Super Felicitous Difference Labeling of Special Types of Graphs

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Abstract

A graph with p vertices and q edges is called super felicitous difference labeling graph if f: V (G) → $\{1,2,\ldots .\, p+q\}$is an injective map so that the induced edge labeling is defined by$f^*(e=uv) = \left(f(u)-f(v)\right) \pmod{p+q}andf\left(v(G)\right) \cup f^*(e){:}\, e \in E(G) = \{1,2,3 \ldots .\, p+q\}$. A graph that admits a **SuperFelicitous Difference Labeling** (SFDL) is called **Super Felicitous Difference Labeling Graph**. In this paper, we investigate super felicitous difference labeling graph of special types of trees like the Jelly fish, thespider graph, the Globe graph Gl(n), the ladder graph,the $H_{n,n}$graph, $S'(P_n)$, the $P_2(+)N_{2n}$ Graph and the Jewel graph.

Keywords: Super Felicitous Difference Labeling (SFDL), Felicitous Difference labeling graph

I.INTRODUCTION

All graphs in this paper represent finite, undirected and simple one. The vertex set and the edge set of a graph G are denoted by V(G) and E(G) respectively. Terms and Notations not defined here are used in the sense of Harary[5].

A graph labeling is an assignment of integers to the vertices or edges or both vertices and edges subject to certain conditions. There are several types of graph labeling and a detailed survey is found in [6]. The notion of felicitous difference labeling was due to V. Lakshmi Alias Gomathi, A. Nagarajan and A. NellaiMurugan[8].

In this paper, we define super felicitous difference labeling graph and show that the Jelly fish, the spider graph, the Globe graphGl(n), the ladder graph,the $H_{n,n}$ graph, $S'(P_n)$graph, the $P_2(+)N_{2n}$and the Jewel Graph are super felicitous difference labeling graph. We use the following definitions in the subsequent section.

II. Preliminaries

Definition 2.1: The Jelly fish graph (m,n) is obtained from a 4 – cycle (v_1, v_2, v_3, v_4) together with an edge v_1, v_3 and appending m pendent edges to v_2 and n pendent edges to v_4.

Definition 2.2:A Spideris a tree having a unique node e with degree greater than 2 and all the other nodes have degrees less than or equal to 2.

A Spiderwith k legs of length n_i: $1 \leq i \leq k$ is denoted by SP ($n_1, n_2, \ldots n_k$). The graph SP (P_n, m)denotes a spider having a path P_n with m pendant vertices attached to one end vertex of P_n. A spider with k legs (paths) each of length n is called a regular spider and is denoted by SP (k, n).

Definition 2.3:A globe is a graph obtained from two isolated vertex are joined by n paths of length two. It is denoted by Gl(n).

Definition 2.4: The ladder L_n (n≥ 2) is the product graph $P_2 \times P_n$which contains 2n vertices and 3n – 2 edges.

Definition 2.5: The graph $H_{n,n}$ is a special bipartite graph with the vertex set

V $(H_{n,n}) = \{u_i, v_i: 1 \leq i \leq n\}$ and edge set $E(H_{n,n}) = \{(u_i, v_i): 1 \leq i \leq n \text{ and } n-i+1 \leq j \leq n\}$.

Definition 2.6: For a graph G, the Splitting graph which is denoted by Spl(G) is obtained by adding to each vertex u, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G.

Definition 2.7: A Path P_n is a walk in which all the vertices are distinct.

Definition 2.8: The Jewel graphJ_n is the graph with vertex set $V(J_n) = \{u, v, x, y, u_i/1 \leq i \leq n\}$ and edge set $E(J_n) = \{ux, uy, xv, yv, uu_i, vv_i/1 \leq i \leq n\}$.

