Mathematical Inventory Management Modeling

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Abstract

For inventory tracking strategies that specifically consider storage space capacity, this research entails creating new mathematical formulas to determine the reorder point and order quantity. The backlog and decreased demands during stock outs are taken into consideration, along with continuous and periodic evaluations. When inventory outnumbers storage space, the cost of over-warehousing at an outside warehouse is calculated on a per-unit basis. The goal is to reduce the entire cost, which includes the expenses associated with ordering, shortages, holding, and over ordering. Consumption and lead time have discontinuous, unpredictable characteristics. To ensure that the results are not dependent on assumptions about demand and lead time posterior distributions, demand is modelled using a likelihood function during periods of variable lead time. The challenges are tackled iteratively since the developed mathematical formulas are so complex. The approach is tested using examples of problems and actual business data. An extensive search is used to identify the challenge instance's optimal solutions. The suggested approach can identify near-optimal solutions for continuous review policies and ideal solutions for continuous review policies. A comparison of continuous review and periodic review methods, as well as a comparison of overstock and lost sales scenarios, reveals important inventory policy insights.

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1. Introduction

Generally, inventories are collections of things that are kept for later use or trade. Any business working with tangible goods, such as producers, distributors, and merchants, must maintain inventories. Keeping and transporting inventory involves certain expenses, but dealing with the issue of running out of stock and the expenditures involved have a significant detrimental effect on the company's performance. Therefore, corporations adopt logistical and mathematical inventory tactics to improve profits while reducing the expense of inventory storage and avoiding scarcity expenses. The most crucial points in inventory management to highlight are when and what amount to order. These two questions' answers describe the inventory management plan and method. In this project, we are estimating a model via which we will propose an ideal method of inventory replenishment to reduce the expenses associated with inventory.

2. Aim

Our goal in this project is to create a mathematical model of an effective inventory strategy for a distribution business so that they may use it to maximise inventory and reduce inventory costs. We are examining the demand-supply relationship of a marketing firm for coffee as an example. Since the number of other elements, such as stirrers, sugar, cups etc., varies depending on that amount of coffee beans, it is enough in this firm to analyse the inventory system using just the inventory of coffee beans. Thus, in our research, we'll outline a workable and ideal technique to reduce inventory costs and boost profits.

3. Literature Survey

Three well-planned and organised resources that give crucial details and expertise regarding inventory theory and management are used as our references. All three components are listed. 1. **Supply Chain Design and Management**:,Case Studies, Strategies, and Concepts edited by David Simchi-Levi, Philip Kaminsky, and the writers of this textbook introduce supply chain management by highlighting its significance and discussing the difficulties, models, and ideas resulting from concerns with efficient supply chain management. Over the last few years, interest in supply chain management has exploded in both business and academia. This growth is mostly attributable to a rise in corporate objectives to cut production costs and the savings that result from efficiently planning and controlling the supply chain.

2.**Modeling Inventory Management System at a Distribution Company**:“Oksana Soshko, VilmarsVjakse, and Yuri Merkuryev's case study, " The case study in this essay focuses on enhancing inventory control at a Latvian distribution firm. The case study focuses on the use of several modelling techniques, including inventory models, optimization models, and simulation models, in inventory management under unpredictable demand.At the end of the paper, we talk about how each model works and what its benefits are for the current issue.

3. **Roberto Andreani's Inventory Theory Book, chapter 19**: Inside this chapter, the writer uses a deterministic continuous review model to try to develop a mathematical solution to the inventory management problems. In our study, this chapter is the primary source we use, but we also use other sources to present a mathematical idea that is wholly original and provides a solution to the difficulties with inventory management.

4. System characterization –

Object

Our object in this project is quantity (Q), which is required to replenish the inventory. In The quantity of coffee beans is the only important factor in this example. Factor; other raw materials and ingredients depend on the amount of coffee beans, so we are taking only a small amount of coffee beans as an object.

Environmental factors

There are many environmental factors in this case that affect the inventory system of the company. There are some factors that affect the supply from the manufacturer. Some factors damage the inventory, like fire and natural disasters. There are many maintenance charges like electricity charges Season and other environmental factors Aside from these advertisements, which influence coffee demand; the population Competition also affects the inventory system. In our model, we are not taking taking environmental factors into account, i.e., assuming that there is no interaction between the environment and the system. So our case study model type is "closed." Also the degree of details is "black box." We are not going into deep structure.

