Group Mean Cordial Labeling of Triangular Snake Related Graphs

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Article Info

Page Number: 12518 - 12531

Publication Issue:

Vol 71 No. 4 (2022)

Abstract

Let G be a (p, q) graph and let A be a group. Let $f : V(G) \rightarrow A$ be a map. For each edge uv assign the label $\left\lfloor \frac{o(f(u))+o(f(v))}{2} \right\rfloor$. Here $o(f(u))$denotes the order of $f(u)$ as an element of the group A. Let I be the set of all integers that are labels of the edges of G. f is called a group mean cordial labeling if the following conditions hold:

(1) For x, y ∈ A, $|v_f(x) - v_f(y)| \leq 1$, where $v_f(x)$ is the number of vertices labeled with x.

(2) For i, j ∈ I, $|e_f(i) - e_f(j)| \leq 1$, where $e_f(i)$ denote the number of edges labeled with i.

A graph with a group mean cordial labeling is called a group mean cordial graph. In this paper, we take A as the group of fourth roots of unity and prove that, Triangular snake, Double triangular snake and Alternate triangular snake are group mean cordial graphs.

Article History

Article Received: *15 September 2022*

Revised: *25 October 2022*

Accepted: *14 November 2022*

Publication: *21 December 2022*

Keywords — Cordial labeling, mean labeling, group mean cordial labeling, Triangular snake, Double triangular snake, Alternate triangular snake.

AMS Subject Classification — 05C78.

I. INTRODUCTION

Graphs considered here are finite, undirected and simple. Terms not defined here are used in the sense of Harary [4] and Gallian [3]. Somasundaram and Ponraj [6] introduced the concept of mean labeling of graphs.

Definition 1.1. [6] A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to 0, 1, 2, ..., q such that when each edge uv is labeled with $\frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edge labels are distinct.

Cahit [2] introduced the concept of cordial labeling.

Definition 1.2. [2] Let f : V (G) → {0, 1} be any function. For each edge xy assign the label |f(x) − f(y)|. f is called a cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Also the number of edges labeled 0 and the number of edges labeled 1 differ by at most 1.

Ponraj et al. [5] introduced mean cordial labeling of graphs.

Definition 1.3. [5] Let f be a function from the vertex set V (G) to {0, 1, 2}. For each edge uv assign the label l $\left\lceil\frac{f(u)+f(v)}{2}\right\rceil$. f is called a mean cordial labeling if $|v_f(i) - v_f(j)| \le 1$ and $\left|e_f(i) - e_f(j)\right| \le 1$ i, j ∈ {0, 1, 2}, where $v_f(x)$ and $e_f(x)$ respectively denote the number of vertices and edges labeled with x (x = 0, 1, 2). A graph with a mean cordial labeling is called a mean cordial graph.

Athisayanathan et al. [1] introduced the concept of group A cordial labeling.

Definition 1.4. [1] Let A be a group. We denote the order of an element a ∈ A by o(a). Let f : V (G) → A be a function. For each edge uv assign the label 1 if (o(f(u)), o(f(v))) = 1or 0 otherwise. f is called a group A Cordial labeling if $|v_f(a) - v_f(b))| \le 1$ and $|e_f(0) - e_f(1)| \le 1$, where vf (x) and ef (n) respectively denote the number of vertices labelled with an element x and number of edges labelled with n(n = 0, 1). A graph which admits a group A Cordial labeling is called a group A Cordial graph.

Motivated by these , we define group mean cordial labeling of graphs.

For any real number x, we denoted by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x. The triangular snake T_n is obtained from a path Pn by replacing each edge of the path by a triangle. The double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path. The Alternate triangular snake $A(T_n)$ is obtained from a path P_n by replacing every alternate edge of the path by a triangle.

II. Main Results

Definition 2.1. Let G be a (p, q) graph and let A be a group. Let $f : V(G) \rightarrow A$ be a map. For each edge uv assign the label $\left\lceil \frac{o(f(u))+o(f(v))}{2} \right\rceil$. Here $o(f(u))$ denotes the order of $f(u)$ as an element of the group A. Let I be the set of all integers that are labels of the edges of G. f is called a group mean cordial labeling if the following conditions hold:

(1) For x, y ∈ A, $|v_f(x) - v_f(y)| \leq 1$, where $v_f(x)$ is the number of vertices labeled with x.

