Relatively Prime Inverse Domination on Jump Graph

C. Jayasekaran [1], L. Roshini [2]
[1]Associate Professor, Department of Mathematics,
Pioneer Kumaraswamy College, Nagercoil - 629003, Tamil Nadu, India.
[2]Research Scholar,Reg. no. 20113132092002, Department of Mathematics,
Pioneer Kumaraswamy College, Nagercoil - 629003, Tamil Nadu, India.
Affliated to Manonmaniam Sundaranar University, Abishekapatti,
Tirunelveli - 627012,Tamil Nadu, India.
email : jayacpkc@gmail.com [1], jerryroshini92@gmail.com [2]

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Abstract
Let G be non-trivial graph. A subset D of the vertex set $V(G)$ of a graph G is called a dominating set of G if every vertex in $V-D$ is adjacent to a vertex in D. The minimum cardinality of a dominating set is called the domination number and is denoted by $\gamma(G)$. If $V-D$ contains a dominating set S of G, then S is called an inverse dominating set with respect to D. In an inverse dominating set S, every pair of vertices u and v in S such that $(deg(u), deg(v)) = 1$, then S is called relatively prime inverse dominating set. The minimum cardinality of a relatively prime inverse dominating set is called relatively prime inverse dominating number and is denoted by $\gamma_{rp}^{-1}(G)$. In this paper we find relatively prime inverse dominating number of some jump graphs.

Keywords:- Domination, Inverse domination, Relatively prime domination.

1 Introduction

By a graph, we mean a finite undirected graph with neither loops nor multiple edges. For graph theoretic terminology, we refer to the book by Chartrand and Lesniak [2]. All graphs in this paper are assumed to be non-trivial. In a graph $G = (V, E)$, the degree of a vertex v is defined to be the number of edges incident with v and is denoted by $deg(v)$. A set D of vertices of graph G is said to be a dominating set if every vertex in $V-D$ is adjacent to a vertex in D. A dominating set D is said to be a minimal dominating set if no proper subset of D is a dominating set. The minimum cardinality of a dominating set of a graph G is called the domination number of G and is denoted by γ(G). Kulli V. R. et al. introduced the concept of inverse domination in graphs [6]. Let D be a minimum dominating set of G. If $V-D$ contains a dominating set S, then S is called the inverse dominating set of G with respect to D. The inverse dominating number γ^{-1}(S) is the minimum cardinality taken over all the minimal inverse dominating set of G. A Fan graph $F_{m,n}$ is defined as the graph join $\overline{K}_m + P_n$, where $\overline{K}_m$ is the empty graph on m nodes and P_n is the path graph on n nodes. The case $m = 1$ corresponds to the usual fan graph, while $m = 2$ corresponds to the double fan[8]. A dumbbell graph, denoted by $D_{a,b,c}$, is a bicycle graph consisting of two vertex- disjoint cycles C_a, C_b and a path $P_{c+3} (c \geq -1)$ joining them having only its end-vertices in common with the two cycles[7]. The book graph is defind as the cartisian product of star graph S_n and path P_2[3].

A set S$\subseteq V$ is said to be relatively prime dominating set if it is a dominating set with at least two elements and for every pair of vertices u and v in S such that $(deg(u), deg(v)) = 1$. The minimum cardinality of a relatively prime dominating set of a graph G is called the relatively prime domination number of G and is denoted by $\gamma_{rpd}(G)$ [4]. The purpose of this paper is to study about the concept of relatively prime inverse domination on jump graphs.

Definition 1.1 *[5]Let D be a minimum dominating set of a graph G. If $V - D$ contains a dominating set S of G, then S is called an inverse dominating set with respect to D. If every pair of vertices u and v in S such that $(deg(u), deg(v)) = 1$, then S is called relatively prime inverse dominating set. The minimum cardinality of a relatively prime inverse dominating set is called a relatively prime inverse dominating number and is denoted by $\gamma_{rp}^{-1}(G)$.*

Definition 1.2 *[1]The jump graph $J(G)$ of a graph G is the graph whose vertices are edges of G, and where two vertices of $J(G)$ are adjacent if and only if they are not adjacent in G. Equivalently the jump graph $J(G)$ of G is the complement of the line graph $L(G)$ of G.*

Example 1.3 *Consider the graphs G and $J(G)$ which are given figure 1.1. Clearly, $\{e_1, e_2\}$ is a minimum dominating set of $J(G)$ and $\{e_4, e_5\}$ is a corresponding minimum inverse dominating set and so $\gamma_{rp}^{-1}J(G) = 2$.*