III. Main Results

Definition 3.1: A graph with p vertices and q edges is called super felicitous difference labeling graph if f: V (G) → $\{1,2, \ldots . p+q\}$is an injective map so that the induced edge labeling is defined by $f^*(e = uv) = (f(u) - f(v)) \pmod{p+q}$ and $f(v(G)) \cup f^*(e): e \in E(G) = \{1,2,3 \ldots . p+q\}$. A graph that admits a **SuperFelicitous Difference Labeling**(SFDL) is called **Super Felicitous Difference Labeling Graph**.

Theorem 3.2: The Jelly fish J(m,2n) is SFDL graph for all m and n .

Proof: Let G be the graph J(m,2n).

Let V (G)= $\left\{ u, v, x, y, u_i, v_j \big/ 1 \leq i \leq m, 1 \leq j \leq 2n \right\}$ and

E (G) =$\{xu, xv, yu, yv, xy\} \cup \left\{ uu_i \big/ 1 \leq i \leq m \right\} \cup \left\{ vv_j \big/ 1 \leq j \leq 2n \right\}$Then |V (G)| = m+n+4 and |E (P_n)| = m+n+5.

Define f: V (G)→ $\{1,2, \ldots . .3(m+n+1)+3, 3(m+n+1)+6\}$ as follows:

$f(u) = 1$

$f(v) = 2$

$f(x) = 3(m+n+1)+3$

$f(y) = 3(m + n + 1) + 6$

$f(u_i) = \{2k + 1, 2 \leq k \leq m + 1$

$f(v_1) = f(u_m) + 3$

$f(v_{2k+1}) = f(v_{2k+1-2}) + 4, \quad 1 \leq k \leq m - 1$

$f(v_2) = f(u_m) + 4$

$f(v_{2k}) = f(v_{2k-2}) + 4, \quad 2 \leq k \leq m - 1$

let f^* be the induced edge labeling of f. Then

$f^*(uu_i) = 2k \qquad\qquad 2 \leq k \leq m + 1$

$f^*(vv_j) = f(v_j) - f(v) 1 \leq j \leq 2n$

$f^*(xu) = 3(m + n + 1) + 2$

$f^*(xv) = 3(m + n + 1) + 1$

$f^*(yu) = 3(m + n + 1) + 5$

$f^*(yv) = 3(m + n + 1) + 4$

$f^*(xy) = 3$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the Jelly fish graph J(m,2n) admits Super felicitous difference labeling graph.

Example 3.2.1: The SFD labeling graph of J(3,6) is given in fig.1

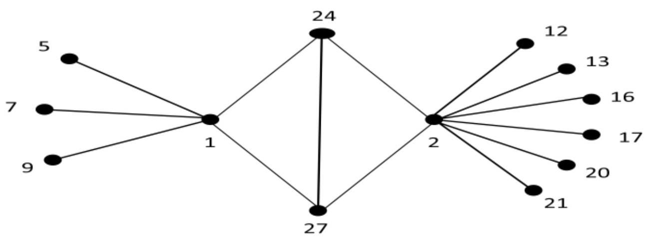


Fig. 1

Theorem 3.3: Spider SP $(P_m, k_{1,n})$ is a SFDL graph for all m,n.

Proof: Let V (SP $(P_m, k_{1,n})$) $=\left\{ {u_i}/{1 \leq i \leq m} \right\} \cup \left\{ {v_j}/{1 \leq j \leq n} \right\}$ and

E (SP $(P_m, k_{1,n})$) $=\left\{ {u_i u_{i+1}}/{1 \leq i \leq m - 1} \right\} \cup \left\{ {v_0 v_j}/{1 \leq j \leq n} \right\}$

Let $u_m = v_0$be the centre vertex of $k_{1,n}$.

