Continuum Model on (FXTD) of Fungal Growth

Nabaa Naeem[1], Prof. Dr. Ali Hussein Shuaa[2]
Education College for Pure Sciences, Wasit University, Iraq
[1]Email: nabaanaeem38@gmail.com
[2]Email: alishuaa@uowasit.ed.iq

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Abstract
In principle, we know that the growth of any plant requires time, effort, and cost that differs from one plant to another, as well as fungus. In this paper, we studied its growth behavior and the effect of each branch on the fungus, then we combined a number of branches represented as mathematical model as partial differential equations (PDEs), approximate numerical solutions, and some mathematical steps, such as non-dimensionalisation, finding stability or steady state and representing it on phase plane, we found approximate results for these types using certain codes in **MATLAB** such as **Pplane8** and **Pdepe**.

Keyword: Lateral tip-hypha, anastomosis and Hyphal, Fungal Growth, Mathematical Model, Tip-hypha anostomosis.

1. **Introduction:** Mathematical modeling is defined as the application of mathematics in addressing real-life problems or problems in mathematics itself , by transforming a life problem into a mathematical problem. Leah-keshet is the first transform the biologist phenomena into mathematical form , here we have transformed the phenomenon of fungi growth into a formula and mathematical equations. Where the table (1) represents the biological type of branches, clarifying the parameters, and version of each branch.

Branching	Biological type	Symbol	Version	Parameters description
	Dichotomous branching	Y	$\delta = \alpha_1 n$	α_1 is the number of tips produced per tip per unit time
	Lateral branching	F	$\delta = \alpha_2 \rho$	α_2 is the number of branches produced per unit length hypha per unit time.
	Tip-hypha anastomosis	H	$\delta = -\beta_2 n\rho$	β_2 is the rate of tip reconnections per unit length hypha per unit time
	Tip-tip anastomosis	W	$\delta = -\beta_1 n^2$	β_1 is the rate of tip reconnections per unit time.
	Tip death	T	$\delta = -\alpha_3 n$	α_3 is the loss rate of tips (constant for tip death)

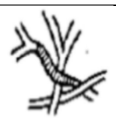
	Tip death due to overcrowding	X	$\delta = -\beta_3\rho^2$	β_3 is the rate at which overcrowding density limitation eliminates branching.
	Hyphal death	D	$d = \gamma_1\rho$	γ_1 is the loss rate of hyphal (constant for hyphal death).

Table 1 Illustrate branching, Biological type, symbol of this type and version.

2. Mathematical Model

Mathematicians saw that they transform the branches of the fungi into letters, and these letters depend on the behavior of the species in terms of ρ = density of the hypha in unit filament length per unit area , n = tip density.

We will study a new type of branching of fungal growth with hyphal death $\mathrm{F, X, T,}$ and D. We can describe hyphal growth by the system below:

$$\frac{\partial \rho}{\partial t} = nv - d\rho$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + \delta(p, n) \qquad (1)$$

Where: $(p, n) = \alpha_2 p - \beta_3 p^2 - \alpha_3 n$

Then this system becomes:

$$\frac{\partial \rho}{\partial t} = nv - \gamma\rho$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + \alpha_2 p - \beta_3 p^2 - \alpha_3 n \qquad (2)$$

2.1. Non-dimensionlision and Stability

Leah-keshet (1982) and Ali H. Shuaa Al-Taie (2011)[2,3], clear up how can put these parameters as dimensionlision less

$$\frac{\partial \rho}{\partial t} = n - \rho$$

$$\frac{\partial n}{\partial t} = -\frac{\partial (nv)}{\partial x} + \alpha p(1 - p) - \beta n \qquad (3)$$

Where: $\alpha = \frac{\alpha_2 v}{\gamma}$ is the parameter α represent the rate of hyphal branching per unit length hypha per unit time. $[\alpha p(1 - p) - \beta n]$ Thus represent the number of branches produced per unit time per unit length of hyphae. [2, 3].

