Micropolar Fluid with Concentration in a Vertical Channel

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Abstract
The general theory of micropolar fluids deviates from that of Newtonian fluids by adding two new variables to the velocity. These variables are microrotations that are spin and microinertia tensors describing the distributions of atoms and molecules inside the microscopic fluid particles. This paper deals them mixed convective flow and heat transfer of micropolar fluid in a vertical channel with asymmetric wall temperatures and concentrations are solved analytically and we compared too numerically. It helps to identify the mathematical modeling validity.

Key words: Micropolar fluid, vertical channel, Newtonian fluids, heat source, heat sink

Introduction

This theory may be applied to the explanation of the phenomena of the flow of colloidal fluids, liquid crystals, polymeric suspensions, animal blood, etc. With this in mind, it developed the theory of a new class of fluids called microfluids [1]. This theory is concerned with a class of fluids that exhibit certain microscopic effects as a result of the local structure and micro motions of the fluid elements [2]. The Navier-Stokes equations are the special cases of those equations that arise in such types of fluids [3]. However, a serious difficulty is encountered when this theory is applied to real nontrivial flow problems; even for the linear theory, a problem contains an unavoidably complex system of 19 partial differential equations with 19 unknowns, and the underlying mathematical problem is not easily amenable to solution [4]. It proposed that a subclass of these fluids called micropolar fluids be considered, in which the skew-symmetric properties of the gyration tensor are imposed, along with a condition of micro isotropy, so that the system of 19 equations is reduced to seven equations and seven unknowns only. Many experiments in this direction have also verified the basic conclusions drawn [5].

We have shown experimentally that the fluids containing minute polymeric additives indicate a considerable reduction of skin friction (about 25–30%), a concept that is well explained by the theory of micropolar fluids. These fluids with microstructure are also capable of representing body fluids. It has shown that the fluid flowing in the brain (CSF) is adequately modelled by micropolar fluids [6]. An excellent review of micropolar fluids and their applications was provided [7]. They extended the theory of micro-fluids to take into account the thermal effects, i.e., heat conduction, convection, and heat dissipation, which is called the theory of thermomicrofluids [8].

Many industrial and technical processes benefit from research into mixed convection flow of micropolar fluids, such as nuclear reactors cooled during emergency shutdown, solar central receivers exposed to winds, electronic devices cooled by fans, heat exchangers placed in a low velocity environment, and so on [9]. The numerical study of the flow and heat transfer characteristics of mixed convection in a micropolar fluid along a vertical flat plate with conduction effects was conducted [10]. They studied an exact analysis of the laminar free convection from a sphere by considering the prescribed surface temperature and surface heat flux with are studied convection of micropolar fluid in a vertical channel [11].

In many transport processes existing in nature and in industrial applications, heat and mass transfer are consequences of buoyancy effects caused by diffusion of heat and chemical species [12]. The phenomenon of natural convective flow is not often caused entirely by the effect of temperature gradients but also by differences in concentrations of dissimilar chemical species. Convection and transfer processes are governed by buoyancy mechanisms arising from both thermal and species diffusion [13]. This present analysis is primarily motivated by the existence of many transport processes in which the transfer of heat and mass occurs simultaneously [14]. The engineering applications include the chemical distillatory processes, the design of heat exchangers, the migration of moisture in heat exchanger devices, petroleum reservoirs, filtration, nuclear waste repositories, the spreading of chemical pollutants in plants, and the diffusion of medicine in blood veins, as mentioned [15].

We intend to investigate fully developed heat and mass transfer by mixed convection of a micropolar fluid in a vertical channel and concentrations [16]. The exact solutions are derived, and the effects of the vortex viscosity parameter and buoyancy ratio on the flow, the effect of heat absorption or generation coefficient on the flow, and heat transfer and mass transfer characteristics, such as the microrotation, velocity, volume flow rates, total heat rate added to the fluid, and total species rate added to the fluid, are examined with a heat source or sink [17].