Define f: V (SP $(P_m, k_{1,n})$) $\rightarrow \{1,2, \ldots .2m + 2n - 3, 2m + 2n - 1\}$ as follows:

$f(v_j) = 2m + 2n - 1 - 2j \qquad 0 \le j \le m-1$

Case (i): when m is odd

$f(u_{2i+1}) = m - 2i \qquad 0 \le i \le \frac{m+1}{2}$

$f(u_{2i}) = m + 2i \qquad 0 \le i \le \frac{m-1}{2}$

Case (ii): when m is even

$f(u_{2i+1}) = (m+1) + 2i \qquad 0 \le i \le \frac{m}{2}$

$f(u_{2i}) = (m-1) - 2i \quad 0 \le i \le \frac{m}{2}$

Then the induced edge labels are distinct. Hence from the above labeling pattern the graph SP $(P_m, k_{1,n})$ admits Super Felicitous Difference labeling graph.

Example 3.3.1: The SFD labeling graph of SP $(P_8, k_{1,5})$ is given in fig. 2.

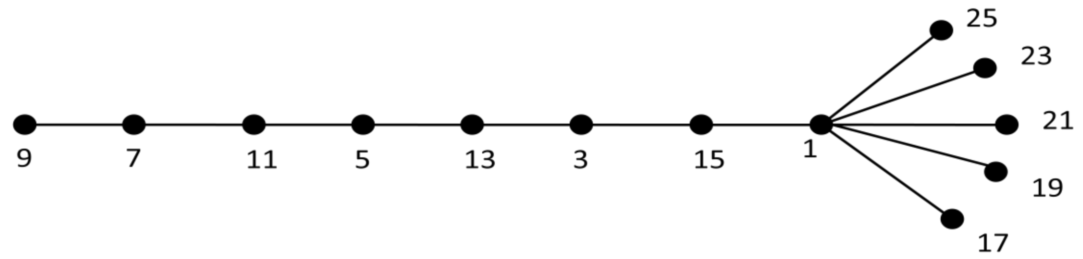


Fig. 2

Theorem 3.4: Globe Gl(n)is SFDL graph for all m and n.

Proof: Let V (Gl(n))$\{u, v, w_i : 1 \le i \le n\}$ and

E (Gl(n)) =$\left\{ {uw_i}/{1 \le i \le n} \right\} \cup \left\{ {vw_i}/{1 \le i \le 2n} \right\}$

Define f: V (Gl(n)) → $\{1,2,\ldots\ldots 3n+2\}$ as follows:

f(u) = 1

f(v) = 2

f(w_i) $= 3n + 2 - 3h,\ 0 \le h \le \frac{2n-2}{2}$

let f^*be the induced edge labels

$$f^*(uw_i) = 3n + 1 - 3h, \quad 0 \le h \le \frac{2n-2}{2}$$

$$f^*(vw_i) = 3n - 3h, \qquad 0 \le h \le \frac{2n-2}{2}$$

Then the induced edge labels are distinct. Hence from the above labeling pattern, the graph Gl(n)admits super felicitous difference labeling graph.

Example 3.4.1: The SFD labeling graph of Gl(6) isgiven in fig 3 respectively.

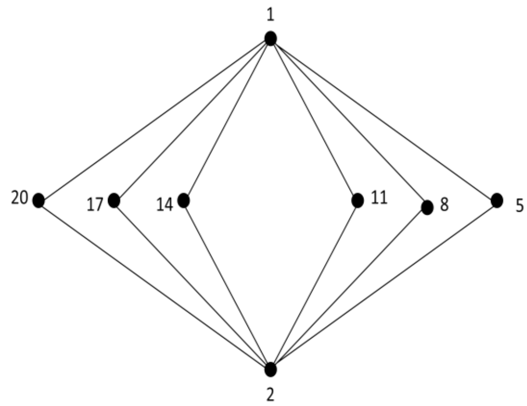


Fig. 3

Theorem 3.5: Ladders $n \le 5$ are Super Felicitous Difference Labeling Graph.