(2) For i, j ∈ I, $|e_f(i) - e_f(j)| \leq 1$, where $e_f(i)$ denote the number of edges labeled with i.

A graph with a group mean cordial labeling is called a group mean cordial graph.

In this paper, we take the group A as the group {1, −1, i, −i} which is the group of fourth roots of unity, that is cyclic with generators i and −i.

Example 2.2. A simple example of a group mean cordial graph is given in Fig. 2.1.

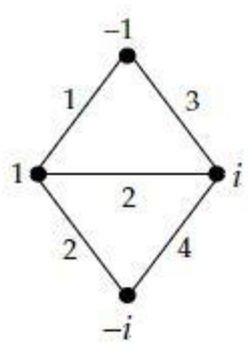

Fig. 2.1. Example of Triangular Snake graph

Theorem 2.1. The Triangular Snake graph, T_n is a group mean cordial graph for every n.

Proof. Let $P_n = u_1u_2 \dots u_n$ be a path. Let $V(T_n) = V(P_n) \cup \{ v_j : 1 \le j \le n-1 \}$. Then $E(T_n) = E(P_n) \cup \{ u_jv_j, u_{j+1}v_j : 1 \le j \le n-1 \}$. The order and size of T_n are 2n-1 and 3n-3 .

Case 1: $n \equiv$ 0,1,2 (mod 4).

Define $f: V(T_n) \to \{1, -1, i, -i\}$ by,

$$f(u_j) = \begin{cases} -1 & if\ j \equiv 1\ (mod\ 4) \\ i & if\ j \equiv 2\ (mod\ 4) \\ -i & if\ j \equiv 3\ (mod\ 4) \\ 1 & if\ j \equiv 0\ (mod\ 4) \end{cases}$$

and

$$f(v_j) = \begin{cases} 1 & if\ j \equiv 1\ (mod\ 4) \\ i & if\ j \equiv 2\ (mod\ 4) \\ -1 & if\ j \equiv 3\ (mod\ 4) \\ -i & if\ j \equiv 0\ (mod\ 4) \end{cases}$$

Case 2: $n \equiv$ 3 (mod 4).

The group mean cordial labeling of T_3 is given in Fig.2.2.

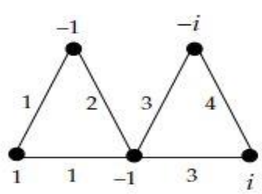

Fig. 2.2. Triangular Snake graph –T_3.

Let n > 3 .

Assign the labels as in case 1 to the vertices u_j (1 ≤ j ≤ n-3) and v_j (1 ≤ j ≤n-4) .

Next label u_{n-2}, u_{n-1}, u_n as $-1, i, 1$ in order and $v_{n-3}, v_{n-2}, v_{n-2}$ as $-1, -i, -i$ in order.

The values of v_f (j) and e_f(s) are tabulated in Tables 2.1 and 2.2.

Nature of n	v_f (1)	v_f (2)	v_f (2)	v_f (3)
$n \equiv 0,2 \pmod 4$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}$	$\frac{n}{2}-1$
$n \equiv 1,3 \pmod 4$, $n \neq 3$	$\frac{n-1}{2}$	$\frac{n+1}{2}$	$\frac{n-1}{2}$	$\frac{n+1}{2}$

TABLE 2.1.

Nature of n	e_f (1)	e_f (2)	e_f (3)	e_f (4)
$n \equiv 0 \pmod 4$	$\frac{3n}{4}-1$	$\frac{3n}{4}-1$	$\frac{3n}{4}-1$	$\frac{3n}{4}$
$n \equiv 1 \pmod 4$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$
$n \equiv 2 \pmod 4$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-6}{4}$
$n \equiv 3 \pmod 4$	$\frac{3n-5}{4}$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$	$\frac{3n-1}{4}$

TABLE 2.2.

Hence Tables 2.1. & 2.2. prove that f is a group mean cordial labeling.