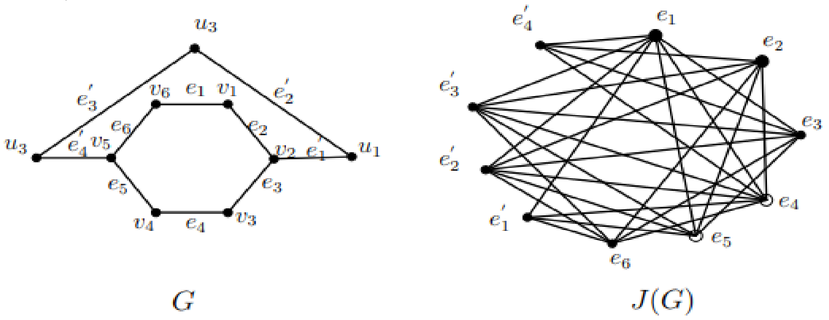


Figure 1.1

2 Known Results

Theorem 2.1 *[6] If G is a graph with no isolate vertices, then the complement $V - S$ of every minimal dominating set S is a dominating set.*

Theorem 2.2 *[?] For a path P_n,* $\gamma_{rp}^{-1}(P_n) = \begin{cases} 2 & if\ \ 3 \le n \le 5 \\ 3 & if\ \ n = 6,7 \\ 0 & otherwise \end{cases}$.

3 Relatively prime inverse domination on jump graph

Theorem 3.1 *For the path graph P_n,* $\gamma_{rp}^{-1}(J(P_n)) = \begin{cases} 2\ if\ n \ge 5 \\ 0\ otherwise \end{cases}$.

Proof. Let $v_1v_2\ldots v_n$ be the path P_n and $J(P_n)$ be the jump graph of P_n. By the definition of jump graph, edges which are adjacent in P_n are not adjacent in $J(P_n)$. Denote $e_i = v_iv_{i+1}, 1 \le i \le n-1$.

Clearly, $V(J(P_n)) = \{e_i/1 \leq i \leq n-1\}$ and $E(J(P_n)) = \{e_ie_j/1 \leq i,j \leq n-1, j \neq i+1, i-1\}$. Let D be a minimum dominating set of $J(P_n)$ and S be minimum relatively prime inverse dominating set of $J(P_n)$ with respect to D. We now consider the following two cases.

Case 1. $2 \leq n \leq 4$

If $n = 2$, then $J(P_n) \cong K_1$ and hence $\gamma_{rp}^{-1}(J(P_n))$ = 0.

If $n = 3$, then $J(P_n) \cong K_1 \cup K_1$ and if $n = 4$, then $J(P_n) \cong K_1 \cup P_2$. In both cases $J(P_n)$ has an isolated vertex and by Theorem 2.1, relatively prime inverse dominating set does not exist and so $\gamma_{rp}^{-1}(J(P_n))$ = 0.

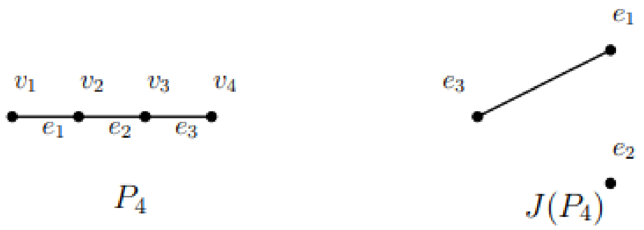


Figure 3.1

Case 2. $n \geq 5$

In $J(P_n)$, the vertex e_{n-1} is adjacent to all vertices except e_{n-2}, and so a minimum dominating set of $J(P_n)$ is $D = \{e_{n-1}, e_{n-2}\}$ and a corresponding inverse dominating set $S = \{e_1, e_2\}$. Also in $J(P_n)$, $deg(e_1) = n-3$ and $deg(e_2) = n-4$ and so $(deg(e_1), deg(e_2)) = (n-3, n-4) = 1$. This implies that S is a minimum relatively prime inverse dominating set of $J(P_n)$ and so $\gamma_{rp}^{-1}(J(P_n)) = 2$.

Thus the theorem follows from cases 1 and 2.