Mathematical formulation

Consider a steady fully developed laminar mixed convection flow of a micropolar fluid between two vertical plates. The vertical plates are separated by a distance b . The inlet temperature is T_0 and inlet concentration is C_0 . The inner surface of the left plate (i.e., at $y = 0$) is kept at a constant temperature T_1 while the inner surface of the right plate (i.e., at $b = y$) is maintained at a constant temperature T_2 . In addition, the concentration of a certain constituent in the solution varies from C_1 on the inner surface of the left plate to C_2 on the inner surface of the right plate. Because the flow is fully developed, the transverse velocity is zero, and the flow depends only on the transverse:

$$(\mu + k)\frac{d^2u}{dy^2} + k\frac{d\nu_3}{dy} + \rho\beta_t g(T - T_0) + \rho\beta_c g(c - C_0) = 0$$

$$\gamma\frac{d^2\nu_3}{dy^2} - 2k\nu_3 - k\frac{du}{dy} = 0$$

$$k\frac{d^2T}{dy^2} \pm Q(T - T_0) = 0$$

$$\frac{d^2c}{dy^2} = 0$$

The fluid properties are assumed to be constant except for density variations in the buoyancy force

term. With introducing Boussinesq approximations, the governing equations can be written as.
The appropriate boundary conditions are:

$$u = 0, \nu_3 = 0, T = T_1, c = C_1 \text{ on } y = 0$$

$$u = 0, \nu_3 = 0, T = T_2, c = C_2 \text{ on } y = b$$

Here *u* is the velocity component in the stream wise direction. *T, c* and v_3 are the fluid temperature, concentration and angular velocity of the micropolar fluid respectively. *k*is the vortex viscosity, and ***j*** is the micro inertia density. Here γ is the spin gradient viscosity and we assume that γ = (μ + 0.5*k*). Property μ is the dynamic viscosity of the fluid and ρ is the fluid density. β_t and β_c are the coefficients for thermal expansion and for concentration expansion, respectively, and *g* is the gravitational acceleration. Note that the boundary condition for the micro rotation at the fluid-solid interface is v_3 = 0, the condition of zero spin, as used. The microstructure does not rotate relative to surface.
Here we introduce non-dimensional variables as follows:

$$U = \frac{ub\rho}{Gr\mu}, \theta = \frac{T - T_0}{T_1 - T_0}, Y = \frac{y}{b}, \phi = \frac{Qb^2}{k}$$

$$U = \frac{\nu_3 b^2 \rho}{Gr\mu}, C = \frac{c - C_0}{C_1 - C_0}, Gr = \frac{g\beta_t (T - T_0) b^3 \rho^2}{\mu^2}.$$

We obtain the following dimensionless governing equations:

$$(1 + K)\frac{d^2U}{dY^2} + K\frac{dH}{dY} + \theta + NC = 0$$

$$\left(1 + \frac{K}{2}\right)\frac{d^2H}{dY^2} - BK\left(2H + \frac{dU}{dY}\right) = 0$$

$$\frac{d^2\theta}{dY^2} \pm Q\theta = 0$$

$$\frac{d^2C}{dY^2} = 0$$

The relevant boundary conditions in dimensionless form are

$$U = 0, H = 0, \theta = 1, C = 1 \text{ on } Y = 0$$

$$U = 0, H = 0, \theta = m, C = n \text{ on } Y = 1$$

The solutions of velocity, temperature, concentration, dimensionless volume flow rate, total species rate and total heat rate for Newtonian fluids (K = 0) with heat sink are

$$U = -\left(\frac{d_1}{\phi}\right)\cosh(\sqrt{\phi}y) - \left(\frac{d_2}{\phi}\right)\sinh(\sqrt{\phi}y) - \frac{N(n-1)Y^3}{2} - a_3Y - a_4$$