Theorem 2.2. Double Triangular Snake graph, $D\ (T_n)$ is a group mean cordial graph for every n.

Proof. Let $P_n = u_1 u_2 \ldots u_n$ be the common path. Let x_j, y_j ($1 \leq j \leq n-1$) be the newly added vertices. Then E(D(T_n)) =E(P_n) ∪ $\{u_j x_j, u_{j+1} x_j, u_j y_j, u_{j+1} y_j \ : \ 1 \leq j \leq n-1\}$. The order and size of D (T_n) are 3n-2 and 5n-5.

Define $f : V(\ D\ (T_n)) \ \rightarrow \ \{1, -1, i, -i\}$ by,

Case 2: $n \equiv 1 \ (mod \ 4)$.

$$f(u_j) = \begin{cases} 1 & if\ j \equiv 1\ (mod\ 4) \\ -1 & if\ j \equiv 2\ (mod\ 4) \\ i & if\ j \equiv 3\ (mod\ 4) \\ -i & if\ j \equiv 0\ (mod\ 4) \end{cases}$$

and

$$f(x_j) = f(y_j) = \begin{cases} i & if\ j \equiv 1\ (mod\ 4) \\ 1 & if\ j \equiv 2\ (mod\ 4) \\ -i & if\ j \equiv 3\ (mod\ 4) \\ -1 & if\ j \equiv 0\ (mod\ 4) \end{cases}$$

Case 2: $n \equiv 2\ (mod\ 4)$.

Assign the labels to the vertices $u_j(1 \le j\ \ n-1)\ and\ \ x_j, y_j(1 \le j \le n-2)$ as in case 1. Then assign $i, -1, -i$ to the vertices u_n, x_{n-1}, y_{n-1} in order.

Case 3: $n \equiv 3\ (mod\ 4)$.

Assign the labels to the vertices $u_j(1 \le j \le n-2)\ and\ \ x_j, y_j(1 \le j \le n-3)$ as in case 1. Next label x_{n-2}, y_{n-2} with -1 ; u_{n-1}, x_{n-1} with i ; u_n with $-i$ and y_{n-1} with 1.

Case 4: $n \equiv 0\ (mod\ 4)$.

Assign the labels to the vertices $u_j(1 \le j \le n-3)\ and\ \ x_j, y_j(1 \le j \le n-4)$ as in case 1. Next assign i to the vertices u_{n-2}, x_{n-2} ; -1 to the vertices $x_{n-3}, y_{n-3}, y_{n-2}$; $-i$ to the verices u_{n-1}, x_{n-1} and assign 1 to the verices u_n, y_{n-1}.

Tables 2.3 & 2.4prove that f is a group mean cordial labeling.

Nature of n	v_f ***(1)***	v ***(2)***	v ***(3)***	v_f ***(4)***
$n \equiv 0\ (\text{mod}\ 4)$	$\frac{3n}{4}$	$\frac{3n}{4}$	$\frac{3n}{4} - 1$	$\frac{3n}{4}$
$n \equiv 1\ (\text{mod}\ 4)$	$\frac{3n+1}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$	$\frac{3n-3}{4}$
$n \equiv 2\ (\text{mod}\ 4)$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$	$\frac{3n-2}{4}$
$n \equiv 3\ (\text{mod}\ 4)$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-1}{4}$	$\frac{3n-5}{4}$

TABLE 2.3.

Nature of n	e_f ***(1)***	e_f ***(2)***	e_f ***(3)***	e_f ***(4)***
$n \equiv 0\ (\text{mod}\ 4)$	$\frac{5n-8}{4}$	$\frac{5n-4}{4}$	$\frac{5n-4}{4}$	$\frac{5n-4}{4}$
$n \equiv 1\ (\text{mod}\ 4)$	$\frac{5n-5}{4}$	$\frac{5n-5}{4}$	$\frac{5n-5}{4}$	$\frac{5n-5}{4}$
$n \equiv 2\ (\text{mod}\ 4)$	$\frac{5n-6}{4}$	$\frac{5n-2}{4}$	$\frac{5n-6}{4}$	$\frac{5n-6}{4}$
$n \equiv 3\ (\text{mod}\ 4)$	$\frac{5n-7}{4}$	$\frac{5n-3}{4}$	$\frac{5n-7}{4}$	$\frac{5n-3}{4}$

TABLE 2.4.