Now, to find steady states when take from system (2)

$$\frac{\partial \rho}{\partial t} = n - \rho = 0 \rightarrow n = \rho$$

And on the other hand

$$\frac{\partial n}{\partial t} = \alpha p(1 - p) - \beta n = 0 \quad \rightarrow \alpha p(1 - p) - \beta p = 0$$

The solution of equation, we'll find the values of (p,n), the steady state are :

(p,n) = (0,0) and $\left(\frac{\alpha-\beta}{\alpha}, \frac{\alpha-\beta}{\alpha}\right)$ therefore, we take Jacobain of these equations [5, 8, 6]

f = n − p

g = αp(1 − p) – βn

$$J(p, n) = \begin{bmatrix} \frac{\partial f}{\partial p} & \frac{\partial f}{\partial n} \\ \frac{\partial g}{\partial p} & \frac{\partial g}{\partial n} \end{bmatrix}$$

$$J(p, n) = \begin{bmatrix} -1 & 1 \\ \alpha - 2\alpha p & -\beta \end{bmatrix}$$

We can classify the cirtical points according to the eigenvalues of this matrix Jacobain at (0,0):

$$J(p, n) = \begin{bmatrix} -1 & 1 \\ \alpha & -\beta \end{bmatrix}$$

Thus | A − λI | = 0 , we get two values of (λ) , let β = 1

$$\lambda_1 = -1 + \frac{1}{2}\sqrt{4\alpha}$$

$$\lambda_2 = -1 - \frac{1}{2}\sqrt{4\alpha}$$

The stability of the steady state is saddle point for all α, β ≥ 0 and α > β. Then we take Jacobain for $\left(\frac{\alpha-\beta}{\alpha}, \frac{\alpha-\beta}{\alpha}\right)$:

$$J\left(\frac{\alpha - \beta}{\alpha}, \frac{\alpha - \beta}{\alpha}\right) = \begin{bmatrix} -1 & 1 \\ -\alpha + 2\beta & -\beta \end{bmatrix}$$

Thus | A − λI | = 0 we get two values of (λ)

$$\lambda_1 = -\frac{1}{2} + \frac{1}{2}\sqrt{8 - 4\alpha}$$

$$\lambda_2 = -\frac{1}{2} - \frac{1}{2}\sqrt{8 - 4\alpha}$$

The stability of the steady state is stable spiral for all α, β ≥ 0 and α > β., see Fig(1).

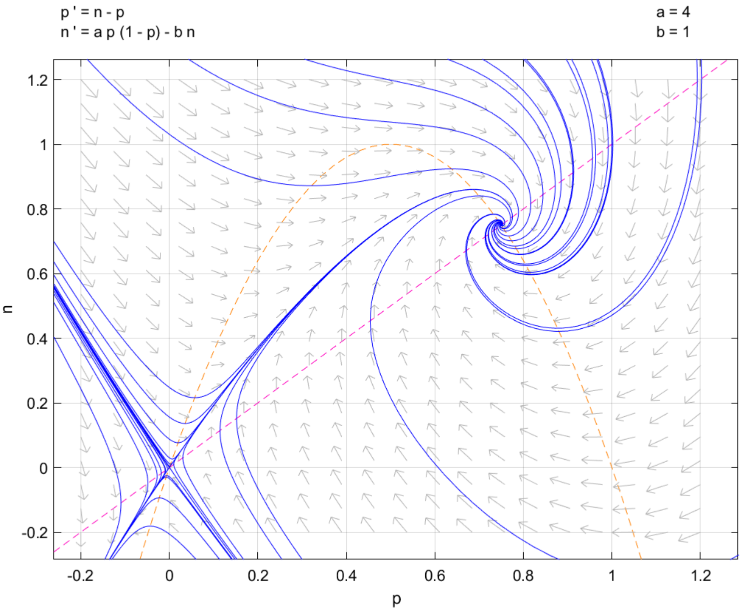


Figure 1: The (p, n) plane for the ordinary differential equation (3), when α = 4, β = 1. The solid blue line corresponds to the model a trajectory connects the saddle point (0, 0) to the stable spiral $\left(\frac{3}{4},\frac{3}{4}\right)$, the dashed lines corresponds the model to the null-cline. Solution are produced using **MATLAB pplane8**.