$$\theta = d_1\cosh(\sqrt{\phi}y) + d_2\sinh(\sqrt{\phi}y)$$

$$C = d_3Y + d_4$$

$$Q_v = -\left(\frac{d_1}{\sqrt{\phi}}\right)\sinh\left(\frac{\sqrt{\phi}}{\phi}\right) - \left(\frac{d_2}{\sqrt{\phi}}\right)\cosh\left(\frac{\sqrt{\phi} - 1}{\phi}\right) - \frac{N(n-1)}{24} - \frac{N}{6} - \frac{a_3}{2} - a_4$$

$$C_s = l_{01}\cosh(\sqrt{\phi}) + l_{02}\sinh(\sqrt{\phi}) + l_{03}$$
$$E = l_{04}\sinh(2\sqrt{\phi}) + l_{05}\sinh^2(\sqrt{\phi}) + l_{06}\cosh(\sqrt{\phi}) + l_{07}\sinh(\sqrt{\phi}) + l_{08}.$$

The solutions of velocity, temperature, concentration, dimensionless volume flow rate, total species rate and total heat rate for Newtonian fluids (K = 0) with heat source are

$$U = \left(\frac{d_1}{\phi}\right)\cosh(\sqrt{\phi}y) + \left(\frac{d_2}{\phi}\right)\sinh(\sqrt{\phi}y) - \frac{N(n-1)Y^3}{2} - \frac{NY^2}{2}a_3 Y - a_4$$
$$\theta = d_1\cos(\sqrt{\phi}y) + d_2\sin(\sqrt{\phi}y)$$
$$C = d_3 Y + d_4$$
$$Q_v = -\left(\frac{d_1}{\sqrt{\phi}}\right)\sin\left(\frac{\sqrt{\phi}}{\phi}\right) - \left(\frac{d_2}{\sqrt{\phi}}\right)\cos\left(\frac{\sqrt{\phi}-1}{\phi}\right) - \frac{N(n-1)}{24} - \frac{N}{6} - \frac{a_3}{2} - a_4$$
$$C_s = l_{01}\cos(\sqrt{\phi}) + l_{02}\sin(\sqrt{\phi}) + l_{03}$$
$$E = l_{04}\sin(2\sqrt{\phi}) + l_{05}\sin^2(\sqrt{\phi}) + l_{06}\cos(\sqrt{\phi}) + l_{07}\sin(\sqrt{\phi}) + l_{08}.$$

All the constants appeared in the above equations can be obtained easily and hence not defined.

Results and discussion

Figures 1 to 12 are for the heat absorption case, and figures 13 and 16 are for the heat generation case. Figures 1 and 2 are shows the microrotation velocity and velocity, respectively, for various vortex viscosity parameters.

Figures 3 and 4 show the microrotation velocity and velocity, respectively, for various buoyancy ratios N. The magnitude of microrotation is enhanced with an increase in the buoyancy ratio. Moreover, increasing the buoyancy ratio tends to accelerate the fluid flow in the vertical channel. Figures 5 and 6 show the effect of the heat absorption coefficient φ on the microrotation velocity and velocity.The magnitude of microrotation velocity and velocity decreases as the heat absorption coefficient φ remains constant for Newtonian fluids (K = 0)and increases for micropolar fluids (K = 1.5).

Figure 7 shows how the dimensionless volume flow rate Q_v varies with the buoyancy ratio N for different vortex viscosity parameters K . Increasing the buoyancy ratio tends to accelerate the fluid flow, thus raising the volume flow rate of the fluid flowing through the vertical channel. Moreover, the dimensionless volume flow rate flowing through the vertical channel tends to decrease as the vortex viscosity parameter is increased. Figure 2.8 depicts the effect of the heat absorption coefficient φ on the dimensionless volume flow rate Q_v .The dimensionless volume flow rate Q_v decreases as φ increases.Increasing the heat sink parameter phi tends to reduce the velocity, thus decreasing the volume flow rate of the fluid.

