Construction of Fuzzy Relational Map in Bipolar Fuzzy Environment with an Application to Study the Causal Influence of Self-Beliefs on the Formation of Self-Concept in Children

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Abstract: Cognitive maps are knowledge structures that demonstrate the causal relationship among the concepts. Cognitive map models based on fuzzy sets have contributed to the development of knowledge processing methods. The fuzzy relational map is a cognitive map model in which concepts are divided into two sets which are disjoint. The fuzzy sets that denote the concepts and the causal links determine the efficiency of the model. The advanced fuzzy sets enable the model to capture the uncertainty in the system and the vagueness in the expert opinions, making it more sophisticated and efficient. Bipolar fuzzy set is an extension of ordinary fuzzy set that can capture the bipolar information involved in the causal relationship. In this work, a fuzzy relational map is constructed based on bipolar fuzzy sets to study the causal influence of self-beliefs on the formation of self-concept in children.

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1. Introduction

A variety of factors contribute to coordination and decision making, such as cooperation and competition, friendship and hostility, shared interests and conflicting interests, likely outcomes and unlikely outcomes, feedforward, and feedback, etc. [1]. Boolean logic and fuzzy logic are both unipolar models. They lack the representational and reasoning capability to model directly how bipolar relationships coexist and interact. This is because the logical values in both cases lie in the positive interval [0,1]. Zhang introduced a new concept called 'bipolar fuzzy sets' [2]. These sets are also known as Yin-Yang bipolar fuzzy sets which illustrate the bipolar behaviour of objects. Here, 'Yang' represents the positive side and 'Yin' depicts the negative side of a system. In the process of representing uncertainty with negative and positive membership degrees, bipolar fuzzy sets (BFSs) have been introduced [2]. Zhang pointed out that "bipolar fuzzy set theory combines both polarity and fuzziness into a unified model which provides a theoretical basis for bipolar clustering, coordination, conflict resolution and decision analysis" [1]. Zhang enumerated some of the key advantages of bipolar fuzzy set theory as follows: "1) it formalizes a unified approach to polarity and fuzziness; 2) it captures the bipolar or double-sided (negative and positive, or effect and side effect) nature of human cognition and perception, and 3) it provides a basis for bipolar cognitive modelling and multiagent decision analysis" [2].

Human decisions are influenced by bipolar or double-sided judgmental thinking, where both positive and negative perceptions are considered. Chen noted that "coexistence, equilibrium, and harmony between the two sides are essential to a person's mental and physical wellbeing as well as to the stability and prosperity of a society" [3]. Self-concept is a survival tool that ensures the existence of human beings in the history of evolution. Self-concept includes both positive and negative beliefs that shape a person into what they want to be. Psychologists and sociologists typically refer to a person's self-concept as her beliefs about herself. Self-beliefs generally appear in the form of bipolar dimensions [4]. Positive data indicates what is possible or true, while negative data shows what is prohibited or likely false. To distinguish positive from negative data, which shows what can be achieved, bipolarity is necessary. Fuzzy relational maps based on Bipolar fuzzy sets are a suitable framework for knowledge representation and a strong mathematical modelling process. They can tackle bipolarity and constructing a systematic model to analyse the problem. In this research work, the nature and dynamics of the psychological construct ‘self-concept’ is analysed with FRM based on bipolar fuzzy sets.

2. Linguistic Interval-valued Bipolar Fuzzy Sets

Bipolar fuzzy set theory has been introduced for bipolar reasoning in the space $\{\forall(x,y)|(x,y) \in [-1,0] \times [0,1]\}$. The value range of the positive membership degree of Bipolar Fuzzy Set (BFS) is [0, 1]. It signifies the degree of satisfaction that an element $x \in X$ has with a property of a bipolar fuzzy set. The value range of negative membership degree is [-1,0] and it signifies the satisfaction degree of $x \in X$ to some implicit counter property of bipolar fuzzy sets. Let X be the universal set. A bipolar fuzzy set is of the form

$$B = \left\{(\mu_B^n(x), \mu_B^p(x))\middle| x \in X\right\} \tag{1}$$

where $\mu_B^n: X \rightarrow [-1, 0]$ and $\mu_B^p: X \rightarrow [0,1]$ are any mappings [1].

Further,

- If $\mu_B^p \neq 0$ and $\mu_B^n = 0$ then x is said to have only positive satisfaction degree to a bipolar set B.
- If $\mu_B^p = 0$ and $\mu_B^n \neq 0$ then x is satisfying only the counter property of a bipolar set B.
- If $\mu_B^p \neq 0$ and $\mu_B^n \neq 0$ then x is said to satisfy both the property and counter property of a bipolar fuzzy set A in some part of X.

Real-valued bipolar representations are limited in their ability to capture high-order fuzziness. For this reason, bipolar fuzzy set theory is needed to account for both polarity and fuzziness. Zhang explained that "bipolar fuzziness refers to fuzziness embedded in a bipolar compound-valued fuzzy variable with a negative pole and a positive pole, where the poles may be real-valued, interval-based, or fuzzy set-based" [2]. Let $S = \{s_t | t = 1, \dots \dots, h\}$ be a linguistic term set (LTS). The linguistic variable s_h has the following characteristics [5].