2.2. Traveling wave solution

We will now discuss the traveling wave solution, we assume that: $(x,t) = \rho(z)$ and, $n(x,t) = N(z)$ where $z = x - ct$, $P(z)$ profile density and propagation rate c of edge of the colony. (z) And, $N(z)$ is a non-negative function of z. The function, $p(x,t)$, $n(x,t)$ are moving waves, moving with a 5 constant speed c in a positive x direction, where $c > 0$, and $\alpha = 1$ to search for a traveling wave solution to the equations in x and t of the system (3).

$\frac{d\rho}{dt} = -c\frac{d\rho}{dx}, \quad \frac{dn}{dt} = -\frac{dN}{dx}, \quad \frac{dn}{dt} = \frac{dN}{dx}$

Thus we can reduce the system (3) to a set of two ordinary differential equation:

$$-\frac{1}{c}[N - P] = 0$$

$$\frac{1}{1-c}[\alpha P(1 - P) - \beta N] = 0$$

The system (3) has two uniform steady state points (0,0) and, $\left(\frac{\alpha-\beta}{\alpha},\frac{\alpha-\beta}{\alpha}\right)$, we can take Jacobian and by using determinate eigenvalue of λ , | A − λI |= 0 to solve the system, we get two of the value of λ, we get (N, P) = (0,0) saddle point and $\left(\frac{3}{4},\frac{3}{4}\right)$ unstable spiral constant for c=-0.5 , $\alpha = 4$ and β=1. This helps us to determine the initial conditions of p and . See Fig (2).

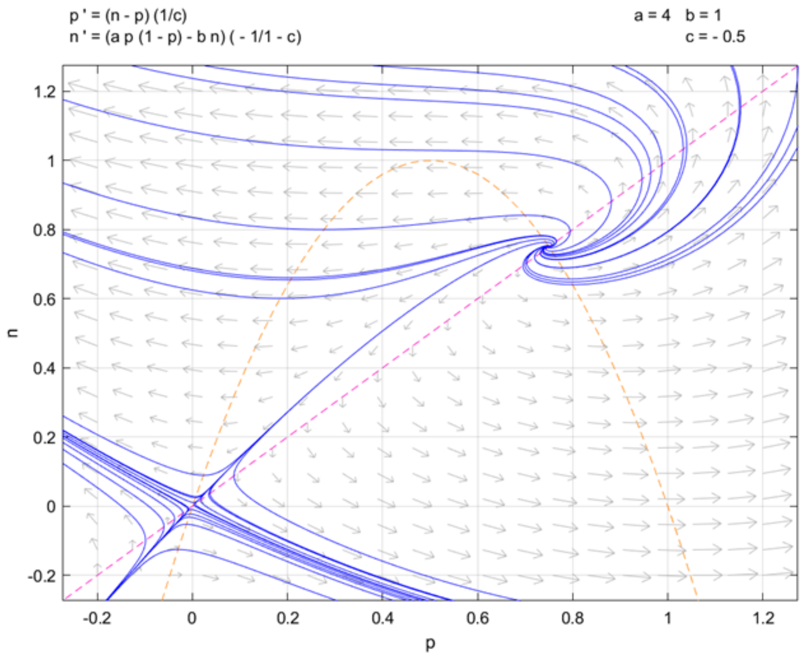


Figure (2): The (n, p) plane, note that a trajectory connects the saddle point in (0, 0) and unstable spiral in point $\left(\frac{3}{4},\frac{3}{4}\right)$.

2.3. Numerical Solution

Because the system (3) cannot be solved exactly so we resort to numerical solution, and here we using **pdepe** code in **MATLAB.** See Fig(3,4,5,6).

