A Cosine Similarity Measure based Ranking of Priority Vectors in Fuzzy Preference Relation

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Abstract

For a Pairwise Comparison Matrix (PCM), it is essential to derive a reliable priority vector collected from group of decision makers. Even though there are many prioritization methods available in literature, no method is proved as the best one to derive the best ranking of the priority vectors. In this chapter, the prioritization method on Cosine Similarity Measure (CSM) [3] is used to find the ranking of the priority vectors of a fuzzy preference relation which gives comparatively closure result with the other methods and to check the consistency of them.

Keywords: Pairwise Comparison Matrix, Multiplicative preference relation, Fuzzy preference relation, Cosine similarity measure.

1. Introduction

Group decision making (GDM)intents to achieve the best alternatives according to the opinions contributed by a group of experts. Preference relations are the most important tools in DM to apply widely[3].]. In practical, it is very hard to derive a collective opinion of each decision maker according to different preference formation [5]. Different methodologies have been found to solve these problems using the appropriate steps as (i) converting various preference formats as uniform structures (ii) aggregating the selection operators used uniform structures and (iii) ranking the alternatives making thedecision according to it.

The characteristics of various preference formats are investigated by Chiclanaet.,al.,[5] and MPRs which can changedifferent preference information to uniform representations and using theOrdered Weighted Average(OWA) geometric operator, the ranking alternatives were also obtained. The transformation function proposed by Chiclana et al.,[7] had been discussed in several studies as a standardagainst many methods may be compared. F. Herrera et al.,[5] prescribed a transformation function amongst the multiplicative and the fuzzy preference relations. Xu [8] defineddissimilar interval preference relations and suggestedin multi attribute decision making to derive overall weights.L. Michailov [6] derived a linear fuzzy preference programming of a FPR to derive a priority vector.

Chiclana et al.[8]obtained a transformation which satisfies the required properties of consistency but it lacks behind according to consistency ratio. The lack of consistency will lead to an unreasonable result. X.R.Chao et. al.,[1] hasproposed a new consistency based method on MPR and FPR which is based on CSM to derive collective priority vector. In case of investigating the consistency of a preference relation, the concept of consistency has to be checked in forms of transitivity, weak transitivity, max-max transitivity, additive and multiplicative transitivity[8,9]. For the new generalized transformation in FPR [2], all the above concepts were checked and satisfied.

Kou and Lin [3] developed a cosine maximization model and derived the priority vector that is equivalent to each and every column of a PCM using the similarity measure for AHP.Extending the similarity measure method, X.R.Chao et. al.,[1] applied CSM to the GDM problem to show the effectiveness and simplicity of the model.

1.2 Multiplicative Preference Relation

In a MPDM problem[5], an expert implemented his/her choices on set of alternatives X={x_1, x_2,……, x_n}as a MPR, $A^k = [a_{ij}^k]$ where $a_{ij}^k \in [1/9, 9]$, k =1,2,….m,describing the number of experts, and $a_{ij}^k . a_{ji}^k = 1 \, \forall i, j$, wherei,j = 1,2,…..n.

1.3 Fuzzy Preference Relation

A FPR[5], $P^k \subset X \times X$ with membership $\mu_{P^k} : X \times X \to [0,1]$ where $\mu_{P^k}(x_i, x_j) = {p_{ij}}^k$stands for the preference degree or intensity of alternativesx_i overx_j.

- ${p_{ij}}^k = \frac{1}{2}$expresses that the difference between $x_i$andx_j is insignificant.
- ${p_{ij}}^k = 1$ expresses that x_i is absolutely preferred to x_j.
- ${p_{ij}}^k > \frac{1}{2}$expresses that x_i is preferred to x_j.

Here P^k is caused to be additive reciprocal if ${p_{ij}}^k + {p_{ji}}^k = 1$ and ${p_{ii}}^k = \frac{1}{2}$

In this suggested work, a transformation function f is characterized from [1/9, 9] to [0, 1] such that f (A^k) = P^k, for all k, where $A^k = [a_{ij}^k]$ in MPR whichentertains the action,

$$f(a_{ij}^k) + f\left(\frac{1}{a_{ij}^k}\right) = 1 \quad \forall i, j$$

i.e $p_{ij}^k + p_{ji}^k = 1$

1.4 Similarity Measure

Similarity is a process by which we can determine relationship between two vectors[3]. Many methods are existing from PCM for the priority vector derivation which comprises eigenvector method(EV), weighted least square method(WLS), additive normalization method(AN), logarithmic least square method(LLS), etc., are discussed to derive the enhanced priority vector [1,3]. The notion of cosine similarity measure is derived to find the PCM as perfectly consistent using the priority vectors.