1) The set is ordered: $s_k \leq s_t \Leftrightarrow k \leq t$
2) Complement of a variable: Negation $(s_k) = s_{h-k}$
3) Max operator: $max \{s_k, s_t\} = s_{\max(k,t)}$
4) Min operator: $min \{s_k, s_t\} = s_{\min(k,t)}$

The linguistic term set S be defined as $S = \{s_1 = not\ at\ all, s_2 = very\ little, s_3 = a\ little, s_4 = medium, s_5 = much, s_6 = very\ much, s_7 = absolutely\}$. Let $\tilde{s} = [s_\alpha, s_\beta]$ where $s_\alpha, s_\beta \in S$. Xu developed the following operational laws [6].

Consider, $\tilde{s} = [s_\alpha, s_\beta]$, $\tilde{s}_1 = [s_{\alpha_1}, s_{\beta_1}]$ and $\tilde{s}_2 = [s_{\alpha_2}, s_{\beta_2}]$ and $\lambda \in [0,1]$

(1) $\tilde{s}_1 \oplus \tilde{s}_2 = [s_{\alpha_1}, s_{\beta_1}] \oplus [s_{\alpha_2}, s_{\beta_2}] = [s_{\alpha_1} \oplus s_{\alpha_2}, s_{\beta_1} \oplus s_{\beta_2}] = [s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}]$ (2)

(2) $\tilde{s}_1 \otimes \tilde{s}_2 = [s_{\alpha_1}, s_{\beta_1}] \otimes [s_{\alpha_2}, s_{\beta_2}] = [s_{\alpha_1} \otimes s_{\alpha_2}, s_{\beta_1} \otimes s_{\beta_2}] = [s_{\alpha_1\alpha_2}, s_{\beta_1\beta_2}]$ (3)

(3) $\lambda\tilde{s} = \lambda \otimes [s_\alpha, s_\beta] = [\lambda s_\alpha, \lambda s_\beta] = [s_{\lambda\alpha}, s_{\lambda\beta}]$ (4)

(4) $\tilde{s}^\lambda = ([s_\alpha, s_\beta])^\lambda = [(s_\alpha)^\lambda, (s_\beta)^\lambda] = [s_{\alpha^\lambda}, s_{\beta^\lambda}]$ (5)

Linguistic interval-valued sets based on bipolar fuzzy sets were proposed by Gao, et. al. [7,8]. The "linguistic interval-valued bipolar fuzzy sets (LIVBFS)" are defined as

$\tilde{B} = \{[s^L_{\theta(x)}, s^U_{\theta(x)}], ([\mu_B^{L-}, \mu_B^{U-}], [\mu_B^{L+}, \mu_B^{U+}])\ x \in X\}$ where $s^L_{\theta(x)}, s^U_{\theta(x)} \in S$, the positive membership function $\mu_B^{L+}, \mu_B^{U+} \in [0,1]$ and the negative membership function $\mu_B^{L-}, \mu_B^{U-} \in [-1,0]$. The set $\tilde{b} = \{[s^L_\theta, s^U_\theta], ([\mu^{L-}, \mu^{U-}], [\mu^{L+}, \mu^{U+}])\}$ denotes a linguistic interval-valued bipolar fuzzy number. Let $\tilde{b}_1 = \{[s^L_{\theta_1}, s^U_{\theta_1}], ([\mu_1^{L-}, \mu_1^{U-}], [\mu_1^{L+}, \mu_1^{U+}])\}$ and $\tilde{b}_2 = \{[s^L_{\theta_2}, s^U_{\theta_2}], ([\mu_2^{L-}, \mu_2^{U-}], [\mu_2^{L+}, \mu_2^{U+}])\}$ be two LIVBFNs and $\lambda > 0$. Then the operations on LIVBFNs are defined as follows.

(1) $$\tilde{b}_1 \oplus \tilde{b}_2 = \left\{[s^L_{\theta_1} \oplus s^L_{\theta_2}, s^U_{\theta_1} \oplus s^U_{\theta_2}], \begin{pmatrix} [-|\mu_1^{L-}||\mu_2^{L-}|, -|\mu_1^{U-}||\mu_2^{U-}|], \\ [\mu_1^{L+} + \mu_2^{L+} - \mu_1^{L+}\mu_2^{L+}, \mu_1^{U+} + \mu_2^{U+} - \mu_1^{U+}\mu_2^{U+}] \end{pmatrix}\right\} \quad (6)$$

(2) $$\tilde{b}_1 \otimes \tilde{b}_2 = \left\{[s^L_{\theta_1} \otimes s^L_{\theta_2}, s^U_{\theta_1} \otimes s^U_{\theta_2}], \begin{pmatrix} [\mu_1^{L-} + \mu_2^{L-} - \mu_1^{L-}\mu_2^{L-}, \mu_1^{U-} + \mu_2^{U-} - \mu_1^{U-}\mu_2^{U-}], \\ [-|\mu_1^{L+}||\mu_2^{L+}|, -|\mu_1^{U+}||\mu_2^{U+}|] \end{pmatrix}\right\} \quad (7)$$