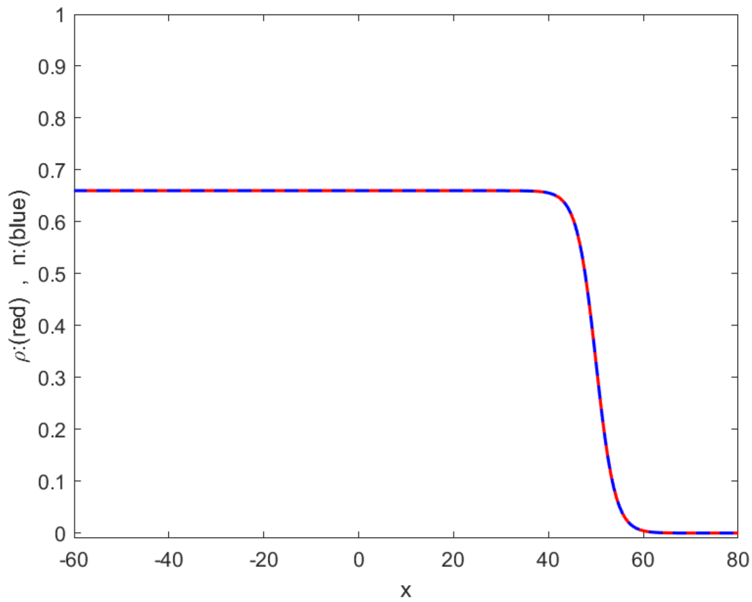


Figure (3): The initial condition of solution to the system (3) with the parameters α = 4 and β = 1.