Figures 9 and 10 show the dimensionless total heat rate added to the fluid E as functions of the buoyancy ratio N and heat absorption coefficient φ for various vortex viscosity parameters K .Increasing the buoyancy ratio tends to accelerate the fluid flow, raising the heat transfer rate

between the wall and the fluid, and thus increasing the total heat rate added to the fluid in the vertical channel. As the heat absorption coefficient (φ) increases, the dimensionless total heat rate added to the fluid (E) decreases. A higher vortex viscosity parameter, on the other hand, results in a reduction in the total heat rate added to the fluid in the vertical channel for both buoyancy ratio N and heat absorption φ .
Figures 11 and 12 show how the dimensionless total species rate added to the fluid SC varies with the buoyancy ratio N and heat absorption coefficient φ for various vortex viscosity parameters K .Increasing the buoyancy ratio accelerates the fluid flow, thus enhancing the mass transfer rate between the wall and the fluid flowing through the vertical channel. As the heat absorption coefficient phi increases, the dimensionless total species rate added to the fluid SC decreases. Increasing the heat sink parameter phi results in a reduction of the flow field and hence reduces the mass transfer rate. However, a higher vortex viscosity parameter leads to a decrease in the dimensionless total heat rate added to the fluid in the vertical channel.

The effects of the heat generation coefficient φ and various vortex viscosity parameters on microrotation velocity and velocity are shown in Figures 13 and 14, respectively. The microrotation velocity decreases in magnitude as the heat generation coefficient phi increases, whereas the vortex viscosity parameter enhances the microrotation velocity.

The effect of buoyancy ratio on the dimensionless volume rate Q_v , dimensionless heat rate E, and dimensionless species rate for heat generation yields the same result as heat absorption and is thus not illustrated graphically. Figures 15, 16, and 17 show the effects of the heat generation coefficient φ on the dimensionless volume flow rate Q_v , the dimensionless heat rate E, and the dimensionless species rate SC. As φ rises, so do Q_v , E, and SC.This is due to the fact that an increase in the heat generation coefficient increases the fluid flow and thus enhances the volume flow rate, heat rate, and species rate. The figures 1–17 show that the velocity and microrotation velocity for micropolar fluid are lower than for Newtonian fluid (K = 0), which was the similar result observed.

The results for all flow field parameters for micropolar fluid were lower than those for Newtonian fluid, which was similar.

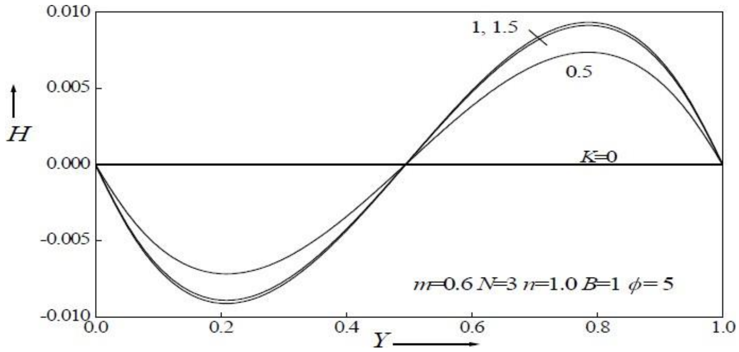

Figure 1: Effects of vortex viscosity parameter on the microrotation velocity.

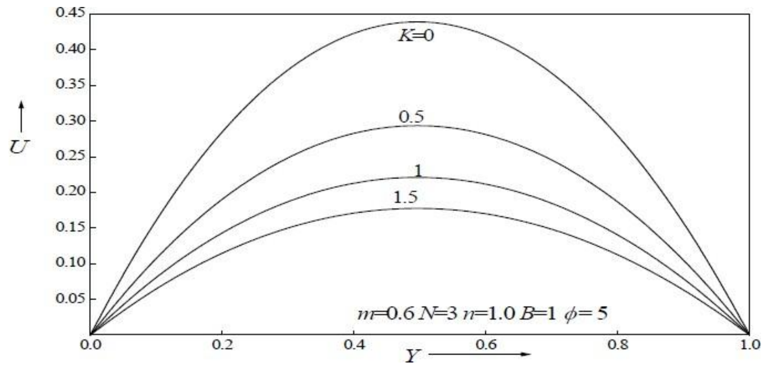

Figure 2: Effects of vortex viscosity parameter on the velocity.