Variables

We are considering three factors:

1. The quantity Q that has to be replenished at a given time
2. The time after which the inventory has to be refilled.
3. The inventory-related total cost TC, which consists of:

(A) Setup costs (K),

(B)Holding costs (h),

(C)Coffee price (c)

Parameters

The following characteristics will be taken into account in this case study:

1. Ordering Cost: This parameter is made up of the operating expense and the overall cost of all the units of the product. The administrative expenses are fixed regardless of the quantity of units, and the price per unit is similarly fixed.

2. Holding cost per unit: This figure reflects the cost of storing an item's inventory per unit. The total cost of holding an inventory is found by dividing the variable number of units in stock by the cost of holding each unit.

3.Demand: We estimate demand by looking at data from previous years, so demand is a parameter in our model.

Dynamic and discrete: The demand and replenishment systems in our model are dynamic mathematical models. Replenishment is discrete since it occurs after a certain amount of time. **Deterministic vs. stochastic:** Since the demand system is based on data from the previous year, our model is deterministic.

Causal Relationship: The total cost (C), time (T), demand (D), and quantity (Q) for the replenishment are discussed in the causal connection. Quantity Q directly affects price, and Q is influenced by time (season), demand, and the amount from the past year. Demand varies with the season since people prefer hot coffee in the winter over cold coffee in the summer. Therefore, our final causal link graph appears as follows:

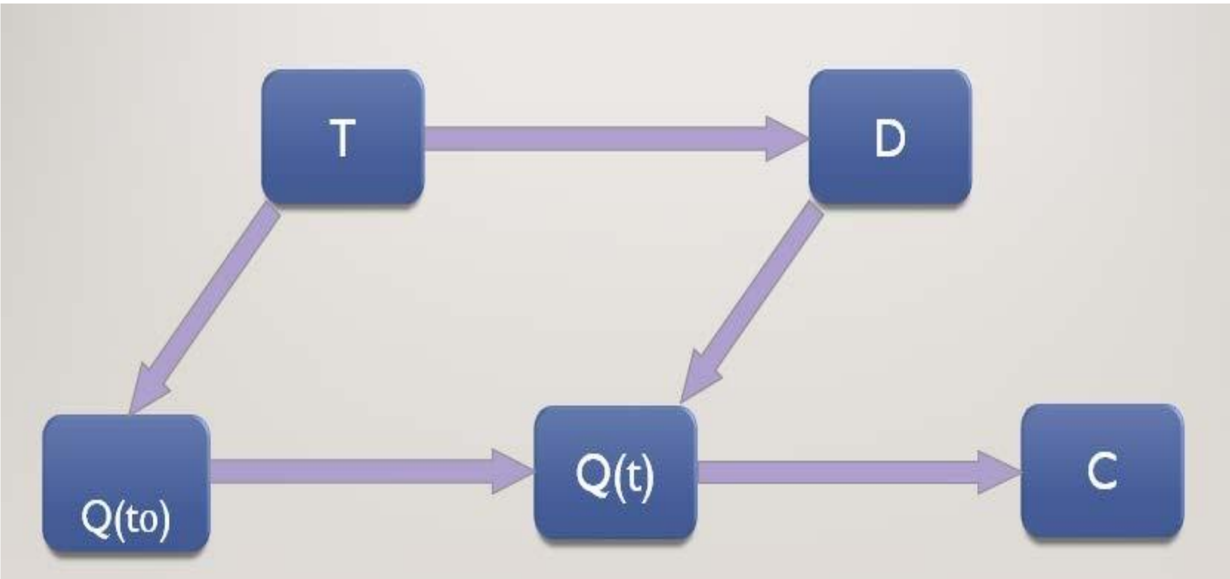

Figure 1: Causal Link Graph

Formulation

Weather and the seasons have a big influence on coffee demand, so they would also have an effect on the stock. Therefore, a year is divided into four seasons. As previously noted, we are making the assumption that demand is known from the data collected the previous year and would remain consistent throughout the full season while varying during the year. The following economic order quantity model, sometimes known as the EOQ model, is a straightforward model that captures this circumstance.