Proof: Assume $n \le 3$

Case (i):

let $V(L_n)$ = $\{u_i v_j = 1 \le i \le n\}$ and

$E(L_n)$ = $\{u_i v_j = 1 \le i \le n\}$

Define f : v(Ln) → {1, 2, …….. 5n-2} is defined as follows

$f(u_1)$ = 1,

$f(v_i)$ = 5n-2-3h $0 \le h \le n-1$

$f(u_2)$ = 2,

$f(u_3)$ = 3,

let f * be the induced edge labelsof f. Then

$f^*(u_i v_j)$ = 5n-2-1

$f^*(u_i v_j)$ = $f^*(f^*(u_{i-1} v_{j-1}) - (2+i)$ $\quad 2 \le i \le 3$

$f^*(u_1 u_2)$ = 5n-4

$f*(u_2v_3) \quad = \quad 5n-10$

$f*(u_1v_2) \quad = \quad 5n-6$

$f*(u_2u_3) \quad = \quad 5n-9$

Then the induced edges are all. Distinct. Hence from the above labeling pattern, the graph L_n admits super felicitous difference labeling graph.

Example 3.5.1: The SFD labeling graph of l_3 is given in fig 4 respectively.

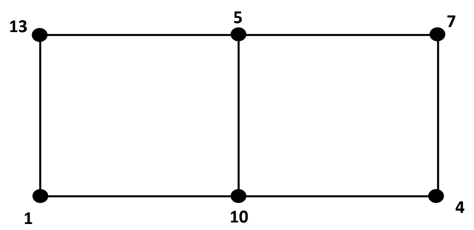


Fig. 4

Case (ii): Assume $n \leq 5$

Let $V(L_n) \quad = \quad \{u_iv_j / 1 \leq i \leq n, 1\}$ and

$E(L_n) = \quad \{u_iv_j / 1 \leq i \leq n\}$

Define f: $v(L_n) \rightarrow \{1, 2, \ldots\ldots 5n - 2\}$ is defined as follows:

$f(u_i) = i \quad 1 \leq i \leq 3$

$f(u_4) = \quad 6$

$f(u_5) = \quad 7$

$f(v_j) \quad = \quad 5n - 2$

$f(v_{2i}) \quad = \quad 5n - 5i \quad 1 \leq i \leq 2$

$f(v_{2i+1}) \quad = \quad f(v_{(2i+1)-1} - 4) \quad 1 \leq i \leq 2$

let f* be the induced edge labels

$f^*(u_1v_1) \quad = \quad 5n-2-i$

$f^*(u_{2i}, v_{2j}) \quad = \quad \{u_{2j+1}, v_{2j+1}) - 4, \quad 0 \leq i \leq 1, 1 \leq i \leq 2$

$f^*(u_{2i+1}, v_{2j+1}) = \quad \{u_{2i}, v_{2j}) - 5, \quad 0 \leq i \leq 1, 1 \leq i \leq 2$

$f^*(v_1, u_2) \quad = \quad 5n - 6$

$f^*(u_2, v_3) \quad = \quad f*(v_1, u_2) - 2$

$f^*(v_3, u_4) \quad = \quad f*(u_2, v_3) - 5$

$f^*(u_4, v_5)$	=	$f^*(u_3, v_4) - 4$
$f^*(u_1, v_2)$	=	$5n - 4$
$f^*(v_2, u_3)$	=	$f^*(u_1, v_2) - 7$
$f^*(u_3, v_4)$	=	$f^*(v_2, u_3) - 4$
$f^*(v_4, u_5)$	=	$f^*(u_3, v_4) - 5$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the graph L_n admits SFDL graph.

Example 3.5.2: The SFD labeling graph of $\boldsymbol{l_5}$ is given in fig 5 respectively.

