Restrained Edge Detour Domination of Network Graphs

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Abstract
Let G be a connected graph with atleast two vertices. An edge detour dominating set S of V is called a restrained edge detour dominating set of a graph if for every vertex not in S is adjacent to a vertex in S and to a vertex in V- S. The minimum cardinality of such a dominating set is called a restrained edge detour domination number of G and is denoted by $\gamma_{red}(G)$. In this paper, we have determined the restrained edge detour domination number for mesh network, torus network, Hexagonal network, Honeycomb network and its properties are studied.

Keywords: edge detour, restrained domination, network graphs.

I. Introduction

The concept of domination was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [5] is currently receiving much attention in literature. For basic definition and terminology we refer to Buckley and Harary. [1] A set $S \subseteq V(G)$ is called a detour set if every vertex v in G lie on a detour joining a pair of vertices of S. The detour number $dn(G)$ of a G is the minimum order of a detour set and any detour set of order $dn(G)$ is called a minimum detour set of G. This concept was studied by Chartrand et al.[2] For a connected graph G, A set $S \subseteq V(G)$ is called a dominating set of G if every vertices in $V(G) - S$ is adjacent to some vertices in S.

The domination number $\gamma(G)$is the minimum order of its dominating and any dominating set of order $\gamma(G)$ is called $\gamma(G)$. A set $S \subseteq V(G)$ is called a detour dominating set of G if S is both a detour and a dominating set of G. It is denoted by γ_d (G).

II. Restrained edge detour domination of network graphs

Definition 2.1: An edge detour dominating set S of V is called a restrained edge detour dominating set of a graph if for every vertex not in S is adjacent to a vertex in S and to a vertex in V- S and is denoted by $\gamma_{red}(G)$.

Definition 2.2: The Cartesian product of two graphs G and H, denoted by $G \times H$, is the graph with vertex set $V(G) \times V(H)$. Two vertices(g, h) and (g', h') are adjacent in $G \times H$ if they are equal in one coordinate and adjacent in the other. The graph $P_m \times P_n$ is called $m \times n$ mesh graph, and is denoted by $M(m, n)$ $where\ m, n \geq 2$.

Definition 2.3: A Torus is a mesh with wrap – around links. A 1-dimensional torus is simply a cycle or ring. Tori are bipartite if and only if all side length are even. The parameters n and m in TR (n, m) designate the side lengths of the network.

Definition 2.4: For n dimensional of the hexagonal mesh HX_n, there are $2n - 1$ vertical lines. The middle vertices of the hexagonal mesh as X_0 and call as the spine of the hexagonal mesh. Left of X_0, we call as $X_1, X_2, , X_3, \dots, X_{n-1}$ lines and the right side as $X_{-1}, X_{-2}, , X_{-3}, \dots, X_{-n+1}$ lines. We label the vertices on X_0 from top to bottom as $v_{1,1}, v_{1,2}, \dots, v_{1,2n-1}$, the vertices on X_1 from top to bottom as $v_{2,1}, v_{2,2}, \dots, v_{2,2n-2}$, and so on, finally the vertices on X_{n-1} from top to bottom as $v_{n,1}, v_{n,2}, \dots, v_{n,n}$. Similarly, we name$X_{-1}$ as $v_{-1,1}, v_{1,2}, \dots, v_{1,2n-1}$,.. and X_{-n+1} as $v_{-n+1,1}, v_{-n+1,2}, \dots, v_{-n+1,n}$. The diameter of HX_n is $2n - 2$.

Definition 2.5: A unit honeycomb network is a hexagon, denoted by $HC(1)$. Honeycomb network of size 2 denoted by $HC(2)$, can be obtained by adding six hexagons around the boundary edges of $HC(1)$. Inductively, honeycomb network $HC(n)$ can be obtained from $HC(n - 1)$ by adding a layer of hexagons around the boundary edges of $HC(n - 1)$.

Definition 2.6: A Silicate network can be constructed from a honeycomb network. Take a honeycomb network $HC(n)$ of dimension n and fix the silicon ions on all the nodes of $HC(n)$. Next divide each edge of $HC(n)$ once and place oxygen ions on the new vertices. Now introduce $6n$ new pendant edges one each at the 2 $degree$ silicon ions of $HC(n)$ and place oxygen ions at the pendant vertices. The resulting network is a silicate network of dimension n, denoted $SL(n)$.

Theorem 2.1 For a mesh network $G = M(m, n)$ $where\ m, n \geq 2$ & $n = m$.

$$\gamma_{red}(G) = \begin{cases} \left\{\left\lceil\frac{m-4}{3}\right\rceil + 2\right\}m & if\ m \geq 4. \\ 2m & if\ m = 3. \\ 3 & if\ m = 2. \end{cases}$$

Proof: Let $V(G) = mn$.