5. EOQ Model

The order quantity that minimises the overall inventory cost and purchasing expenses is known as the EOQ model. This concept makes the assumption that there is a continuous demand for the goods (a kg per day). For an infinite period of time, this model is valid for continuous demand.

Cost Optimization in EOQ model-

Optimal Quantity (Q*) =sqrt(2aK/h)

Holding cost per cycle = $h*Q_{avg}*t$; $Q_{avg} = Q/2$; t = Q/a

Corresponding cycle time (t*)= Q*/a= sqrt(2K/ah)

Ordering cost per cycle = c∗Q

Total cost per unit time (T) = TC/((Q/a))

Total Cost per cycle (TC) = K+c∗Q+(h∗Q2)/(2∗a)

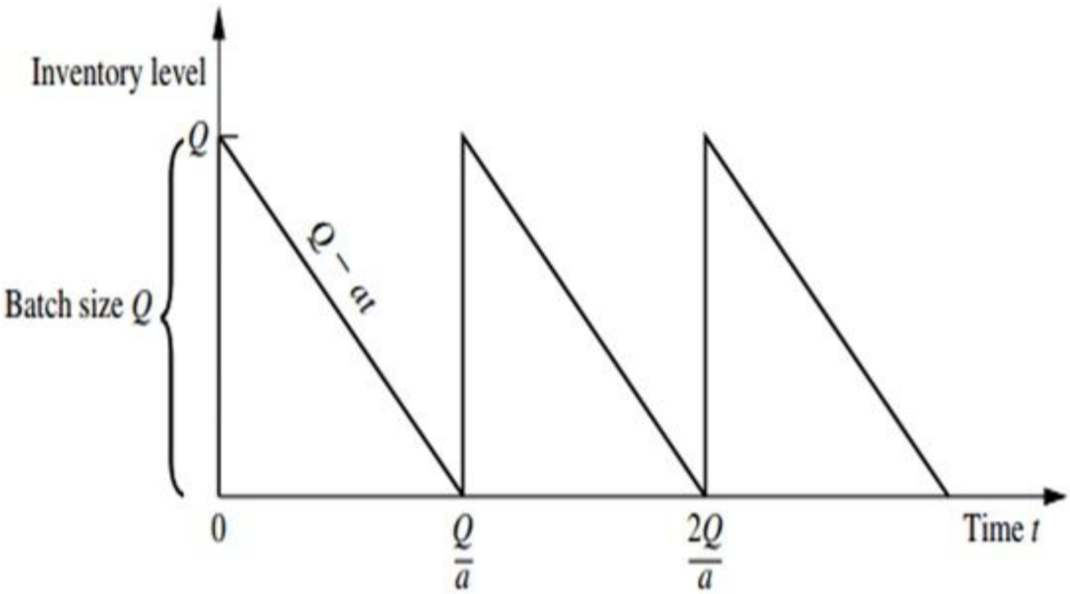

Figure 2: Cost Optimization in EOQ model

We use the EOQ model for every season, and at the conclusion of each season (apart from the most recent one), we looked at all of the different courses or tactics and evaluated the expenses involved with each of them. Our approach to the issue would be to identify the best course of action at the lowest cost.

6. ALGORITHM

We employed a dynamic programming method in this model. Using this method, we compared the costs of each path and selected the one with the lowest cost. We believe that inventories should have been zeroed out during the previous season (termination point).
For the four seasons, there are 29 subpaths (P1 to P29) and 11 nodes (C0 to C10) (see Figures 1 and 2). These can change depending on how many seasons there are. Several subpaths depict potential scenarios for how the starting inventory level may stop or remain at the end of each season. At which nodes do the nodes represent the total minimum cost of potential subpaths with the same season-ending finishing point? This means that the least expensive potential subpaths leading to Ci are included, whereas Co incurs no costs.

Example:

C1 = C0 + minimum(Cost of P1, Cost of P2) = 0 + minimum(Cost of P1, Cost of P2)

C2 = C0 + minimum(Cost of P1) = 0 + Cost of P1

For C3 cost

C3 node have four possible subpaths, two coming from C1 node and other two coming from C2

So - C3 = min[{C2+min(Cost of P7, Cost of P9)}, {C1+min(Cost of P4, Cost of P5)}]

It follows the same process for the available ones.
A different approach to locating the optimistic path is to carry out the exact same operation by using the recursion method. Through increasing node expenses and sliding backwards at the beginning point, we may reduce the expense of subpaths by beginning from the finishing point (C10) (C0). In this instance, C10 is free.