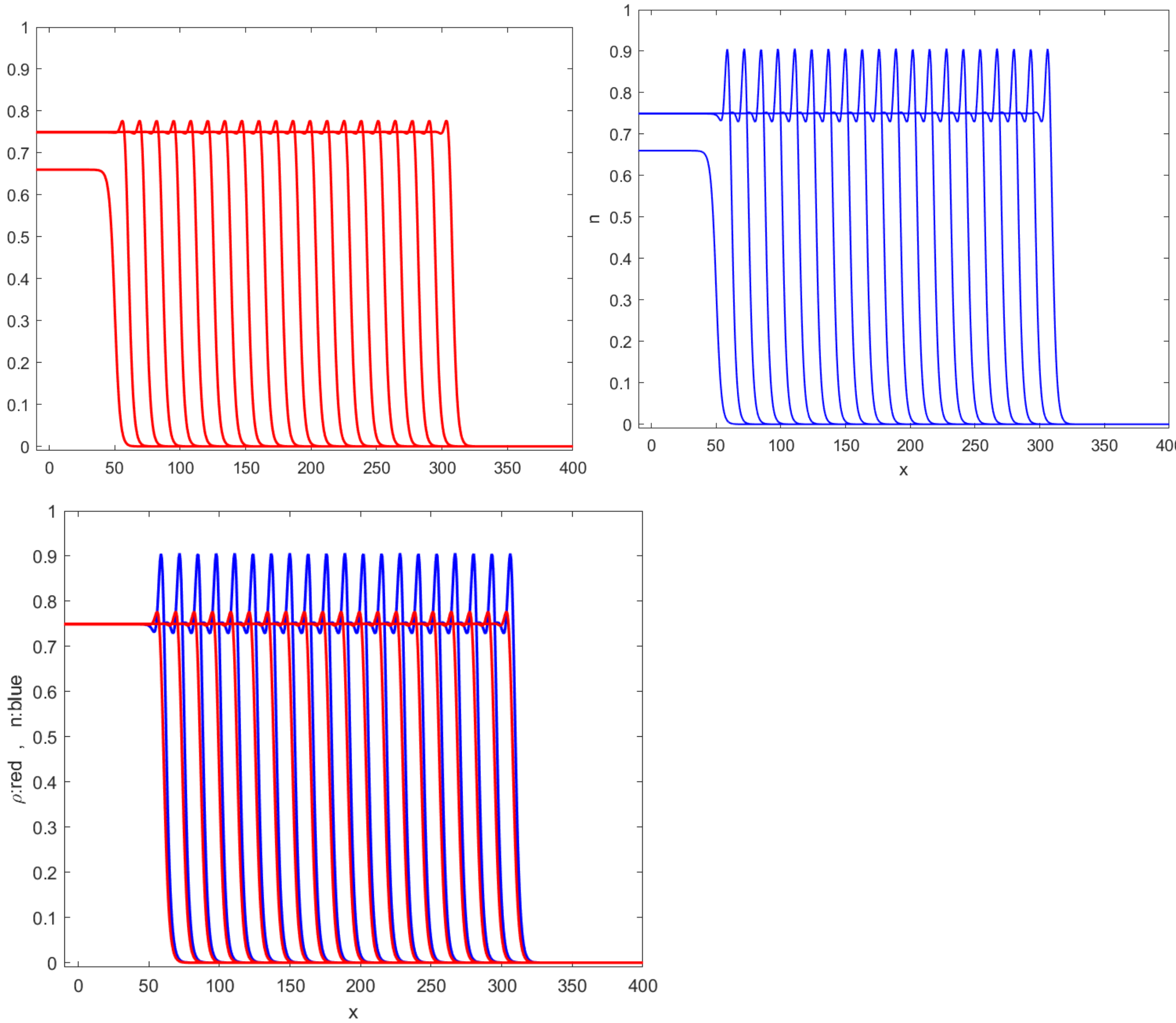


Figure (4,5,6): Solution to the system (3)with parameters $\alpha = 4$ and $\beta = 1$ and time $t = 1, \ldots, 200$, where blue line represented tips (n), and red line represented branches (p).

With take values of $\alpha = 0.5$, that is very clear the traveling wave solution start from left to right and still the same wave. From this operation we get the relationship between traveling wave solution (c) and parameter (α). With taking $\beta=1$,$v = 1$, we can show that the table(2). Where α increasing then the traveling waves solution c is increasing, see Fig(7).

α	0.5	1	2	3	4	5	6	7	8	9	10
c	0.15	0.35	0.69	1.36	1.94	2.45	2.92	3.34	3.74	4.11	4.45

Table 2: the value of α and c for solution of the type FXTD

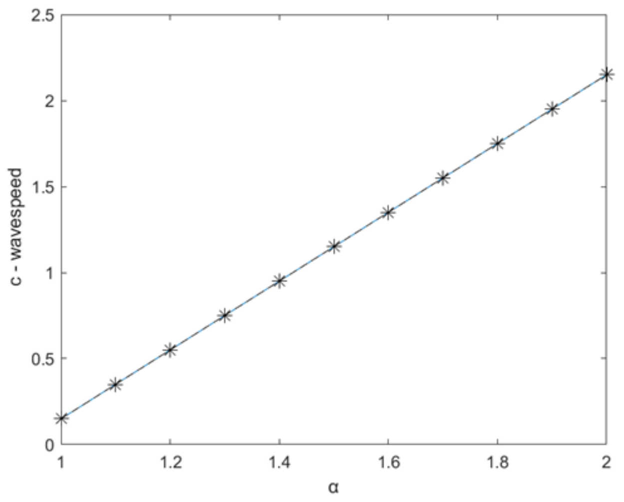


Figure 7: The relation between wave speed c and α values

Now, we get the relation between traveling waves solution c and β values with taking α ,v = 1 , we can show that the table(3), where β increasing then the traveling waves solution c is decreasing, see Fig(8)

β	0.5	1	2	3	4	5	6	7	8	9	10
c	1.69	1.37	1.09	0.84	0.62	0.39	0.10	0.083	0.07	0.061	0.04

Table 3: The value of β and c for solution of the type FXTD.

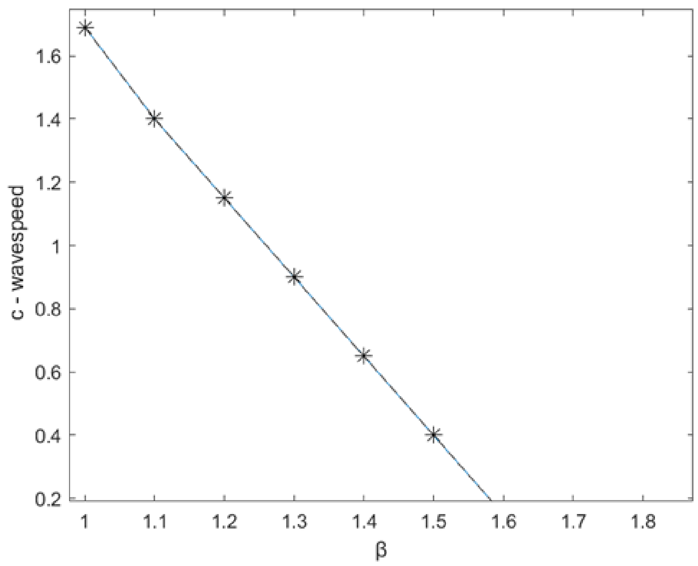


Figure 8: The relation between wave speed c and β values.

