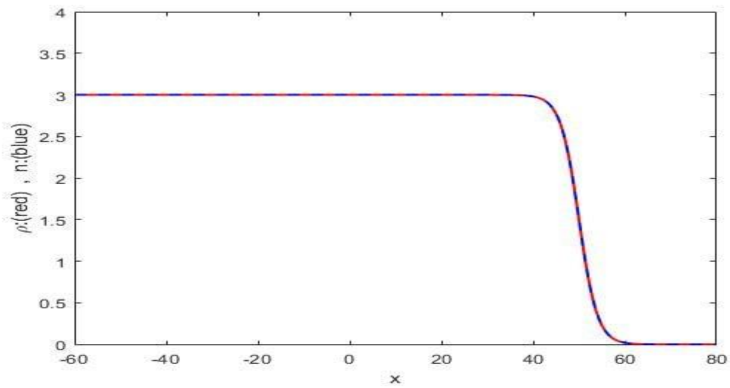

Figure 3. the initial condition of solution to the system (3) with the parameter α = 2 and β = 1

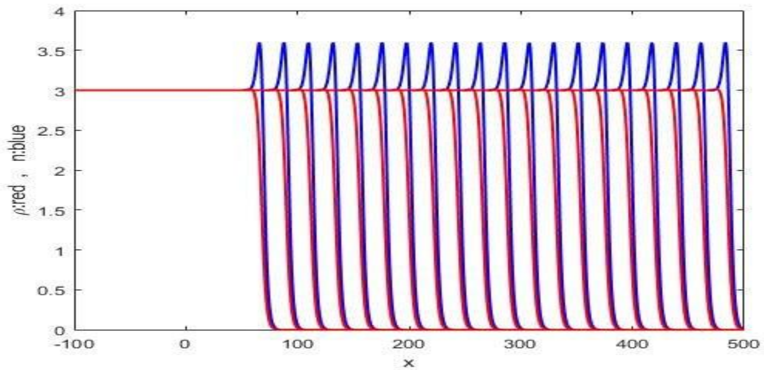

Figure 4: solution to the system (3) with parameters s α = 2 and β = 1 The wave speed c= 13.1145. where the blue line represent tips (n) ,the red line represent branches (p)

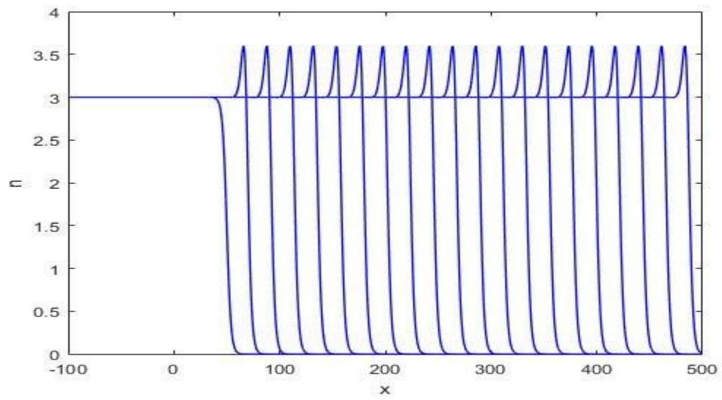

Figure 5: Tips were represented by the blue line (n).

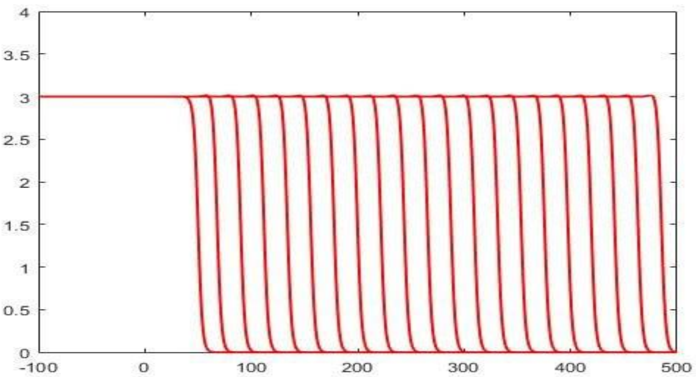

Figure 6: Branches were represented by the red line (p).

The association between traveling wave solution c and parameter α was determined in this study, where traveling wave increasing anytime the values of α increase [2,7] See Fig(7)

Table 2: the value α and c for solution of the type FYTHD

α	0.5	1	2	3	4	5	6	7	8	9	10
c	0.6	2	13.11	32.86	61	97.54	142.33	195.36	256.63	326.12	403.54

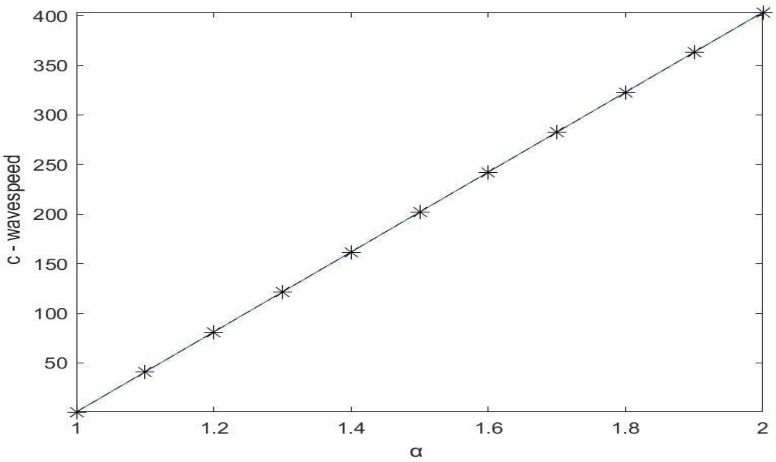

Figure 7: Illustrates the relation between the value α, and waves speed c

Now we consider the relationship between the parameter β and the traveling wave speed c, where the traveling wave decreases as the values of β rise. See Fig (8)