1.5 Cosine Similarity Measure

The commonly used similarity measure is the cosine similarity measure. Cosine similarity is a degree of similarity concerning two vectors of n dimensions by finding the cosine similarity is a method of normalizing length of document during comparison. Cosine similarity measure is modeled a priority vector in AHP. For any kind ofdifficult decision making situation, represented CSM is used in AHP since it is simple in process. Such a CSM is used and conversed in retrieving the information models and AHP model.

Let the two vectors be $v_i = (v_{i1}, v_{i2}, ...v_{in})^T$ and $v_j = (v_{j1}, v_{j2}, ...v_{jn})^T$, then the CSM between the two vectors v_i and v_j is represented as

$$CSM(v_i, v_j) = \frac{\sum v_{ik} v_{jk}}{\sqrt{\sum v_{ik}^2} . \sqrt{\sum v_{jk}^2}}$$

For the two types of preference structures MPR and FPR, the PCM is said be perfectly consistent if the following situations are to be satisfied [3]:

- For Multiplicative Preference Relations, $a_{ij}^{(k)} = \frac{w_i^k}{w_j^k}$, i=1,2,3,…n
- for Fuzzy Preference Relations, $p_{ij}^{(k)} = \frac{w_i^k}{w_i^k + w_j^k}$, , i=1,2,3,…n, , j=1,2,3,…n

where $(a_{ij}^{(k)})_{nxn}$, $k = 1,2,...k_m$ and $(p_{ij}^{(k)})_{nxn}$, $k = k_{m+1}, k_{m+2}, ...$.

Using the similarity measure technique in AHP, the priority vector obtained is very similar to each column of a PCM according to CSM [1].

2. **Priority vectors by applying the generalized transformation function that transforms MPR into FPR**

If $A^k = [a_{ij}^k]$ is a MPR for set of alternatives X then the corresponding FPR, $P^k = [p_{ij}^k]$ is obtained as an extension work of [2] by the generalized transformation function[11] as,

$$p_{ij}^k = f(a_{ij}^k) = \frac{1}{\alpha}\left\{\beta\left(\frac{1}{1+a_{ji}^k}\right) - 1\right\} \text{, ……………………………….(1)}$$

if $\beta = \alpha + 2$, $8 \leq \alpha \leq \infty, \alpha$ is an integer.

A new generalized transformation function[11] is used to satisfy various properties of consistency of preference relation as suggested in [7]. For designing the transformation, an aggregation OWA operator based on fuzzy majority is used and defined the quantifier-guided dominance and non-dominance choice degrees for multiplicative preference relations. After measuring the consistency ratio fuzzy majority scheme in AHP technique, it is valued as strongly consistent by the proposed generalized transformation function comparing the existing transformation[2]. Therefore, by applying this new transformation function in a pairwise comparison matrix, it is probable to estimate a preference of alternatives with strongly consistent solutions in decision making processes.

Further, in this proposed paper, the new generalized transformation in decision making processes is found as perfectly consistent by using cosine similarity measure.

3. Perfect consistency in cosine similarity measure

For Multiplicative Preference Relations (MPRs) and Fuzzy Preference Relations (FPRs), the PCM is said to be perfectly consistent if the conditions given below are satisfied.

For multiplicative preference relations, $a_{ij} = \frac{w_i}{w_j}$, , i, j=1,2,3,…n

For fuzzy preference relations, $p_{ij} = \frac{w_i}{w_i + w_j}$, i=1,2,3,…n, j=1,2,3,…n

In a pairwise comparison matrix A,

A is perfectly consistent if and only if, $CSM(w, a_j) = CSM(w, b_j) \cong 1$

where $w = (w_1, w_2, ..., w_n)^T$, $a_j = (a_{1j}, a_{2j}, ..., a_{nj})^T$ and $b_j = (b_{1j}, b_{2j}, ..., b_{nj})^T$.

If A is not perfectly consistent, then $0 \le C_j < 1$, where C_j represents cosine similarity value.

4. Numerical example

To validate the strength of the alternatives used by the proposed transformation function (2.6) as perfectly consistent, we consider the following randomized PCM of MPR adopted from Gang Kou &Changsheng Lin [3],

$$A = \begin{pmatrix} 1 & 5 & 3 & 7 & 6 & 6 & 1/3 & 1/4 \\ 1/5 & 1 & 1/3 & 5 & 3 & 3 & 1/5 & 1/7 \\ 1/3 & 3 & 1 & 6 & 3 & 4 & 6 & 1/5 \\ 1/7 & 1/5 & 1/6 & 1 & 1/3 & 1/4 & 1/7 & 1/8 \\ 1/6 & 1/3 & 1/3 & 3 & 1 & 1/2 & 1/5 & 1/6 \\ 1/6 & 1/3 & 1/4 & 4 & 2 & 1 & 1/5 & 1/6 \\ 3 & 5 & 1/6 & 7 & 5 & 5 & 1 & 1/2 \\ 4 & 7 & 5 & 8 & 6 & 6 & 2 & 1 \end{pmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