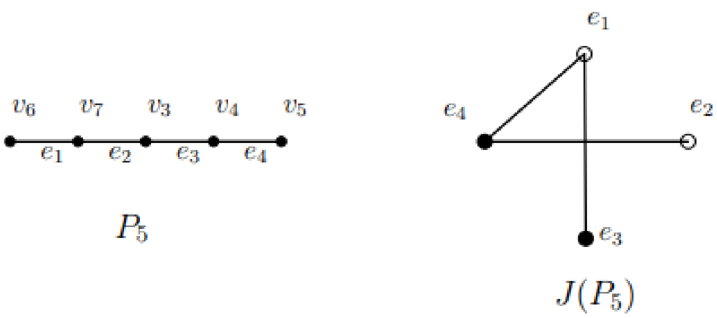


Figure 3.2

Theorem 3.2 *For the fan graph* $F_{1,n}$, $\gamma_{rp}^{-1}(J(F_{1,n})) = \begin{cases} 3 \ if \ n \geq 4 \\ 0 \ otherwise \end{cases}$.

Proof. Let $v_1v_2....v_n$ be the path P_n. Add a new vertex v_0 which is adjacent to v_i, $1 \leq i \leq n$. The resultant graph is the fan graph $F_{1,n}$ with vertex set $V(F_{1,n}) = \{v_0, v_i/1 \leq i \leq n\}$ and edge set $E(F_{1,n}) = \{v_0v_i, v_iv_{i+1}, v_0v_n/1 \leq i \leq n-1\}$. Let $J(F_{1,n})$ be the jump graph of the fan graph $F_{1,n}$. Denote the edges v_iv_{i+1} by e_i and v_0v_j by e_j', $1 \leq i \leq n-1, 1 \leq j \leq n$. Clearly, $V(J(F_{1,n})) = \{e_i, e_j'/1 \leq i \leq n-1, 1 \leq j \leq n\}$ and $E(J(F_{1,n})) = \{e_ie_j, e_ie_k'/1 \leq i \neq j \leq n-1, 1 \leq k \leq n$, and

$i \neq j$ or $j \neq i+1, i-1\}$. Let D be a minimum dominating set of $J(F_{1,n})$ and S be a corresponding minimum relatively prime inverse dominating set of $J(F_{1,n})$. We now consider the following two cases.

Case 1. $n = 2$

Clearly, $J(F_{1,2})$ is the totally disconnected graph $3K_1$. By Theorem 2.1, inverse domination set does not exist and so $\gamma_{rp}^{-}1(J(F_{1,2}))$ = 0.

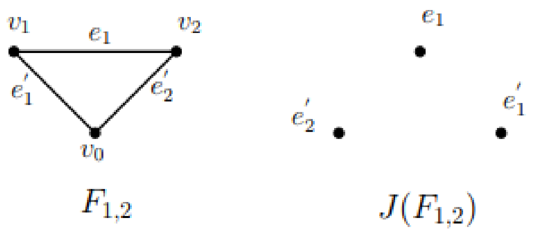


Figure 3.3

Case 2. $n = 3$

In this case $J(F_{1,3}) \cong 2K_2 \cup K_1$. By Theorem 2.1, relatively prime inverse dominating set does not exist and so $\gamma_{rp}^{-1}(J(F_{1,3}))$ = 0.