(3) $$\lambda\tilde{b}_1 = \left\{[\lambda s^L_{\theta_1}, \lambda s^U_{\theta_1}], \begin{pmatrix} [-|\mu_1^{L-}|^\lambda, -|\mu_1^{U-}|^\lambda], \\ [1 - (1 - \mu_1^{L+})^\lambda, 1 - (1 - \mu_1^{R+})^\lambda] \end{pmatrix}\right\} \quad (8)$$

(4) $$\left(\tilde{b}_1\right)^{\lambda}=\left\{\left[\left(s_{\theta_1}^{L}\right)^{\lambda},\left(s_{\theta_1}^{U}\right)^{\lambda}\right],\begin{pmatrix}\left[-1+|1+\mu_1^{L-}|^{\lambda},-1+|1+\mu_1^{U-}|^{\lambda}\right],\\ \left[(\mu_1^{L+})^{\lambda},(\mu_1^{R+})^{\lambda}\right]\end{pmatrix}\right\}$$ (9)

Let $\tilde{b}_j=\left\{\left[s_{\theta_j}^{L},s_{\theta_j}^{U}\right],\left(\left[\mu_j^{L-},\mu_j^{U-}\right],\left[\mu_j^{L+},\mu_j^{U+}\right]\right)\right\}$ $(j=1,2,\dots\dots,n)$ be a set of LIVBFNs. Consider $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. The "linguistic interval-valued bipolar fuzzy weighted arithmetic aggregation operator" is defined as follows.

$$LIVBFWA_w\left(\tilde{b}_1,\tilde{b}_1,\dots\dots,\tilde{b}_1\right)=\oplus_{j=1}^{n}\left(w_j\tilde{b}_j\right)$$

$$=\left\{\left[\sum_{j=1}^{n}w_j\,s_{\theta_j}^{L}\,\sum_{j=1}^{n}w_j\,s_{\theta_j}^{U}\right],\begin{pmatrix}\left[-\prod_{j=1}^{n}\left|\mu_j^{L-}\right|^{w_j},-\prod_{j=1}^{n}\left|\mu_j^{U-}\right|^{w_j}\right],\\ \left[1-\prod_{j=1}^{n}\left(1-\mu_j^{L+}\right)^{w_j},1-\prod_{j=1}^{n}\left(1-\mu_j^{U+}\right)^{w_j}\right]\end{pmatrix}\right\}$$ (10)

Let $\tilde{b}_j=\left\{\left[s_{\theta_j}^{L},s_{\theta_j}^{U}\right],\left(\left[\mu_j^{L-},\mu_j^{U-}\right],\left[\mu_j^{L+},\mu_j^{U+}\right]\right)\right\}$ $(j=1,2,\dots\dots,n)$ be a set of LIVBFNs. Consider $w_j>0$ and $\sum_{j=1}^{n}w_j=1$. The "linguistic interval-valued bipolar fuzzy weighted geometric aggregation operator" is defined as follows.

$$LIVBFWG_w\left(\tilde{b}_1,\tilde{b}_1,\dots\dots,\tilde{b}_1\right)=\otimes_{j=1}^{n}\left(w_j\tilde{b}_j\right)$$

$$=\left\{\left[\otimes_{j=1}^{n}\left(s_{\theta_j}^{L}\right)^{w_j},\otimes_{j=1}^{n}\left(s_{\theta_j}^{L}\right)^{w_j}\right],\begin{pmatrix}\left[-1+\prod_{j=1}^{n}\left(1+\mu_j^{L-}\right)^{w_j},-1+\prod_{j=1}^{n}\left(1+\mu_j^{U-}\right)^{w_j}\right],\\ \left[\prod_{j=1}^{n}\left(\mu_j^{L+}\right)^{w_j},\prod_{j=1}^{n}\left(\mu_j^{U+}\right)^{w_j}\right]\end{pmatrix}\right\}$$ (11)

3. Fuzzy Relational Map based on Linguistic Interval-valued Bipolar Fuzzy Sets

Human decision making is based on double-sided (both positive and negative) judgmental thinking. Bipolar fuzzy set theory provides a theoretical basis for cognitive modelling and multi-agent decision analysis which has decision making at its core. Zhang pointed out that "positive and negative causal relationships should not be combined in a usual summation if they are not counteracting at the same time, originating from different sources, or following different paths". Bipolar transitive causal paths are the key knowledge structures in cooperative or competitive decision analysis. They can be utilized to help coordinate interaction between agents, integrate conflict, and resolve it. Zhang proposed that "the bipolar algebra ensures timely summation of bipolar causal effects along all paths, rather than propagating stimuli through only the maximal negative and positive causal pathways" [2]. The bipolar properties of linguistic fuzzy relations can be used to integrate the perspectives of multiple decision-makers. As positive and negative causal relations are both captured in the unified model, decisions about negative large effects (NL) and positive large effects (PL), negative small effects (NS) and positive large effects (PL), negative large effects (NL) and positive large effects (PL), etc, can all be distinguished by the strength of the variable. Moreover, distributed decision-makers can use bipolar features to identify strategies for cooperation and coordination. In addition, this is done to expand global desired effects while limiting side effects, infer compromised solutions and arrive at competitive strategies [2].