For the multiplicative preference relation (2) the corresponding fuzzy preference matrices using the transformation function (1) is given as follows,

$$P = \begin{pmatrix} 0.5 & 0.9167 & 0.8125 & 0.9688 & 0.9464 & 0.9464 & 0.1875 & 0.125 \\ 0.0833 & 0.5 & 0.1875 & 0.9167 & 0.8125 & 0.8125 & 0.0833 & 0.0313 \\ 0.1875 & 0.8125 & 0.5 & 0.9464 & 0.8125 & 0.875 & 0.9464 & 0.0833 \\ 0.0313 & 0.0833 & 0.0536 & 0.5 & 0.1875 & 0.125 & 0.0313 & 0.0139 \\ 0.0536 & 0.1875 & 0.1875 & 0.8125 & 0.5 & 0.2917 & 0.0833 & 0.0536 \\ 0.0536 & 0.1875 & 0.125 & 0.875 & 0.7083 & 0.5 & 0.0833 & 0.0536 \\ 0.8125 & 0.9167 & 0.0536 & 0.9688 & 0.9167 & 0.9167 & 0.5 & 0.2917 \\ 0.875 & 0.9688 & 0.9167 & 0.9861 & 0.9464 & 0.9464 & 0.7083 & 0.5 \end{pmatrix} \quad \ldots\ldots..(3)$$

For a FPR, $P = [p_{ij}]$ with the alternatives set $X = \{x_1, x_2, \ldots x_n\}$, the corresponding normalized matrix is obtained using the relation (3),

$$b_{ij} = \begin{pmatrix} 1 & 11.0048 & 4.3333 & 31.0513 & 17.6567 & 17.6567 & 0.2308 & 0.1429 \\ 0.0909 & 1 & 0.2308 & 11.0048 & 4.3333 & 4.3333 & 0.0909 & 0.0323 \\ 0.2308 & 4.3333 & 1 & 17.6567 & 4.3333 & 7 & 17.6567 & 0.0909 \\ 0.0323 & 0.0909 & 0.0566 & 1 & 0.2308 & 0.1429 & 0.0323 & 0.0141 \\ 0.0566 & 0.2308 & 0.2308 & 4.3333 & 1 & 0.4118 & 0.0909 & 0.0566 \\ 0.0566 & 0.2308 & 0.1429 & 7 & 2.4282 & 1 & 0.0909 & 0.0566 \\ 4.3333 & 11.0048 & 0.0566 & 31.0533 & 11.0048 & 11.0048 & 1 & 0.4118 \\ 7 & 31.0513 & 11.0048 & 70.9424 & 17.6567 & 17.6567 & 2.4282 & 1 \end{pmatrix} \quad \ldots(4)$$

Using the weight vector equation,

$$\hat{\omega}_1^* = \frac{\sum_{k=1}^{n} b_{1j}}{\sqrt{\sum_{k=1}^{n}\left(\sum_{j=1}^{n} b_{kj}\right)^2}} = \frac{83.0765}{200.8184} = 0.4137 \qquad \hat{\omega}_2^* = \frac{\sum_{k=1}^{n} b_{2j}}{\sqrt{\sum_{k=1}^{n}\left(\sum_{j=1}^{n} b_{kj}\right)^2}} = \frac{21.1163}{200.8184} = 0.1052$$

Similarly, $\hat{\omega}_3^* = 0.2605$; $\hat{\omega}_4^* = 0.00797$; $\hat{\omega}_5^* = 0.03193$; $\hat{\omega}_6^* = 0.0548$;

$\hat{\omega}_7^* = 0.3480$; $\hat{\omega}_8^* = 0.7906$.

$$C^* = \sqrt{\sum_{i=1}^{n}\left(b_{i1} + b_{i2} + b_{i3} + b_{i4}\right)^2} = 200.8184$$

Using the equation of the final priority vector it is calculated as,

$$\omega_1^* = (0.4968)(0.4137) = 0.2055$$

$\omega_2^* = 0.0523$; $\omega_3^* = 0.1294$; $\omega_4^* = 0.0040$; $\omega_5^* = 0.01586$; $\omega_6^* = 0.0272$

$\omega_7^* = 0.1729$ and $\omega_8^* = 0.3928$

Therefore the final priority vectors are,

$$\omega^* = (\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*, \omega_4^*, \omega_5^*, \omega_6^*, \omega_7^*, \omega_8^*)^T$$

$$= (0.2056, 0.0523, 0.1294, 0.0040, 0.01586, 0.0272, 0.1729, 0.3928)^T$$

Table 1.1 presents the priority vectors as well as ranking orders derived along with the methods EV,WLS, AN, and LLS. It shows that the proposed ranking of the priority vectors are similar with most of the existing methods.