Theorem 2.3. The Alternate Triangular Snake, A(T_n)is a group mean cordial graph when n is odd.

Proof. Let $P_n = u_1u_2 \dots u_n$ be a path.

Case 1: The Alternative Triangular snake starts with triangle.

Let $V(A(T_n)) = V(P_n) \cup \{v_j : 1 \le j \le \frac{n-1}{2}\}$. Then $E(A(T_n)) = E(P_n) \cup \{u_j v_{\frac{j+1}{2}} : j \equiv 1 \pmod 2\} \cup \{u_j v_{\frac{j}{2}} : j \equiv 0 \pmod 2\}$.

The order and size of this graph are $\frac{3n-1}{2}$ and 2n-2.

Subcase 1.1: $n \equiv 1 \pmod 8$.

Define $f : V(A(T_n)) \to \{1, -1, i, -i\}$ by,

$$f(u_j) = \begin{cases} 1 & if\ \ j \equiv 0,1,2 \pmod 8 \\ -1 & if\ \ j \equiv 4,5 \pmod 8 \\ i & if\ \ j \equiv 3,6,7 \pmod 8 \end{cases}$$

and

$$f(v_j) = \begin{cases} -1 & if\ \ j \equiv 1 \pmod 4 \\ -i & if\ \ j \equiv 0,2,3 \pmod 4 \end{cases}$$

Subcase 1.2: $n \equiv 3 \pmod 8$.

Label the vertices $u_j (1 \le j \le n-2), v_j (1 \le j \le \frac{n-1}{2})$ as in subcase 1.1. Then label u_{n-1} with i and $u_n$with $-i$.

Subcase 1.3: $n \equiv 5 \pmod 8$.

Label the vertices $u_j (1 \le j \le n-4), v_j (1 \le j \le \frac{n-5}{2})$ as in subcase 1.1. Next define $f(u_{n-3}) = 1$; $f(u_{n-2}) = -1$; $f(u_{n-1}) = f(u_n) = i$ and $f\left(v_{\frac{n-3}{2}}\right) = f\left(v_{\frac{n-1}{2}}\right) = -i$.

Subcase 1.4: $n \equiv 7 \pmod 8$.

Label the vertices $u_j (1 \le j \le n-6), v_j (1 \le j \le \frac{n-7}{2})$ as in subcase 1.1. Next define $f(u_{n-5}) = f(u_{n-4}) = i$; $f(u_{n-3}) = f(u_{n-2}) = -i$; $f(u_{n-1}) = 1$; $f(u_n) = -1$ and $f\left(v_{\frac{n-5}{2}}\right) = f\left(v_{\frac{n-3}{2}}\right) = -1$ and $f\left(v_{\frac{n-1}{2}}\right) = 1$.

By this labelling, in each case we get $e_f(s) = \frac{n-1}{2}$, $s \in \{1, -1, i, -i\}$.

The vertex condition is also satisfied by the following Table 2.5

Nature of n	v_f *(1)*	*v (2)*	*v (3)*	v_f *(4)*
$n \equiv 1 \ (\text{mod } 8)$	$\frac{3n+5}{8}$	$\frac{3n-3}{8}$	$\frac{3n-3}{8}$	$\frac{3n-3}{8}$
$n \equiv 3 \ (\text{mod } 8$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$
$n \equiv 5 \ (\text{mod } 8)$	$\frac{3n+1}{8}$	$\frac{3n-7}{8}$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}$
$n \equiv 7 \ (\text{mod } 8)$	$\frac{3n+3}{8}$	$\frac{3n+3}{8}$	$\frac{3n-5}{8}$	$\frac{3n-5}{8}$

TABLE 2.5.

Case 2: The Alternative Triangular snake starts pendent edge.

Let $V\big(A(T_n)\big) = V(P_n) \cup \{v_j : 1 \le j \le \frac{n-1}{2}\}$.Then $E\big(A(T_n)\big) = E(P_n) \cup \{u_j v_{\frac{j-1}{2}} : j \equiv 1 \ (mod\ 2) \cup \{u_j v_{\frac{j}{2}} : j \equiv 0 \ (mod\ 2)\}$.