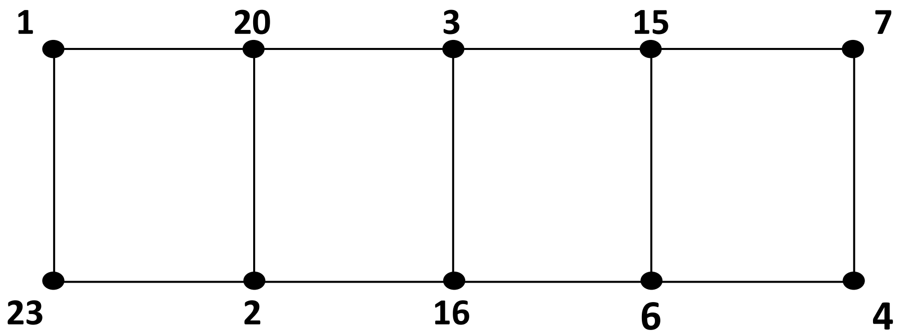


Fig. 5

Theorem 3.6: $H_{n,n}$is Super Felicitous Difference Labeling graph for all n.

Proof: Let V ($H_{n,n}$)= $\{u_i: 1 \le i \le n\} \cup \{v_j: 1 \le j \le n\}$and

E ($H_{n,n}$) =$\{ u_i v_j: 1 \le i \le n \text{ and } 1 \le j \le n\}$.

Define f: V ($H_{n,n}$) → $\left\{1,2, \ldots\ldots \frac{n^2+5n}{2}\right\}$ as follows

$f(u_i) = i \quad 1 \le i \le n$

$f(v_1) = \frac{n^2+5n}{2}$

$f(v_{j+1}) = f\left(v_{j+1-1}\right) - (j+1), 1 \le j \le n$

Then the induced edge labels are all distinct. Hence from the above labeling pattern the graph admits $H_{n,n}$ SFDL graph.

Example 3.6.1: The SFD labeling graph of $H_{5,5}$ is given in fig 6 respectively.

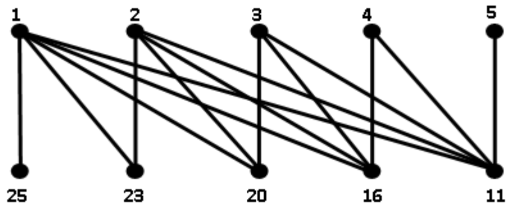


Fig. 6

Theorem3.7: The splitting graph of star Spl (K_1, n) is SFDL graph for all n.

Proof: Let V(G) = $\{u, v, u_i, v_i / 1 \leq i \leq n\}$ and

E(G)= $\{uu_i, uv_i, vv_i / 1 \leq i \leq n\}$

Then $|V(G)|=2n + 2$, and $|E(G)| = 3n$

Define f: V(G) $\rightarrow$ {1, 2, …….. 5n+2} as follows.

f (u) = 4n + 1

f (u_i) = n+i $1 \leq i \leq n$

f (v_i) = i $1 \leq i \leq n$

f (v) = 5n + 2

Letf * be the induced edge labeling of f. Then

$f^*(uu_i)$ = 3n – j $0 \leq j \leq n - 1$

$f^*(vv_i)$ = 5n + 1 - j $0 \leq j \leq n - 1$

$f^*(uv_i)$ = 4n - j $0 \leq j \leq n - 1$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the splittinggraphs admits SFDL graph.

Example 3.7.1: The SFD labeling graph of Spl(K_1,4) is given in fig 7 respectively.

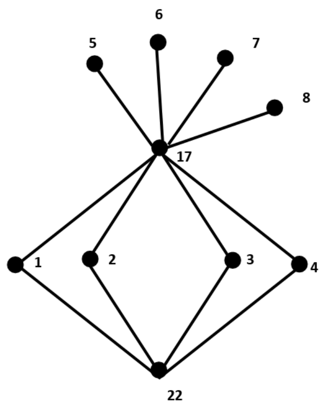


Fig. 7

Theorem 3.8: $P_2(+)N_{2n}$is a SFDL graph.

Proof: Let V ($P_2(+)N_{2n}$)= $\{u, v\} \cup \left\{ {u_i}/_{1 \leq i \leq n} \right\}$ and

E ($P_2(+)N_{2n}$) =$\{(u, v)\} \cup \ \{(u, u_i) \cup (v, u_i)/1 \leq i \leq n\}$.