Cases for first two seasons

Figure 3

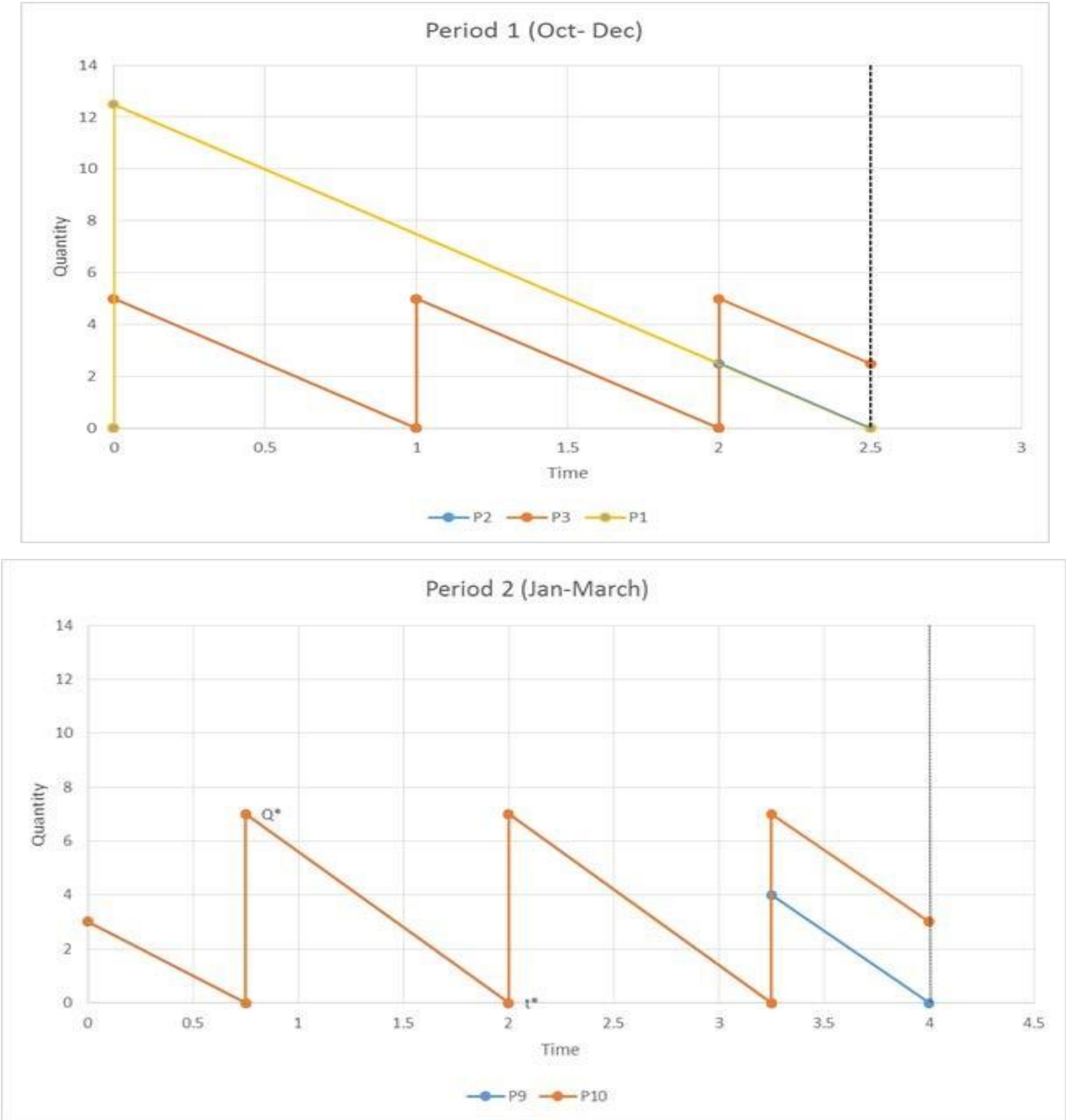

Figure 3: Period of 2 seasons

Algorithm Diagram

Figure 4- All the possible paths

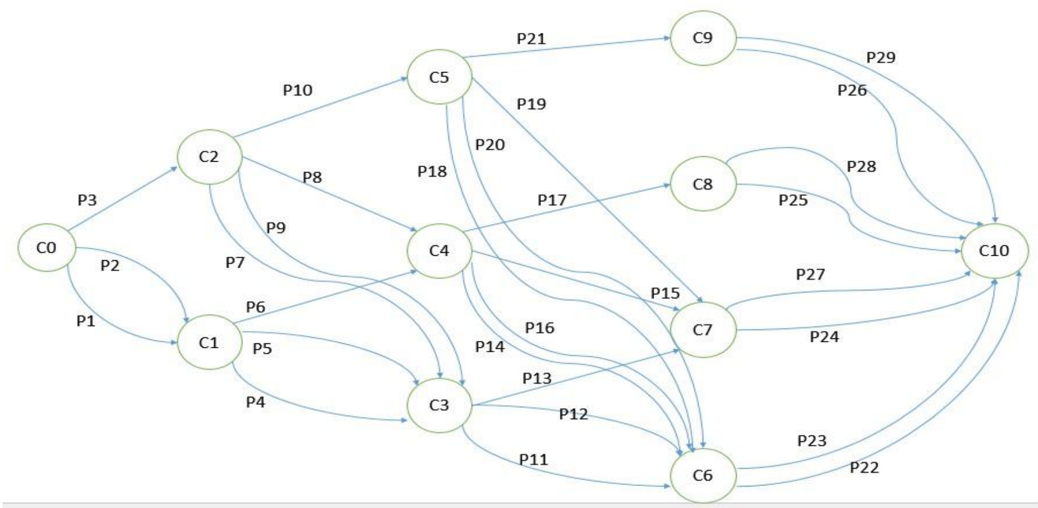

Figure 4: Algorithm Diagram

Our model employs numerical values for the variables (demand) and parameters to determine the best course of action.

Parameters for given problem

- Cost (c) =500 Rs. per kg
- Setup Cost (K) =15000 Rs.
- Holding Cost (h) =0.85 Rs. per kg per day

•Demand rate-

Table 1: Demand rate

Demand Rate	**Period**
60 Kg/Day	July-Sept.
45 Kg/Day	April-June
70 Kg/Day	Jan.-March
50 Kg/Day	Oct.-Dec.

7. Results

The optimal paths would be P3, P10, P20, and P23 when we use the dynamic programming methodology with numeric data for all the factors and parameters. Our model's results are as follows:

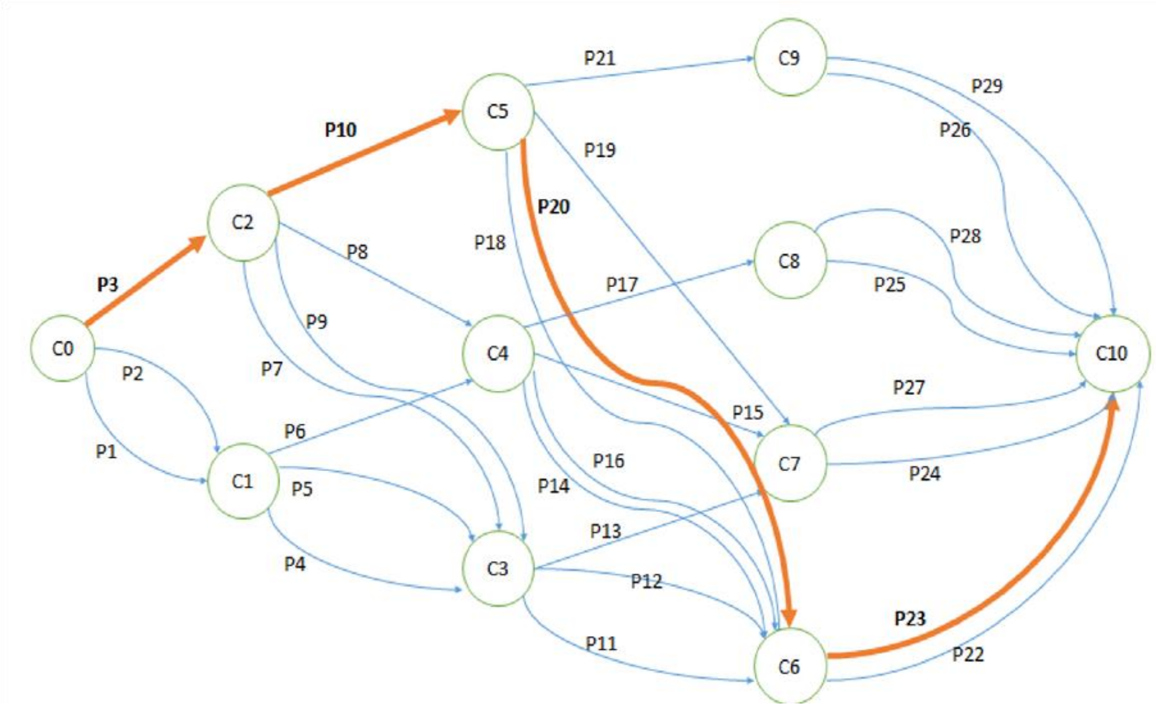

Figure 5: Algorithm Diagram Results

Figure 6- Optimum Path (in red lines)

Conclusions after numerical computations-

Cost of the solution: Rs 10847921 (using the optimum path: P3-P10-P20-P23)

Without the model, the cost is Rs. 11112541,

and the saving is Rs. 264620.

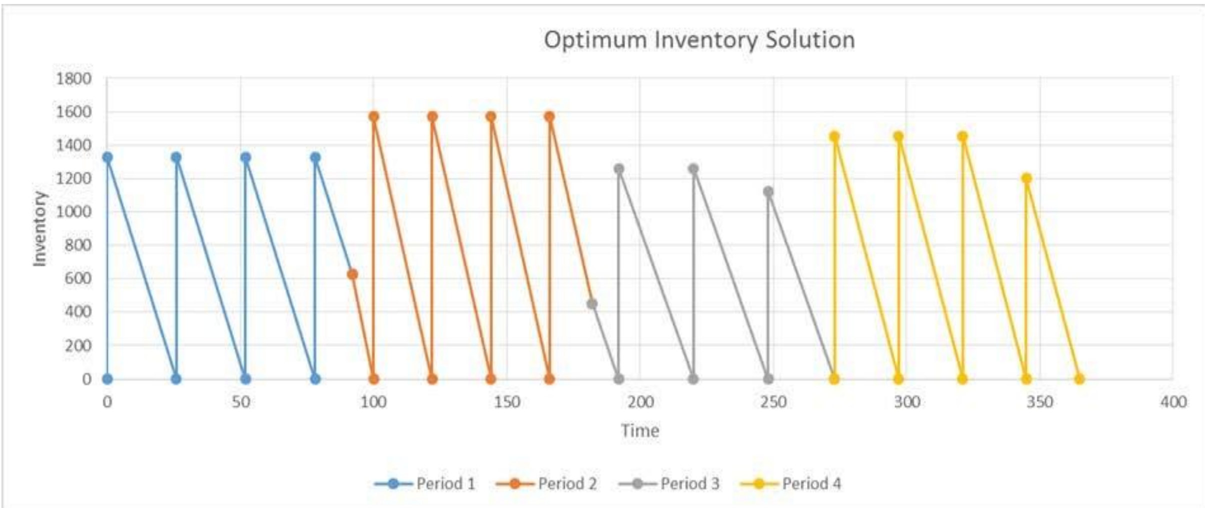

Figure 6- Optimal Inventory Solution

8. Limitations

We attempted to calculate a model that would be a solution for all production and distribution businesses that have inventory problems and must reduce their inventory management expenses. However, there are several drawbacks to our model, including the following:

- This method can only be used with rates that have constant demand.
- As the percentage of periods increases, the algorithm's difficulty would also rise.
- To obtain the optimal solution, numerical values of the factor are needed.
- This model necessitates high holding costs and low setup costs, so it might not be suitable for some production firms with high start-up costs.

9. Conclusions

As previously noted, the two most crucial and vital problems for inventory management—when to purchase and the amount to purchase—are addressed by the finest methods and guidelines. Our approach is a modest attempt to identify the most viable, workable, and lucrative solutions to these two issues.So we observe from the outcomes that the corporation makes a considerable profit when our model is used. Therefore, effective inventory management is essential to the company's development and profitability. This dynamic and mathematical model and technique that we have demonstrated in our project appears to have the extremely beneficial property of being applicable to any production and distribution organisation with a comparable inventory structure and demand.

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