The order and size of this graph are $\frac{3n-1}{2}$ and 2n-2.

Subcase 2.1: $n \equiv 1 \ (mod \ 8)$.

Define $f: V\big(A(T_n)\big) \to \{1, -1, i, -i\}$ by,

$$f(u_j) = \begin{cases} 1 & if \ \ j \equiv 2,7 \ \ (mod\ 8) \\ -1 & if \ \ j \equiv 0,5,6\ (mod\ 8) \\ i & if \ \ j \equiv 1,3,4\ (mod\ 8) \end{cases}$$

and

$$f(v_j) = \begin{cases} -i & if \ \ j \equiv 0,1,2 \quad (mod\ 4) \\ 1 & if \ \ j \equiv 3 \qquad\quad (mod\ 4) \end{cases}$$

Subcase 2.2: $n \equiv 3 \ (mod \ 8)$.

Label the vertices $u_j(1 \le j \le n-2), v_j(1 \le j \le \frac{n-3}{2})$ as in subcase 2.1. Then label u_{n-1} with $-i$ and u_n with -1 and $v_{\frac{n-1}{2}}$ with 1.

Subcase 2.3: $n \equiv 5 \ (mod \ 8)$.

Label the vertices $u_j(1 \le j \le n-4), v_j(1 \le j \le \frac{n-5}{2})$ as in subcase 2.1. Next define $f(u_{n-3}) =$

i; $f(u_{n-2}) = -1$; $f(u_{n-1}) = 1$; $f(u_n) = -i$ and $f\left(v_{\frac{n-3}{2}}\right) = -i$; $f\left(v_{\frac{n-1}{2}}\right) = 1$.

Subcase 2.4: $n \equiv 7 \pmod 8$.

Label the vertices $u_j (1 \le j \le n-6), v_j (1 \le j \le \frac{n-7}{2})$ as in subcase 2.1. Next define $f(u_{n-5}) = f(u_{n-1}) = i$; $f(u_{n-3}) = 1$; $f(u_{n-2}) = -i$; $f(u_{n-4}) = f(u_n) = -1$ and $f\left(v_{\frac{n-5}{2}}\right) = -i$; $f\left(v_{\frac{n-3}{2}}\right) = f\left(v_{\frac{n-1}{2}}\right) = 1$.

Here also, $e_f(s) = \frac{n-1}{2}$, $s \in \{1, -1, i, -i\}$.

The vertex condition is proved by the following Table 2.6.

Nature of n	v_f ***(1)***	v ***(2)***	v ***(3)***	v_f ***(4)***
$n \equiv 1 \pmod 8$	$\frac{3n-3}{8}$	$\frac{3n-3}{8}$	$\frac{3n+5}{8}$	$\frac{3n-3}{8}$
$n \equiv 3$ (mod 8	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$	$\frac{3n-1}{8}$
$n \equiv 5 \pmod 8$	$\frac{3n+1}{8}$	$\frac{3n-7}{8}$	$\frac{3n+1}{8}$	$\frac{3n+1}{8}$
$n \equiv 7 \pmod 8$	$\frac{3n+3}{8}$	$\frac{3n-5}{8}$	$\frac{3n+3}{8}$	$\frac{3n-5}{8}$

TABLE 2.6.

Hence Alternate Triangular Snake is a group mean cordial graph when n is odd.

Theorem 2.4. The Alternate Triangular Snake A(T_n)is a group mean cordial graph when n is even.

Proof. If the triangular snake graph starts with a triangle and n is even, then |V(A(T_n))| = $\frac{3n}{2}$ and |E(A(T_n)| = 2n – 1.

If the triangular snake graph starts with a pendent edge and n is even, then |V(A(T_n))| = $\frac{3n}{2} - 1$ and |E(A(T_n)| = 2n – 3.

Let $P_n = u_1 u_2 \ldots u_n$ be a path.

Case 1: The alternate triangular snake graph starts with a triangle.