Suppose that there are n and m concepts in the domain and range spaces of a bipolar fuzzy relational map. The FRM begins the inference process with the design of the bipolar fuzzy initial state vector at instant $t = 0$. The initial state vector with 'n' nodes is denoted as $\widetilde{D}^0 = \left(\tilde{d}_1^0, \tilde{d}_2^0, \dots \dots, \tilde{d}_n^0\right)$ where each $\tilde{d}_i^0, (1 \le i \le n)$ is an ordered pair with positive and negative membership degree. That is,

$$\widetilde{D}^0 = \left(\tilde{d}_1^0, \tilde{d}_2^0, \dots \dots, \tilde{d}_n^0\right) = \left(\left(\left(\tilde{d}_1^0\right)^n, \left(\tilde{d}_1^0\right)^p\right), \left(\left(\tilde{d}_1^0\right)^n, \left(\tilde{d}_1^0\right)^p\right), \dots \dots, \left(\left(\tilde{d}_1^0\right)^n, \left(\tilde{d}_1^0\right)^p\right)\right)$$

The influence of concept x_i on the two poles of concept x_j is denoted by the dual weight $\tilde{e}_{ij}$. Here $\tilde{e}_{ij} = \left(\tilde{e}_{ij}^n, \tilde{e}_{ij}^p\right) \in [-1, 0] \times [0,1]$ and $\tilde{e}_{ij}^n$ indicates the negative poles' causal relation between the concepts x_i and x_j, while $\tilde{e}_{ij}^p$ indicates the positive poles' causal relation between the concepts x_i and x_j. In the initial FRM, it can receive two pole data from its input concepts, and the state is calculated iteratively until convergence is achieved. The kth iteration state is represented by the state vector $\widetilde{D}^k = \left(\tilde{d}_1^k, \tilde{d}_2^k, \dots \dots, \tilde{d}_n^k\right)$ where $\tilde{d}_i^k \in D$ $(1 \le i \le n)$. The value of each node is calculated by the following equation [9]:

$$\widetilde{D}^{k+1} = \left(f\left(\left|\left(\tilde{r}_j^k\right)^n\right| + \sum_{j=1}^m \left|\left(\tilde{r}_j^k\right)^n\right| \tilde{e}_{ji}^n\right), f\left(\left(\tilde{r}_j^k\right)^p + \sum_{j=1}^m \left(\tilde{r}_j^k\right)^p \tilde{e}_{ji}^p\right)\right) \quad (12)$$

where

$$\tilde{r}^k = \left(f\left(\left|\left(\tilde{d}_i^k\right)^n\right| + \sum_{i=1}^n \left|\left(\tilde{d}_i^k\right)^n\right| \tilde{e}_{ij}^n\right), f\left(\left(\tilde{d}_i^k\right)^p + \sum_{i=1}^n \left(\tilde{d}_i^k\right)^p \tilde{e}_{ij}^p\right)\right)$$

Inference in Linguistic Interval-Valued Bipolar Fuzzy Relational Maps (LIVBFRM)

Suppose that the nodes and causal relationships in a bipolar FRM is represented with interval-valued bipolar fuzzy numbers. Let $\tilde{b}_j = \left\{\left[s_{\theta_j}^L, s_{\theta_j}^U\right]\left(\left[\mu_j^{L-}, \mu_j^{U-}\right], \left[\mu_j^{L+}, \mu_j^{U+}\right]\right)\right\}$ $(j = 1,2, \dots \dots, n)$ be a set of LIVBFNs where $\left(\left[\mu_j^{L-}, \mu_j^{U-}\right], \left[\mu_j^{L+}, \mu_j^{U+}\right]\right)$ is the bipolar interval-value assigned to the linguistic term $\left[s_{\theta_j}^L, s_{\theta_j}^U\right]$. The value of each node in case of interval-valued bipolar fuzzy sets is computed using the equation:

$$\widetilde{D}^{k+1} = d_i^{k+1} = \left\{\left[\mu_p^{L-}(d), \mu_p^{U-}(d)\right], \left[\mu_p^{L+}(d), \mu_p^{U+}(d)\right]\right\}_i^{k+1}$$

$$d_i^{k+1} = \begin{pmatrix} f\left(\left\{\left[\mu_p^{L-}(d), \mu_p^{U-}(d)\right]\right\}_i^k \oplus \left(\oplus_{j=1}^m \left\{\left[\mu_p^{L-}(r), \mu_p^{U-}(r)\right]\right\}_j^k \otimes \left[e_{ji}^{L-}, e_{ji}^{U-}\right]\right)\right), \\ f\left(\left\{\left[\mu_p^{L+}(d), \mu_j^{U+}(d)\right]\right\}_i^k \oplus \left(\oplus_{j=1}^m \left\{\left[\mu_p^{L+}(r), \mu_p^{U+}(r)\right]\right\}_j^k \otimes \left[e_{ji}^{L+}, e_{ji}^{U+}\right]\right)\right) \end{pmatrix} \quad (13)$$

where

$$\left\{\left[\mu_p^{L-}(r), \mu_p^{U-}(r)\right]\right\}_j^k = f\left(\left\{\left[\mu_p^{L-}(r), \mu_p^{U-}(r)\right]\right\}_j^k \oplus \left(\oplus_{i=1}^n \left\{\left[\mu_p^{L-}(d), \mu_p^{U-}(d)\right]\right\}_i^k \otimes \left[e_{ij}^{L-}, e_{ij}^{U-}\right]\right)\right)$$

and

$$\left\{\left[\mu_p^{L+}(r), \mu_p^{U+}(r)\right]\right\}_j^k = f\left(\left\{\left[\mu_p^{L+}(r), \mu_j^{U+}(r)\right]\right\}_j^k \oplus \left(\oplus_{i=1}^n \left\{\left[\mu_p^{L+}(d), \mu_p^{U+}(d)\right]\right\}_i^k \otimes \left[e_{ij}^{L+}, e_{ij}^{U+}\right]\right)\right)$$