Define f: V ($P_2(+)N_{2n}$) → $\{1,2, \ldots ..2n + 2, 2n + 3\}$ by

$f(u) = 3n + 3$ $\qquad$ $f(v) = 2n + 2$

$f(u_i) = i$ $\qquad 1 \leq i \leq n$

The induced edges are as follows

$f^*(uv) = n + 1$

$f^*(uu_i) = 3n + 3 - i, \qquad 1 \leq i \leq n$

$f^*(vu_i) = 2n + 2 - i, \qquad 1 \leq i \leq n$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the graph $P_2(+)N_{2n}$ admits SFDL graph.

Example 3.8.1: The SFD labeling graph of $P_2(+)N_8$are given in fig 8 respectively.

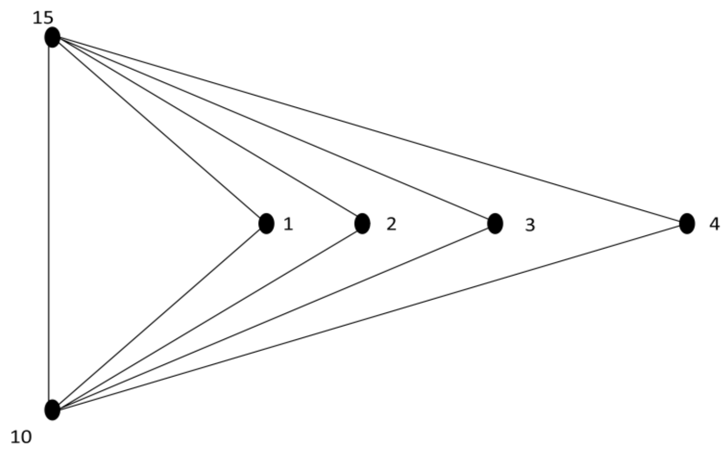


Fig. 8

Theorem 3.9: Jewel graph$J_n$is a SFDL graph.

Proof: Let V (J_n)= $\{u, v, x, y, w_i/1 \leq i \leq n\}$ and

E (J_n) =$\{ux, vx, uy, vy, uw_i, vw_i/1 \leq i \leq n\}$. Then $|$V (G)$|$=n + 4, and $|$E (G)$|$ = 2n+4.

Define f: V (J_n) → $\{1,2, \ldots \ldots 3n + 8\}$ by

$f(u) = 1$ $\qquad$ $f(v) = 2$

$f(x) = 3n + 8$

$f(y) = 3n + 5$

$f(w_i) = f(y) - 3i \qquad 1 \leq i \leq n$

Let f^*be the induced edge label of f. then

$f^*(ux) = 3n + 7$

$f^*(vx) = 3n + 6$

$f^*(vy) = 3n + 3$

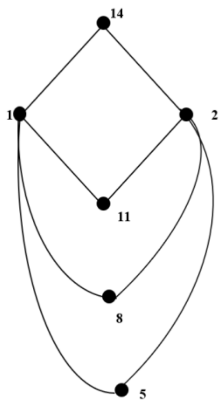
$f^*(uy) = 3n + 4$

$f^*(uw_i) = f^*(uy) - 3i, \qquad 1 \leq i \leq n$

$f^*(vw_i) = f^*(vy) - 3i, \qquad 1 \leq i \leq n$

Then the induced edge labels are all distinct. Hence from the above labeling pattern, the graph J_n admits SFDL graph.

Example 3.9.1: The SFD labeling graph of $J_2$are given in fig 9 respectively.



IV. Conclusion

In this paper, we investigated the Super Felicitous Difference Labeling of some special types of graphs. We have already investigated graphs which are SFDL graph only for certain cases[3]and have planned to investigate the SFD labeling of some special cases of cycle related graphs in our next paper.

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