Let $V\big(A(T_n)\big) = V(P_n) \cup \{v_j : 1 \le j \le \frac{n}{2}\}$. Then $E\big(A(T_n)\big) = E(P_n) \cup \{u_j v_{\lceil \frac{j}{2} \rceil} : j \equiv$

$1 \ (mod\ 2) \cup \{u_j v_{\frac{j}{2}} : j \equiv 0 \ (mod\ 2)\}$.

Subcase 1.1: $n \equiv 0 \ (mod\ 6)$.

Define $f: V(A(T_n)) \rightarrow \{1, -1, i, -i\}$ by,

$$f(u_j) = \begin{cases} 1 & if\ j \equiv 1, 6, 7, 12, 14, 18, 22\ (mod\ 24) \\ -1 & if\ j \equiv 2, 5, 13, 17, 19, 21 \quad (mod\ 24) \\ i & if\ j \equiv 3, 8, 9, 15, 16, 20 \quad (mod\ 24) \\ -i & if\ j \equiv 0, 4, 10, 11, 23 \quad (mod\ 24) \end{cases}$$

and

$$f(v_j) = \begin{cases} 1 & if\ j \equiv 6,10 \quad (mod\ 12) \\ -1 & if\ j \equiv 3,4,5 \quad (mod\ 12) \\ i & if\ j \equiv 1,7,11\ (mod\ 12) \\ -i & if\ j \equiv 0,2,8,9 \ (mod\ 12) \end{cases}$$

By this labeling, we get $e_f(1) = e_f(3) = e_f(4) = \frac{n}{2}$ and $e_f(2) = \frac{2n}{4} - 1$. Table 2.7. proves that the vertex condition is satisfied.

Nature of n	v_f *(1)*	v *(2)*	v *(3)*	v_f *(4)*
$n \equiv 0$ (mod 24)	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}$
$n \equiv 6$ (mod 24)	$\frac{3n-2}{8}$	$\frac{3n+6}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$
$n \equiv 12$(mod 24)	$\frac{3n+4}{8}$	$\frac{3n+4}{8}$	$\frac{3n-4}{8}$	$\frac{3n-4}{8}$
$n \equiv 18$(mod 24)	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$	$\frac{3n-6}{8}$

TABLE 2.7.

Subcase 1.2: $n \equiv 2 \ (mod\ 6)$.

Label the vertices $u_j (1 \le j \le n-2), v_j (1 \le j \le \frac{n}{2} - 1)$ as in subcase 1.1. Next label the remaining vertices according to the following subcases.

Subcase 1.2(a): $n \equiv 8,14,20 \ (mod\ 24)$.

Assign the labels 1, i and −i to the vertices $u_{n-1}, u_n\ and\ v_{\frac{n}{2}}$ in order.

Subcase 1.2(b): $n \equiv 2 \ (mod\ 24)$.

Assign the labels 1, i and −1 to the vertices $u_{n-1}, u_n\ and\ v_{\frac{n}{2}}$ in order.

Here, for $n \equiv 2\ (mod\ \ 24)$, $v_f(1) = v_f(-1) = v_f(i) = \frac{3n+2}{8}$ and $v_f(-i) = \frac{3n-6}{8}$.

For $n \equiv 8(mod\ \ 24)$, $v_f(1) = v_f(-1) = v_f(i) = v_f(-i) = \frac{3n}{8}$.

For $n \equiv 14\ (mod\ \ 24)$, $v_f(1) = \frac{3n+6}{8}$ *and* $v_f(-1) = v_f(i) = v_f(-i) = \frac{3n-2}{8}$.

For $n \equiv 20\ (mod\ \ 24)$, $v_f(1) = v_f(i) = \frac{3n+4}{8}$ *and* $v_f(-1) = v_f(-i) = \frac{3n-4}{8}$.

Also, for $n \equiv 2\ (mod\ \ 24)$, $e_f(1) = e_f(2) = e_f(3) = \frac{n}{2}$ and $e_f(4) = \frac{2n}{4} - 1$.

For $n \equiv 8,14,20\ (mod\ \ 24)$, $e_f(1) = e_f(2) = e_f(4) = \frac{n}{2}$ and $e_f(3) = \frac{2n}{4} - 1$.

Subcase 1.3: $n \equiv 4\ (mod\ \ 6)$.