Table 3: the value of β and c for solution of the type FYTHD

β	0.5	1	2	3	4	5	6	7	8	9	10
c	8.67	8.04	7.52	7.03	6.63	6.3	5.78	5	4.53	4.02	3.5

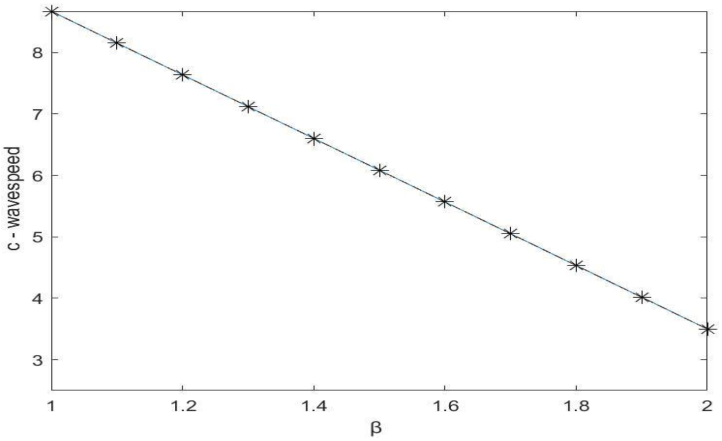

Figure 8: The relation between traveling wave c and parameter .

Consider a connection between the value v and the traveling wave speed c, where the wave speed rises as v rises.See Fig (9).

Table 4: the value of v and c for solution of the type FYTHD

v	0.5	1	2	3	4	5	6	7	8	9	10
c	0.79	2.04	5.52	10.04	15.43	21.57	28.38	35.8	43.82	52.3	61.41

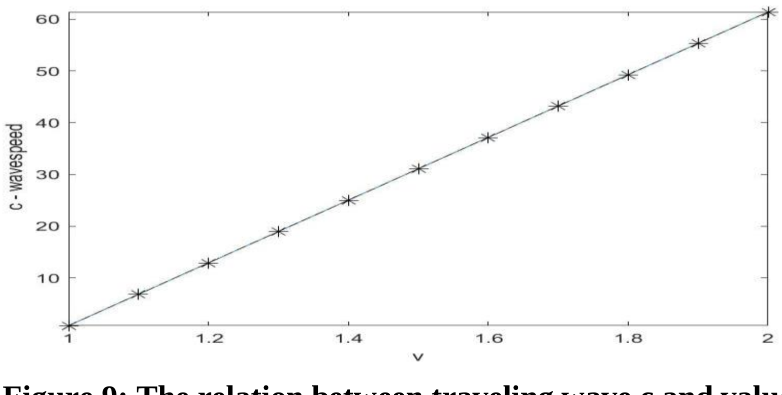

Figure 9: The relation between traveling wave c and value v

Finally, we examine the relationsbetween traveling wave c and value d, noting that the wave speed decreases as d values rise. view Fig (10)

Table 5: the value of d and c for solution of the type FYTHD

d	0.5	1	2	3	4	5	6	7	8	9	10
c	4.58	2.04	0.86	0.51	0.36	0.27	0.21	0.18	0.15	0.13	0.11

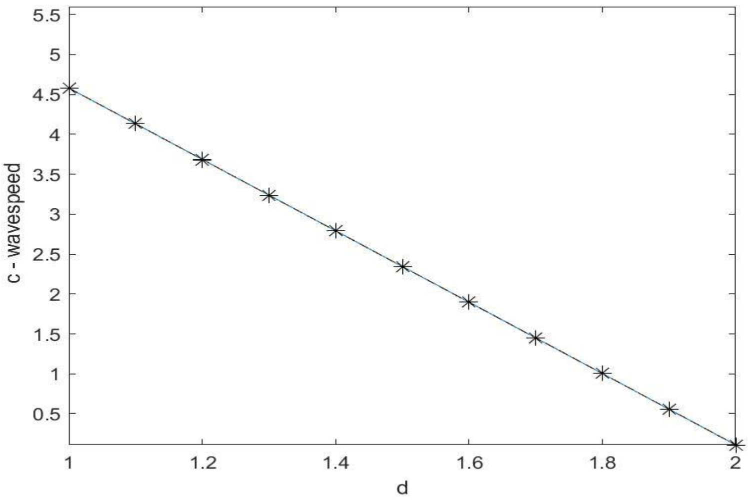

Figure 10: The relation between traveling wave c and value d.

5-Conclusion

Based on the aforementioned findings, we deduced that the traveling wave c increases whenever the values of α rise,see Fig (6) and we see that the traveling wave decreases as the values of β grow, see Fig (7) and we observe that the traveling wave grows as the values of v grow ,see Fig (8), and lastly, if the values of d grow, the traveling wave decreases .see Fig (9) . Since: $\alpha = \frac{\alpha_1}{\gamma}$ and $\beta = \frac{\alpha_3}{\gamma}$,There fore the growthrate is increasing with α_1 while keeping γ fixed , and the growthrate decreasing with α_3 , while keeping γ fixed.

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