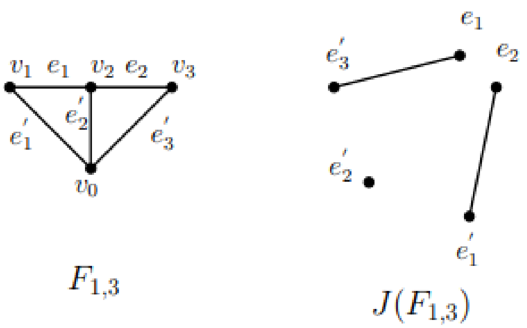


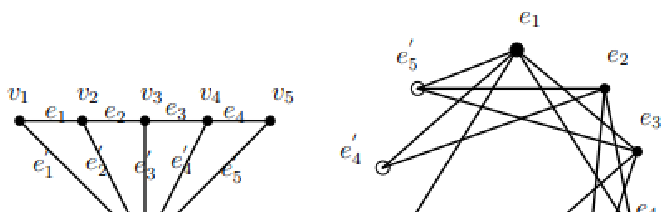
Figure 3.4

Case 3. $n \geq 4$

In $J(F_{1,n})$, the vertex e_1 is adjacent to all vertices except e_2, e_1', e_2'; e_1' is adjacent to all vertices except $e_1, e_i', 2 \leq i \leq n$; e_2' is adjacent to $e_i, 3 \leq i \leq n$ except all other vertices. In $J(F_{1,n})$, a minimum dominating set D = $\{e_1, e_1', e_2'\}$ and a corresponding minimum inverse dominating set S = $\{e_{n-1}, e_{n-1}', e_n'\}$. Also in $J(F_{1,n})$, $deg(e_{n-1}) = 2n-5$, $deg(e_{n-1}') = n-3$ and $deg(e_n') = n-2$. Now $(n-3, n-2) = 1$. Clearly, $2n-5$ is odd and neither a multiple of $n-2$ nor a multilpe of $n-3$ and so $(2n-5, n-2) = (2n-5, n-3) = 1$. Thus, S is a minimum relatively prime inverse dominating set of $F_{1,n}$ and so $\gamma_{rp}^{-1}(J(F_{1,n}))$ = 3.

Thus the theorem follows from cases 1, 2 and 3.

Example 3.3 *Cosider the graphs $F_{1,5}$ and $J(F_{1,5})$ which are given in figure 3.3. Clearly $\{e_1, e_1', e_2'\}$ is a minimum dominating set of $J(F_{1,5})$ and $\{e_4, e_4', e_5'\}$ is a corresponding minimum inverse dominating set. Hence,*



$\gamma_{rp}^{-1}(J(F_{1,5}))$ = 3.

Theorem 3.4 *Let G be a dumbbell graph $D_{a,b,c}$. Then $\gamma_{rp}^{-1}(J(G))$ = 2 if $n \geq 3$.*

Proof. Consider the two cycles $u_0u_1u_2\ldots u_{n-1}$ and $v_0v_1v_2\ldots v_{n-1}$. Joining the vertices u_0 and v_0. The resulting graph G is the dumbbell graph Db_n with vertex set $V(G)$ = $\{u_i, v_i/0 \leq i \leq n-1\}$ and edge set $E(G)$ = $\{u_0v_0, u_0u_i, u_ju_{j+1},\ v_0v_i,\ v_jv_{j+1}\ /i = 1, n-1$ and $1 \leq j \leq n-1\}$. Let $J(G)$ be the jump graph of the graph G. Denote the edges $u_0v_0 = e_0$, $u_0u_1 = e_1$, $u_0u_{n-1} = e_n$, $u_iu_{i+1} = e_{i+1}$, $v_0v_1 = e_1'$, $v_0v_{n-1} = e_n'$ and $v_iv_{i+1} = e_j'$, $1 \leq i \leq n-2$. Clearly $V(J(G))$ = $\{e_0, e_i, e_i'/1 \leq i \leq n\}$ and $E(J(G))$ = $\{e_0e_i, e_0e_i', e_je_k, e_j'e_k'/2 \leq i \leq n-1, 1 \leq j, k \leq n$, and $j \neq k-1, k, k+1\}$. Let D be a minimum dominating set of $J(G)$ and S be a minimum relatively prime inverse dominating set with respect to D of $J(G)$. In $J(G)$, $deg(e_0) = 2n-4, deg(e_1) = deg(e_n) = deg(e_1') = deg(e_n') = 2n-3$, $deg(e_i) = deg(e_i') = 2n-2, 2 \leq i \leq n-1$. The degree sequence of $J(G)$ is $(2n-2, 2n-2, \ldots, (2n-4)$ times, $2n-3, 2n-3, 2n-3, 2n-3, 2n-4)$ and so $J(G)$ contains distinct degrees $2n-2, 2n-3$ and $2n-4$ with $2n-2$ and $2n-4$ are even. In $J(G)$, the vertex e_1 is adjacent to all vertices except e_0, e_2 and e_n, the vertex e_2' is adjacent to all vertices except e_1' and e_3' and so $D = \{e_1, e_2'\}$ is a minimum dominating set. Clearly, a minimum inverse dominating set in $J(G)$ with respect to D is $S = \{e_1', e_2\}$ and $(deg(e_1'), deg(e_2)) = (2n-3, 2n-2) = 1$. This shows that $\gamma_{rp}^{-1}(J(G)) = 2$.