The following steps describe a pseudo algorithm for constructing and analysing FRMs.

Step 1: With the help of experts, the factors that form the domain and range spaces of the FRM are selected.

Step 2: The linguistic term set (LTS) is constructed based on the matching linguistic assessments. The table 2 shows the linguistic terms and the values that correspond to their membership. The values are adopted from [7,8].

Step 3: Relationships are created by participants based on their experience and self-awareness. Participants assessed the causal relations between concepts in the FRM model using interval-valued linguistic evaluations.

Step 4: The linguistic evaluations are converted into the interval-valued bipolar fuzzy numbers. The self-assessments of all the participants are combined using the arithmetic aggregation operator given in equation (10) by taking equal weights.

Step 5: The aggregated values are arranged in the form of a matrix that illustrates the edge strength of the causal relationship between the factors of the domain and range spaces of FRM. The resultant matrix is the relational matrix of the FRM in terms of interval-valued bipolar fuzzy numbers.

Step 6: The input values of the elements of the domain space were defined by the expert.

Step 7: The FRM was simulated using equation (13) based on the initial state vector of the concepts until a steady-state was reached. Since the dynamic system of the FRM involved negative values, hyperbolic tangent function was used as activation function.

Step 8: Converging values of the resultant state vector are obtained after several iterations of the concepts of domain and range spaces.

Step 9: The resultant stable vector contains interval values. The fuzzy interval is defuzzied.

Step 10: Based on the defuzzified weight of each concept in the stable vector, the problem is analysed.

4. Description of the problem

Self-concept is a mental representation of the 'self' that is shaped by beliefs about oneself and the responses of others. It is a system of attitudes, beliefs, and evaluative judgments about one's self that is complex, organized, and yet dynamic. Wehrle and Fasbender described more precisely that "self-concepts harbour a person's knowledge of who or what he/she is (one's self-beliefs) and a person's evaluation of how one feels about oneself; an evaluation in which people link valences to their self-beliefs (positive or negative self-evaluations)" [10]. Showers defined that "a self-concept structure is a set of beliefs about one's self that emerges and is organized according to their accessibility" [11]. Showers further indicated that "the organisational structure of self-concept moderates the accessibility to specific beliefs about one's self-knowledge" [12]. In general, self-concept answers the question, "Who am I?" and it can be described as a process of learning about who one is [10].

Self-concept consists of multiple (contextualized) identities that contribute to the overall self-concept; each of these has positive or negative affective connotations [10]. Depending on how individuals structure their self-concepts, they may have either a positive or negative self-concept, or they may have both positive and negative self-concepts [11]. Showers noted that "people who organize their self-concepts in positive ways report feeling positive about themselves despite having many negative beliefs about themselves, and vice versa" [13]. The organization of self-beliefs as well as their valence determines the self-assessment of an individual based on the distribution of positive and negative beliefs about various self-concepts [11]. This idea of self-concept is used to identify the acceptable characteristics defining a positive self-concept, as well as those attributes that constitute a negatively framed self-concept, which is the polar or opposing attribute [14].

In this study, FRM is used to quantitatively model the differences in self-concept across individuals. Five belief domains are chosen as elements of domain space based on an expert's opinion. Beliefs that influence self-concept about these spheres of cognition are collected. Self-concept develops throughout the lifespan of an individual and is influenced by many factors. The psychologist Bruce A. Bracken believed that self-concept consisted of six independent traits: 1) **Academic**: Success or failure in school, 2) **Affect**: Awareness of emotional states, 3) **Competence**: Ability to meet basic needs, 4) **Family**: Connection and involvement in family, 5) **Physical**: Looks, health, physical condition, and overall appearance and 6) **Social**: Ability to interact with others [15]. The six dimensions of self-concept are taken to the elements of the range space of the FRM. The complete list of the elements of domain and range spaces of the FRM is provided in table 1. The FRM model is constructed based on the responses provided by the primary school children to the self-concept questionnaire.

Factors of Domain Space **Beliefs that influence Self-concept**	**Factors of Range Space** **Dimensions of Self-concept**
B_1: Survival	C_1: Physical
B_2: Connection	C_2: Emotional

B_3: Recognition	C_3: Social
B_4: Responsibility	C_4: Family
B_5: Vulnerability	C_5: Academic
	C_6: Competence

Table 1: Factors of bipolar FRM

The present study is part of a research program on the societal influences on the human belief system of children in primary school within the context of Tamil culture. Fifty students, 25 boys and girls, were asked to participate in this study. The group of participants were children of age group around 10years studying in class V. The participants were provided with a self-assessment questionnaire which included 60 belief statements depicting the relationship between self-beliefs and self-concept. The participants were asked to mark the statements based on their belief that how much is true by taking a continuous range of values from the set { S_1 = Not at all, S_2 = Very little, S_3 = Little, S_4 = Medium, S_5 = Much, S_6 = Very much, S_7 = Absolutely}. The causal relationship between self-beliefs and self-concept is analysed using FRM with interval-valued bipolar fuzzy sets.