We can easily verify $A(T_4)$ is a group mean cordial graph.

Let n>4. Label the vertices $u_j(1 \leq j \leq n-4), v_j(1 \leq j \leq \frac{n}{2} - 2)$ as in subcase 1.1. Next label the remaining vertices according to the following subcases.

Subcase 1.3(a): $n \equiv 4,10,22\ (mod\ 24)$.

Assign the labels 1, i, −i and −1 to the vertices $u_{n-3}, u_{n-2}, u_{n-1}$ and u_n in order. Also label the vertices $v_{\frac{n}{2}-1}$, $v_{\frac{n}{2}}$ with 1, −i in order.

Subcase 1.3(b): $n \equiv 16\ (mod\ 24)$.

Assign the labels −1, i, 1 and –i to the vertices $u_{n-3}, u_{n-2}, u_{n-1}$ and u_n in order. Also label the vertices $v_{\frac{n}{2}-1}$, $v_{\frac{n}{2}}$ with i, −i in order.

Here, for $n \equiv 4\ (mod\ \ 24)$, $v_f(1) = v_f(-i) = \frac{3n+4}{8}$ and $v_f(-1) = v_f(i) = \frac{3n-4}{8}$.

For $n \equiv 10(mod\ \ 24)$, $v_f(1) = v_f(-1) = v_f(-i) = \frac{3n+2}{8}$ and $v_f(i) = \frac{3n-6}{8}$.

For $n \equiv 16(mod\ \ 24)$, $v_f(1) = v_f(-1) = v_f(i) = v_f(-i) = \frac{3n}{8}$.

For $n \equiv 22(mod\ \ 24)$, $v_f(-1) = v_f(i) = v_f(-i) = \frac{3n-2}{8}$ and $v_f(1) = \frac{3n+6}{8}$.

Also, for $n \equiv 4,10,22\ (mod\ \ 24)$, $e_f(1) = e_f(3) = e_f(4) = \frac{n}{2}$ and $e_f(2) = \frac{2n}{4} - 1$.

For $n \equiv 16\ (mod\ \ 24)$, $e_f(2) = e_f(3) = e_f(4) = \frac{n}{2}$ and $e_f(1) = \frac{2n}{4} - 1$.

Case 2: The alternate triangular snake graph starts with a pendant edge.

Let $V\big(A(T_n)\big) = \{u_j : 1 \leq j \leq n-1\} \cup \{ v_j : 1 \leq j \leq \frac{n}{2} - 1\}$.

Then $E(A(T_n)) = \{u_j u_{j+1}: 1 \le j \le n-3\} \cup \{ u_j v_{\lceil \frac{j}{2} \rceil}: j \equiv 1 \ (mod\ 2) \cup \{ u_j v_{\frac{j}{2}}: j \equiv 0 \ (mod\ 2) \} \cup \{x, y\}$.

Subcase 2.1: $n \equiv 2 \ (mod \ 6)$.

Label the vertices $u_j (1 \le j \le n-2), v_j (1 \le j \le \frac{n}{2} - 1)$ as in subcase 1.1. The labels of x,y are defined as follows:

$f(x) = i$, for all n.

$$f(y) = \begin{cases} 1 & if \ \ j \equiv 2 \qquad (mod\ 24) \\ -i & if \ \ j \equiv 8,14,20\ (mod\ 24) \end{cases}$$

Here, we get $e_f(1) = e_f(3) = e_f(4) = \frac{2n}{4} - 1$ and $e_f(2) = \frac{n}{2}$.

Table 2.8. proves the vertex cordial condition.

Nature of n	v_f ***(1)***	v ***(2)***	v ***(3)***	v_f ***(4)***
$n \equiv 2$ (mod 24)	$\frac{3n+2}{8}$	$\frac{3n-6}{8}$	$\frac{3n+2}{8}$	$\frac{3n-6}{8}$
$n \equiv 8$ (mod 24)	$\frac{3n}{8} - 1$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}$
$n \equiv 14$(mod 24)	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$
$n \equiv 20$(mod 24)	$\frac{3n-4}{8}$	$\frac{3n-4}{8}$	$\frac{3n+4}{8}$	$\frac{3n-4}{8}$

TABLE 2.8

Subcase 2.2: $n \equiv 4\ (mod \ 6)$.