Now, we take relation between traveling waves solution c and v values with taking α = β = 1, we can show that the table(4), where v increasing then the traveling waves solution c is increasing, see Fig(9)

v	0.5	1	2	3	4	5	6	7	8	9	10
c	0.12	0.70	1.01	1.36	1.93	2.45	2.91	3.34	4.1	5.61	7.20

Table 4: the value of v and c for solution of the type FXTD

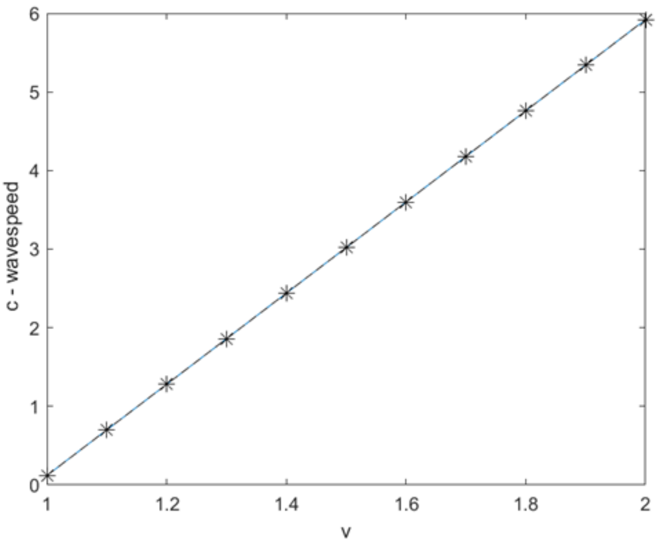


Figure 9: The relation between wave speed c and v values.

Also, we take relation between traveling waves solution c and d values with taking α = β =v = 1, we can show that the table(5), where v increasing then the traveling waves solution c is decreasing, see Fig(10)

d	0.5	1	2	3	4	5	6	7	8	9	10
c	0.67	0.50	0.49	0.31	0.2	0.11	0.05	0.03	0.02	0.01	0.009

Table 5: the value of d and c for solution of the type FXTD

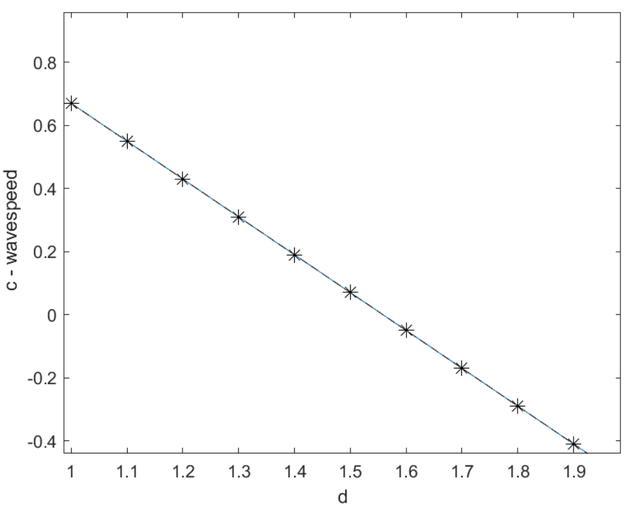


Figure 10: The relation between wave speed c and d values

3. The Conclusions

In this paper it was concluded that there is a relationship between traveling wave solution (c) and parameter $\alpha = \frac{\alpha_2 v}{\gamma}$ where traveling wave (c) increase whenever the values of α increase. From the relationship between traveling wave solution (c) and parameter (α) we see that (α) increasing then the traveling waves solution (c) is increasing, And we noted that the relation between traveling waves solution (c) and (β) values, where (β) increasing then the traveling waves solution (c) is decreasing ,also the relation between traveling waves solution (c) and (v) values, we've seen that (v) increasing then the traveling waves solution (c) is increasing too, finally, we take relation between traveling waves solution (c) and (d) values ,(v) increasing then the traveling waves solution (c) is decreasing[3].

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