Linguistic Term Set	Corresponding IVBFN
$[S_1, S_2]$	[0.0, 0.1], [-0.5, -0.4]
$[S_2, S_3]$	[0.1, 0.2], [-0.6, -0.5]
$[S_3, S_4]$	[0.2, 0.3], [-0.7, -0.6]
$[S_4, S_5]$	[0.3, 0.4], [-0.2, -0.1]
$[S_5, S_6]$	[0.4, 0.5], [-0.3, -0.2]
$[S_6, S_7]$	[0.5, 0.6], [-0.4, -0.3]

Table 2: Interval-valued linguist term sets and the corresponding IVBFN

5. Analysis of the problem using LIVBFRM

The relational matrix with interval-valued bipolar fuzzy numbers entries is the dynamical system that illustrates the underlying relationship between self-beliefs and self-concept. The relational matrices that represent the opinions of boys and girls are constructed independently. The relational matrix that describes the self-concept of boys is given in table 3 and that of girls in table 4. The input values of the factors provided by the expert are given in table 5. The relational matrices are analysed using the FRM inference process as described in the above steps. The defuzzified values of the resultant stable vector are presented in table 6. Fuzzy intervals with symmetrical membership functions are said to be regular. In the case of a regular fuzzy interval, all the three widely used defuzzification methods provide equal results. Zhao and Govind defined that "the defuzzified value of a regular fuzzy interval based on the 'Center of Area' method is equal to the middle point of the corresponding 'mean value' interval" [16]. This idea of defuzzification is adopted to defuzzify the converging values of the stable vector in the inference process.

	C_1	C_2	C_3	C_4	C_5	C_6
B_1	[0.371, 0.474], [-0.366, -0.252]	[0.336, 0.441], [-0.380, -0.271]	[0.321, 0.425], [-0.354, -0.243]	[0.284, 0.388], [-0.379, -0.265]	[0.224, 0.328], [-0.428, -0.314]	[0.354, 0.457], [-0.373, -0.261]
B_2	[0.332, 0.436], [-0.397, -0.287]	[0.326, 0.430], [-0.366, -0.253]	[0.331, 0.435], [-0.375, -0.265]	[0.345, 0.448], [-0.330, -0.218]	[0.392, 0.495], [-0.383, -0.273]	[0.275, 0.380], [-0.422, -0.312]
B_3	[0.326, 0.431], [-0.404, -0.295]	[0.348, 0.452], [-0.381, -0.273]	[0.348, 0.452], [-0.406, -0.298]	[0.319, 0.422], [-0.359, -0.245]	[0.268, 0.372], [-0.362, -0.245]	[0.332, 0.436], [-0.386, -0.274]
B_4	[0.339, 0.443], [-0.374, -0.265]	[0.314, 0.418], [-0.344, -0.232]	[0.277, 0.381], [-0.397, -0.285]	[0.336, 0.441], [-0.414, -0.306]	[0.295, 0.400], [-0.432, -0.323]	[0.360, 0.465], [-0.420, -0.314]
B_5	[0.376, 0.480], [-0.368, -0.260]	[0.299, 0.403], [-0.421, -0.311]	[0.283, 0.386], [-0.332, -0.216]	[0.304, 0.411], [-0.434, -0.331]	[0.377, 0.480], [-0.378, -0.267]	[0.271, 0.376], [-0.395, -0.283]

Table 3: Relational Matrix of bipolar FRM (Boys) in terms of IVBFN

	C_1	C_2	C_3	C_4	C_5	C_6
B_1	[0.362, 0.466], [-0.370, -0.258]	[0.310, 0.415], [-0.437, -0.329]	[0.263, 0.367], [-0.408, -0.295]	[0.275, 0.380], [-0.385, -0.273]	[0.258, 0.363], [-0.443, -0.334]	[0.351, 0.465], [-0.387, -0.280]
B_2	[0.388, 0.492],	[0.343, 0.447],	[0.273, 0.379],	[0.351, 0.453],	[0.391, 0.495],	[0.322, 0.427],

	[-0.397, -0.294]	[-0.397, -0.286]	[-0.424, -0.318]	[-0.349, -0.236]	[-0.395, -0.287]	[-0.413, -0.305]
B_3	[0.342, 0.446], [-0.387, -0.277]	[0.321, 0.426], [-0.413, -0.305]	[0.269, 0.374], [-0.473, -0.370]	[0.326, 0.430], [-0.400, -0.287]	[0.232, 0.335], [-0.400, -0.283]	[0.313, 0.417], [-0.402, -0.292]
B_4	[0.407, 0.510], [-0.371, -0.265]	[0.350, 0.455], [-0.407, -0.299]	[0.302, 0.406], [-0.365, -0.252]	[0.324, 0.430], [-0.418, -0.313]	[0.319, 0.423], [-0.409, -0.300]	[0.354, 0.459], [-0.403, -0.297]
B_5	[0.341, 0.446], [-0.422, -0.313]	[0.367, 0.470], [-0.392, -0.284]	[0.339, 0.442], [-0.378, -0.264]	[0.374, 0.480], [-0.428, -0.327]	[0.400, 0.503], [-0.372, -0.264]	[0.263, 0.369], [-0.414, -0.305]