Take any two vertices which are not adjacent with each other. These two vertices form an edge detour domination. Now take the required vertices to achieve the restrained edge detour domination, with all the vertices in the dominating set should be adjacent with any one of the vertices in the set.

For $m = 2$, its enough to take any two vertices which are not adjacent to achieve the edge detour domination. For the restrained edge detour domination we need adjacent vertices. So it is obviously $\gamma_{red}(G) = 3, if\ m = 2$.

For $m = 3$, its enough to take only 3 vertices to achieve the edge detour domination, but inorder to get the restrained edge detour domination we need atleast $2m$ vertices to satisfy the condition. $ie)\ 2 \times 3 = 6\ vertices$. Hence $\gamma_{red}(G) =\ 6, if\ m = 3$.

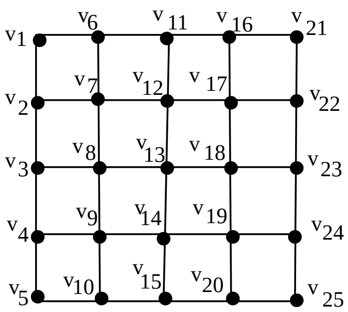

Figure 2.1 Mesh Network $M(5 \times 5)$.

Edge detour $= \{v_1, v_{25}\}$.

Edge detour dominating set $= \{v_1, v_2, v_3, v_4, v_5, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}\}$.

Restrained edge detour dominating set $= \{v_1, v_2, v_3, v_4, v_5, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{21}, v_{22}, v_{23}, v_{24}, v_{25}\}$.

$= \left\{\left\lceil\frac{m-4}{3}\right\rceil + 2\right\}m = \left\{\left\lceil\frac{5-4}{3}\right\rceil + 2\right\}5$

$$= \left\{\left\lceil\frac{1}{3}\right\rceil + 2\right\}5 = 3 \times 5 = 15.$$

Hence, $\gamma_{red}(M(5 \times 5)) = 15$.

For all the mesh networks greater than 3 we get the following result as follows

$$\gamma_{red}(G) = \left\{\left\lceil\frac{m-4}{3}\right\rceil + 2\right\} m \ if \ m \geq 4.$$

Result 2.1 For an enhanced mesh network $G = M(m, n)$ $where\ m, n \geq 2\ \&\ n = m.$

$$\gamma_{red}(G) = \begin{cases} \left\{\left\lceil\frac{m-4}{3}\right\rceil + 2\right\} m & if \ m \geq 4. \\ 2m & if \ m = 3. \\ 3 & if \ m = 2. \end{cases}$$

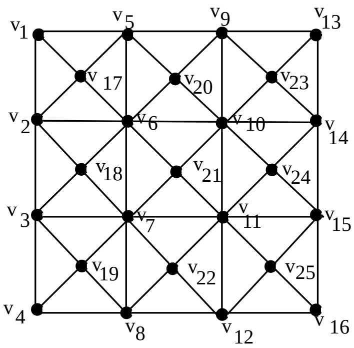

Figure 2.2 Enhanced Mesh Network $M(4 \times 4)$.

Proof: The proof follows same as the above theorem 2.1.

Result 2.1: The maximum detour distance between the vertices of the network graphs is equal to the total number of vertices -1.

Corollary: The above result is not always true.

If we take the first vertex and the last vertex of the graph, then it follows the result 2.1 as the maximum detour distance is $V(G) - 1$.

But it is not the case when we take all the vertices other than first and last.

In both the cases, the vertices covers all the edges but with different edge detour number.

Observation 1: $\gamma_{ed}T(m,n) = \gamma_{ed}HC(n) = \gamma_{ed}M(m,n) = 2.$

Observation 2: $\gamma_{ed}HX(n) = \begin{cases} 3 & n, m = 2. \\ 2 & n, m \neq 2. \end{cases}$

Theorem 2.2 The torus network $G = TR(n, m)$ $where\ m, n \geq 2.$ $\gamma_{red}(G) = 3n - 6\ , n \geq 2.$

Proof: Let $V(G) = nm.$

Torus network is same as mesh network with the condition that every rows and columns form a cycle. Proof follows same from the above theorem.

For $n \geq 2$, we can find a set of vertices which satisfies the restrained edge detour domination in such a way that all the vertices in the dominating set are adjacent with atleast one of the vertices in the set.

Hence, $\gamma_{red}(G) = 3n - 6$.