Label the vertices $u_j (1 \le j \le n-4), v_j (1 \le j \le \frac{n}{2} - 2)$ as in subcase 1.1. Next label the remaining vertices according to the following subcases

Subcase 2.2(a): $n \equiv 4\ (mod \ 24)$.

Assign the labels 1, i and −1 to the vertices u_{n-3}, u_{n-2} and $v_{\frac{n}{2}-1}$ in order. Label x, y with −1, −i in order.

Subcase 2.2(b): $n \equiv 10,16,22\ (mod \ 24)$.

Assign the labels s 1, i and −i to the vertices $u_{n-3}, u_{n-2}$and $v_{\frac{n}{2}-1}$ in order. Label x, y with –I, -1 in order.

The following table 2.9. and the values of $e_f(s)$ show that f is a group mean cordial labeling for the Subcase 2.2.

Nature of n	v_f *(1)*	*v (2)*	*v (3)*	v_f *(4)*
$n \equiv 4 \pmod 4$	$\frac{3n-4}{8}$	$\frac{3n+4}{8}$	$\frac{3n-4}{8}$	$\frac{3n-4}{8}$
$n \equiv 10 \pmod 4$	$\frac{3n-6}{8}$	$\frac{3n+2}{8}$	$\frac{3n-6}{8}$	$\frac{3n+2}{8}$
$n \equiv 16 \pmod 4$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}-1$	$\frac{3n}{8}$
$n \equiv 22 \pmod 4$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$

TABLE 2.9.

For $n \equiv 10,16,24 \ (mod \ 24)$, $e_f(1) = e_f(3) = e_f(4) = \frac{n}{2}$ -1and $e_f(2)$= $\frac{n}{2}$.

For $n \equiv 4 \ (mod \ 24)$, $e_f(2) = e_f(3) = e_f(4) = \frac{n}{2} - 1$ -1and $e_f(1)$= $\frac{n}{2}$.

Subcase 2.3: $n \equiv 0 \ (mod \ 6)$.

Label the vertices $u_j(1 \le j \le n-6), v_j(1 \le j \le \frac{n}{2} - 3)$ as in subcase 1.1. Next label the remaining vertices according to the following subcases

Subcase 2.3.(a): $n \equiv 0, 12, 18 \ (mod \ 24)$.

Assign the labels 1, i, −i and −1 to the vertices $u_{n-5}, u_{n-4}, u_{n-3}$ and u_{n-2} in order. Also label the vertices $v_{\frac{n}{2}-2}, v_{\frac{n}{2}-1}$ with 1, −i in order. Label x, y with i, −1 in order.

Subcase 2.2(c): $n \equiv 6 \ (mod \ 24)$.

Assign the labels i, −i, 1 and 1 to the vertices $u_{n-5}, u_{n-4}, u_{n-3}$ and u_{n-2} in order. Also label the vertices $v_{\frac{n}{2}-2}, v_{\frac{n}{2}-1}$ with −1, −i respectively. Label x, y with i, −1 respectively.

By this labeling, we get $e_f(1) = e_f(3) = e_f(4) = \frac{n}{2}$ -1and $e_f(2)$= $\frac{n}{2}$.

The values of $v_f(j)'s$ are tabulated in Table 2.10.

Nature of n	v_f *(1)*	*v (2)*	*v (3)*	v_f *(4)*
$n \equiv 0 \pmod{24}$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}$	$\frac{3n}{8}-1$
$n \equiv 6 \pmod{24}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$	$\frac{3n-2}{8}$
$n \equiv 12 \pmod{24}$	$\frac{3n-4}{8}$	$\frac{3n+2}{8}$	$\frac{3n-4}{8}$	$\frac{3n-4}{8}$
$n \equiv 18 \pmod{24}$	$\frac{3n+2}{8}$	$\frac{3n+2}{8}$	$\frac{3n-6}{8}$	$\frac{3n-6}{8}$

TABLE 2.10.

Hence Alternate Triangular Snake is a group mean cordial graph when n is even.

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