Table 4: Relational Matrix of bipolar FRM (Girls) in terms of IVBFN

Nodes of Domain Space	B_1	B_2	B_3	B_4	B_5
IVBFN	[0.3, 0.4], [-0.2, -0.1]	[0.4, 0.5], [-0.3, -0.2]	[0.3, 0.4], [-0.2, -0.1]	[0.4, 0.5], [-0.3, -0.2]	[0.3, 0.4], [-0.2, -0.1]

Table 5: Input values of FRM inference process

Nodes of Domain Space		B_1	B_2	B_3	B_4	B_5	
Boys	Fuzzy Interval	[0.383, 0.501]	[0.394, 0.509]	[0.388, 0.505]	[0.386, 0.503]	[0.385, 0.502]	
	Defuzzified Value	0.442	0.452	0.446	0.445	0.444	
Girls	Fuzzy Interval	[0.380, 0.497]	[0.407, 0.517]	[0.378, 0.516]	[0.405, 0.516]	[0.408, 0.518]	
	Defuzzified Value	0.438	0.462	0.437	0.461	0.463	
Nodes of Range Space		C_1	C_2	C_3	C_4	C_5	C_6

Boys	Fuzzy Interval	[0.383, 0.501]	[0.394, 0.509]	[0.388, 0.505]	[0.386, 0.503]	[0.385, 0.502]	[0.385, 0.502]
	Defuzzified Value	0.434	0.421	0.414	0.417	0.414	0.418
Girls	Fuzzy Interval	[0.395, 0.501]	[0.378, 0.487]	[0.345, 0.462]	[0.373, 0.483]	[0.367, 0.478]	[0.366, 0.479]
	Defuzzified Value	0.448	0.432	0.404	0.428	0.423	0.422

Table 6: Converging values of range space nodes of bipolar FRM

6. Results and Discussion

The stable vector obtained in the FRM inference process is called the fixed point or converging values of the dynamical system. The stable vector or fixed point of the system in the case of an FRM is a pair of vectors, one corresponding to the domain space and another to the range space. In this study, the stable vector consists of only the positive membership values as the negative membership values eventually become zero in all the cases as per the arithmetic operations applied to the interval-valued bipolar fuzzy sets. The defuzzified converging values of factors of the domain and range spaces are provided in table 6. The resultant vectors at each step of the iteration of the FRM inference process represent the change in the reasoning or cognition of the dynamical system over time. The converging values of the resultant vector denote the most and the least impactful or significant.

The belief cluster B_2: Connection attains the highest value during the process of convergence and it implies that this is the most impactful belief while the belief cluster B_1: Survival is the least powerful in the case of boys. The belief cluster B_5: Vulnerability is the most influential, while the belief cluster B_3: Recognition is the weakest among girls. It is clear from this information that the influential beliefs of self-concept differ in boys and girls. Boys perceive C_1:Physical to be the most significant factor of self-concept, whereas C_3: Social and C_5: Academic factors are of relatively minor importance. When it comes to girls, C_1: Physical is the most representative factor of their self-concept, while C_3: Social is the least important factor. Although influencing beliefs differ between boys and girls, the explicit and implicit dimensions of early self-concept are similar. The pictorial representation (Figures 1 & 2) of the converging values of the factors of the FRM gives a better view of the casual relationship between self-beliefs and self-concept.