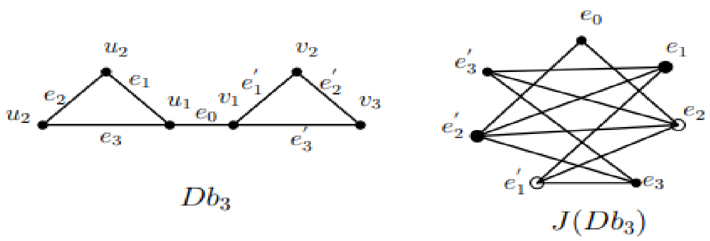


Figure 3.6

Theorem 3.5 *For the book graph B_n, $\gamma_{rp}^{-1}(J(B_n)) = 2$.*

Proof. Let $v_0, v_1, \ldots, v_n$ and $u_0, u_1, \ldots, u_n$ be the two copies of star $K_{1,n}$ with central vertices v_0 and u_0, respectively. Join u_i with v_i for $0 \leq i \leq n$. The resultant graph is the book graph B_n with vertex set $V(B_n) = \{u_0, v_0, u_i, v_i / 1 \leq i \leq n\}$ and edge set $E(B_n) = \{u_0v_0, u_iv_i, u_0u_i, v_0v_i / 1 \leq i \leq n\}$. Let $J(B_n)$ be the jump graph of the graph B_n. Denote the edges u_0v_0 by e_0, u_0u_i by e_i, v_0v_i by e_i', u_iv_i by e_i'', $1 \leq i \leq n$. Clearly $V(J(B_n)) = \{e_0, e_i, e_i', e_i' / 1 \leq i \leq n\}$ and $E(J(B_n)) = \{e_0e_i'', e_ie_j'', e_ie_k', e_i'e_j'', e_l''e_m' / i \neq j, k, l \neq m, 1 \leq i, j, k, l, m \leq n\}$. In $J(B_n)$, $deg(e_0) = n, deg(e_i) = deg(e_i') = 2n-1$ and $d(e_i'') = 3n-2, 1 \leq i \leq n$. Let D be a minimum dominating set of $J(B_n)$ and S be the minimum relatively prime inverse dominating set of $J(B_n)$. In $J(B_n)$, the vertex e_i is adjacent to all verteices except $e_0, e_i'', e_1, e_2, \ldots, e_{i-1}, e_{i+1}, \ldots, e_n$ and the vertex e_i'' is adjacent to all vertices except e_i, e_i' and so $D = \{e_i, e_i''\}$ is a minimum dominating set and a minimum inverse dominating set $S = \{e_j, e_j''\}$ where $1 \leq i \neq j \leq n$. Now in $J(B_n)$, $(d(e_j), deg(e_j'')) = (2n-1, 3n-2) = 1$, since $3n-2$ can't be a multiple of $2n-1$. This implies that S is a minimum relatively prime inverse dominating set of $J(B_n)$. Hence, $\gamma_{rp}^{-1}(J(B_n)) = 2$.

Example 3.6 *Consider the book graphs B_4 and its jump graphs $J(B_4)$ which are given in figure 3.7. Clearly, $\{e_1, e_1''\}$ is a minimum dominating set of $J(B_4)$ and $\{e_2, e_2''\}$ is a corresponding minimum inverse dominating set. Hence $\gamma_{rp}^{-1}(J(B_4)) = 2$.*

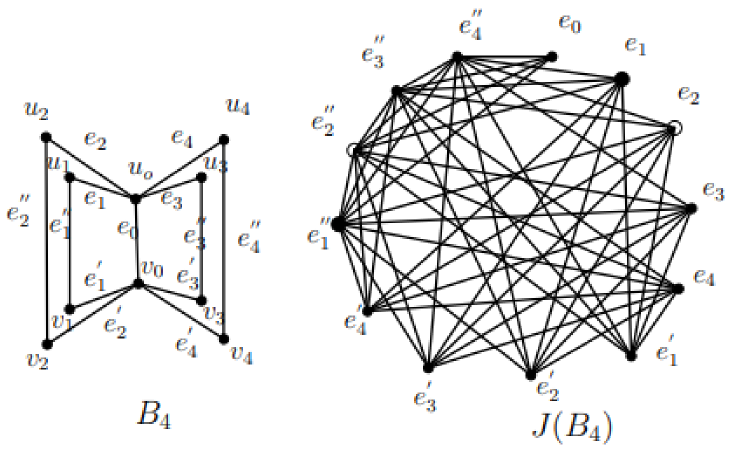


Figure 3.7

4 conclusion

In this paper, we have found the relatively prime inverse domination on line graph of some standard graphs like fan graph, dumbell graph, book graph, comb graph, sunlet graph, pan graph, and ladder graph .

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