Table 1.1 Priority vectors and ranking orders

Priority	**EV**	**Rank of EV**	**WLS**	**Rank of WLS**	**AN**	**Rank of AN**	**LLS**	**Rank of LLS**	**CM**	**Rank of CM**	**Proposed method**	**Rank of proposed method**
ω_1	0.173	3	0.143	**2**	0.174	3	0.172	**2**	0.179	3	0.2056	**2**
ω_2	0.054	**5**	0.054	**5**	0.066	**5**	0.063	**5**	0.072	5	0.0523	**5**
ω_3	0.188	2	0.121	3	0.170	**4**	0.149	**4**	0.159	4	0.1294	**4**
ω_4	0.018	**8**	0.030	**8**	0.019	**8**	0.019	**8**	0.019	8	0.0040	**8**
ω_5	0.031	**7**	0.047	6	0.035	**7**	0.036	**7**	0.037	7	0.01586	**7**
ω_6	0.036	**6**	0.047	**6**	0.045	**6**	0.042	**6**	0.048	6	0.0272	**6**
ω_7	0.167	4	0.089	4	0.179	2	0.167	**3**	0.182	2	0.1729	**3**
ω_8	0.333	**1**	0.469	**1**	0.312	**1**	0.349	**1**	0.303	1	0.3928	**1**

5.

The cosine similarity of FPR and MPR is obtained and verified to be perfectly consistent using the generalized transformation function. In this section, the numerical example is used to demonstrate the strength of the proposed transformation function as perfectly consistent [1,7]. Considering the following randomized PCM,

$$P^1 = \begin{pmatrix} 1 & 1/2 & 1/3 & 4 \\ 2 & 1 & 1/4 & 5 \\ 3 & 4 & 1 & 1/7 \\ 1/4 & 1/5 & 7 & 1 \end{pmatrix}$$

the corresponding fuzzy preference matrices(FP^k) using the transformation as given as follows,

$$FP^1 = \begin{pmatrix} 0.5 & 0.2917 & 0.1875 & 0.8750 \\ 0.7083 & 0.5 & 0.1250 & 0.9167 \\ 0.8125 & 0.8750 & 0.5 & 0.0313 \\ 0.1250 & 0.0833 & 0.9688 & 0.5 \end{pmatrix}$$

Using the necessary conditions [3] in the equation,

$$CSM(w,a_j) = \frac{\sum w_k a_{kj}}{\sqrt{\sum w_k^{\ 2}}.\sqrt{\sum a_{kj}^{\ 2}}}$$

The results, according toMultiplicative Preference Relations,

$$CSM(w,a_j) = \frac{\frac{1^2}{6.25} + \frac{2^2}{6.25} + \frac{3^2}{6.25} + \frac{\left(1/4\right)^2}{6.25}}{\sqrt{1^2 + 2^2 + 3^2 + \left(1/4\right)^2}\sqrt{\left(1/6.25\right)^2 + \left(2/6.25\right)^2 + \left(3/6.25\right)^2 + \left(0.25/6.25\right)^2}} \cong \frac{2.25}{2.25} \cong 1$$

according toFuzzy Preference Relations,

$$CSM(w,p_j) = \frac{\frac{0.5^2}{0.5+2.1458} + \frac{0.7083^2}{0.7083+2.1458} + \frac{0.8125^2}{0.8125+2.1458} + \frac{0.1250^2}{0.1250+2.1458}}{\sqrt{0.5^2 + 0.7083^2 + 0.8125^2 + 0.1250^2}\sqrt{0.18898^2 + 0.2482^2 + 0.2747^2 + 0.055^2}} \cong \frac{0.5004}{0.5010} \cong 1$$

therefore,

$$CSM(w,a_j) = CSM(w,p_j) \cong 1.$$

According to [1], each column of a pair wise comparison matrix and the derived priority vector, in particular, the CSM should be equal to 1 if and only if the pair wise comparison matrix is perfectly consistent. Since $CSM(w,a_j) = CSM(w,p_j) \cong 1$,it is obvious that the preference relation in the proposed transformation is perfectly consistent based on CSM[1].

5. Conclusion

It is a crucial argument in Analytic Hierarchy Process (AHP) to acquire a reliable priority vector which is possessed from expert's perception. In this recommended paper, CSM is accustomed to analyze the reliability of MPR and FPR. For the derived priority vector, the CSM should be equal to 1 iffthe PCM is perfectly consistent. By checking with the values numerically it is evident that, in the proposed transformation, the collective priority vector is a PCM whichpromise the priority vector is very closure to perfectly consistency for individualexperts. By applying the proposed transformation in PCM the priority vectors are considered as perfectly consistent.

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