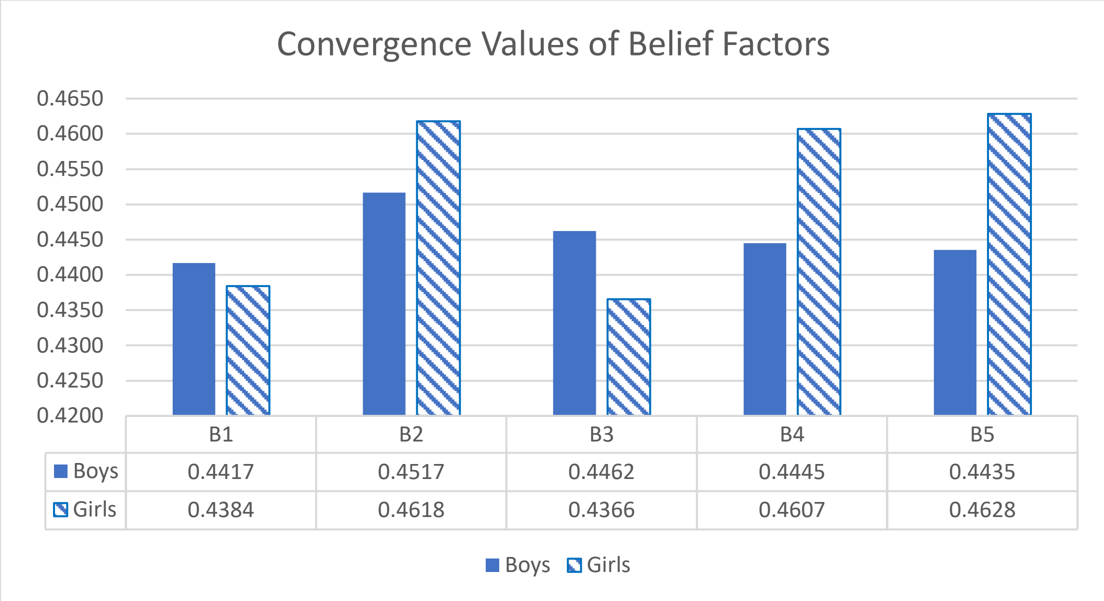

Figure 1: Comparison of Converging values of Self-belief factors

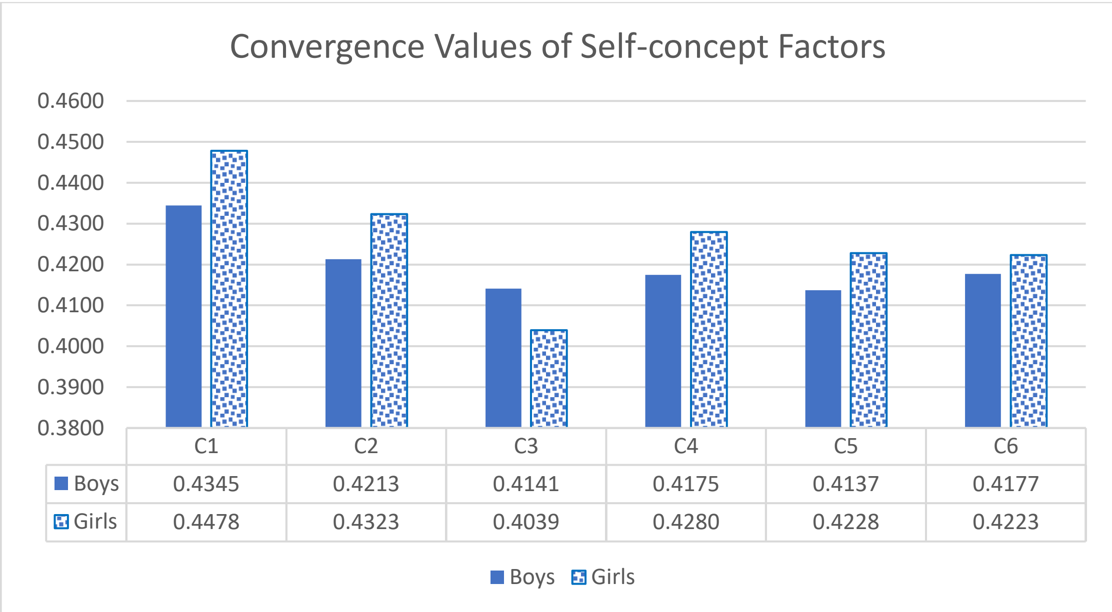

Figure 2: Comparison of Converging values of self-concept factors

This result is consistent with theories that children have a generally positive sense of self-concept and much of their focus is on physical and physical interests. The result of an empirical study conducted in 1975 on nearly 2,000 children and adolescents was that striking sex differences emerge only during adolescence [17]. Further, this result is consistent with the theory of Abraham Maslow that children gradually become aware of other factors of self-concept as they get older. This also suggests that self-concept may be intricately linked to how the children negotiate the many challenges they encounter and how they can navigate through them.

7. Conclusion

Creating and analyzing complex systems requires linguistic terms that represent human perception accurately. The fuzzy sets have been useful in handling uncertainty and vagueness which is an inherent part of human cognition in real-world problems. The advanced fuzzy sets have several advantages on account of their applications in real-life problems. Bipolar fuzzy sets are one of the most significant inventions in the history of fuzzy set theory. Bipolar fuzzy sets have shown advantages in handling vagueness and uncertainty over ordinary fuzzy sets. The unique feature of the membership value of a bipolar fuzzy set which includes negative membership values enhances the set to achieve realistic results. The bipolar fuzzy set theory is characterized by its ability to perform bipolar fuzzy linguistic description, aggregation and numerical computation. The qualitative and quantitative models involving bipolar fuzziness exhibit the following valuable characteristics: This approach 1) captures the bipolar or dual nature of human perception and cognition; 2) models the interactions and coexistence of bipolar relationships directly by providing the representational and reasoning capabilities; 3) provides a theoretical basis for bipolar decision analysis by combining fuzziness and polarity into a unified model; 4) creates a unified computational basis by combining unipolar and bipolar fuzzy systems; and 5) provides a framework for bipolar cognitive modelling and multiagent decision making. In this present study, the concept of bipolar fuzzy sets has been applied to fuzzy relational maps. To capture the maximum amount of information interval-valued linguistic terms and interval-valued bipolar fuzzy sets are employed. The application of interval-valued bipolar fuzzy sets has certainly improved the efficiency of the model and provided accurate results as it includes the bipolar information of the data.

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