Edge detour $= \{v_1, v_{25}\}$

Edge detour dominating set $=\{v_1, v_7, v_{13}, v_{19}, v_{25}, v_5, v_{10}, v_{16}, v_{21}\}$

Restrained edge detour dominating set $= \{v_1, v_7, v_{13}, v_{19}, v_{25}, v_5, v_{10}, v_{16}, v_{21}\}$

$= 3n - 6 = (3 \times 5) - 6 = 15 - 6 = 9.$

Hence $\gamma_{red}(TR(5 \times 5)) = 9$.

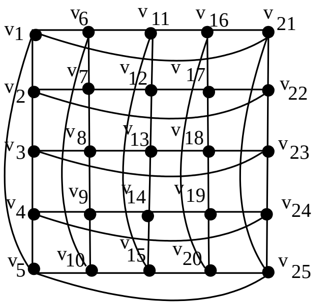

Figure 2.3 Torus Network $TR(5 \times 5)$.

Theorem 2.3: For a honeycomb network $G = HC(n), n \geq 2$.
$$\gamma_{red}(G) = \begin{cases} 2n^2 & , n = 3 \\ 2n^2 + 2n & , n \neq 3. \end{cases}$$

Proof: Let $V(G) = 6n^2$. The edge detour dominating set of $HC(n) = 2$.

For $n = 3$, The edge detour dominating set is same whereas the restrained edge detour dominating set is $2n^2$. We can get $2n^2 + 2n$ vertices but $2n^2$ is the minimal restrained edge detour dominating set.

For $n \neq 3$, The minimum restrained edge detour dominating set is $2n^2 + 2n$. Hence the proof.

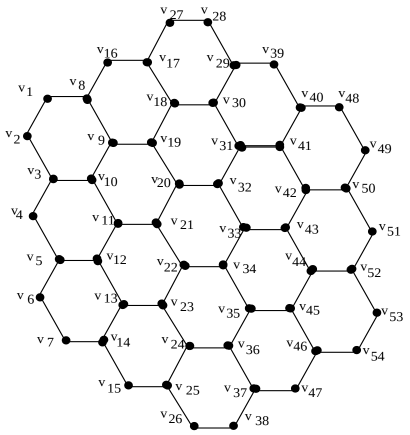

Figure 2.4 Honeycomb Network $HC(3)$.

Edge detour = $\{v_1, v_{54}\}$

Edge detour dominating set = $\{v_1, v_3, v_5, v_7, v_{17}, v_{19}, v_{21}, v_{23}, v_{25}, v_{29}, v_{31}, v_{33}, v_{35}, v_{37}, v_{40}, v_{42}, v_{44}, v_{46}\}$.

Restrained edge detour dominating set = $\{v_1, v_3, v_5, v_7, v_{17}, v_{19}, v_{21}, v_{23}, v_{25}, v_{29}, v_{31}, v_{33}, v_{35}, v_{37}, v_{40}, v_{42}, v_{44}, v_{46}\}$

Hence $\gamma_{red}(HC(3)) = 2n^2 = 2 \times 3^2 = 2 \times 9 = 18$.

Theorem 2.3: For a Silicate network $G = SL(n)$.

$$\gamma_{red}(G) = \begin{cases} 9 & , n = 1 . \\ 9(2n-1) + 12(n-2) & , n > 1. \end{cases}$$

Proof: Let $V(G) = 15n^2 + 3n \ and \ E(G) = 36n^2$.

Edge detour dominating set = $6n$.

We need all the outer vertices of silicate network to cover all the edges with the maximum detour. The proof follows the same, by choosing the dominating set which are adjacent with $V - S$ and S to satisfy the restrained edge detour domination.

For $n = 1$, we get $\gamma_{red}\big(SL(1)\big) = 9$.

For $n > 1$, we get get $\gamma_{red}\big(SL(n)\big) = 9(2n-1) + 12(n-2)$.

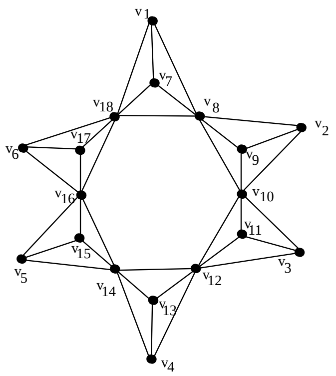

Figure 2.5 Silicate Network $SL(1)$.

Edge detour = $\{v_1, v_2, v_3, v_4, v_5, v_6\}$

Edge detour dominating set = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_8, v_{12}, v_{16}\}$.

Restrained edge detour dominating set = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_8, v_{12}, v_{16}\}$.

Hence $\gamma_{red}(SL(1)) = 9$.

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