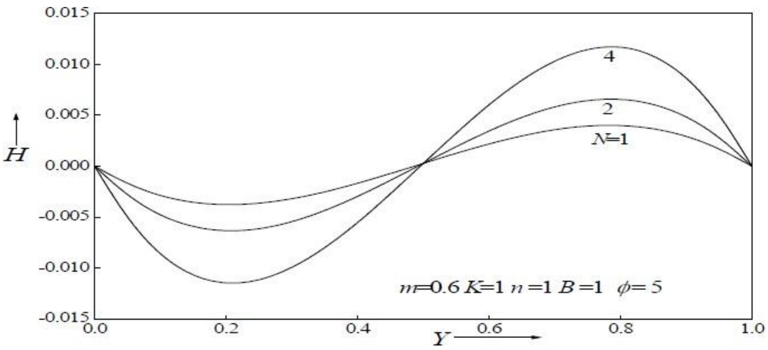

Figure 3: Effects of buoyancy ratio on microrotation velocity

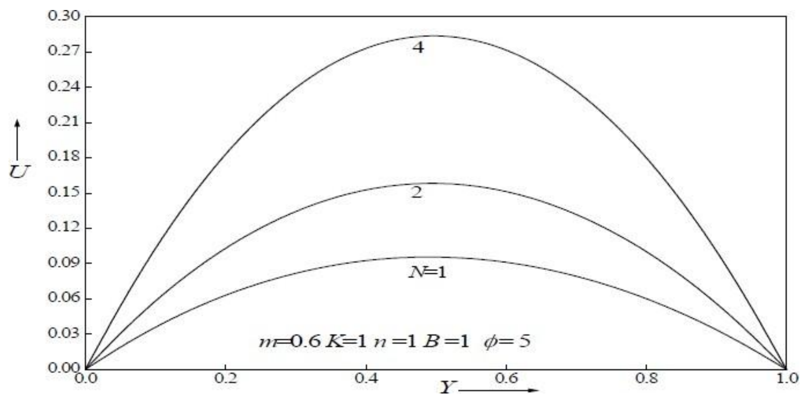

Figure 4: Effects of buoyancy ratio on velocity.

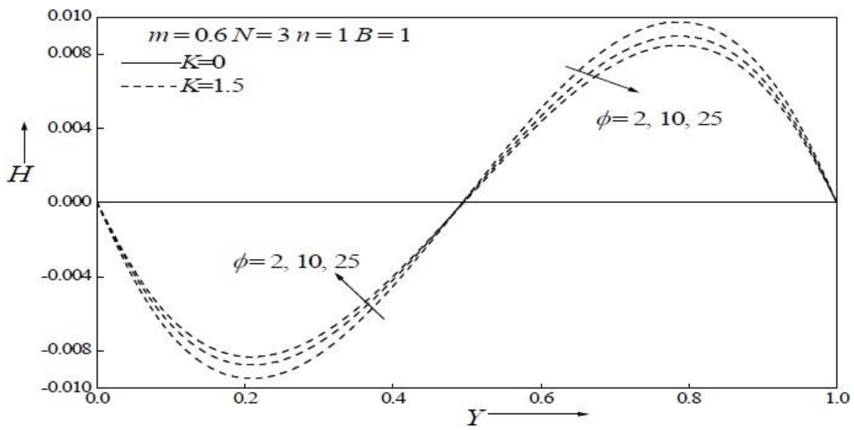

Figure 5: Effects of heat absorption coefficient parameter φ on the microrotationvelocity.

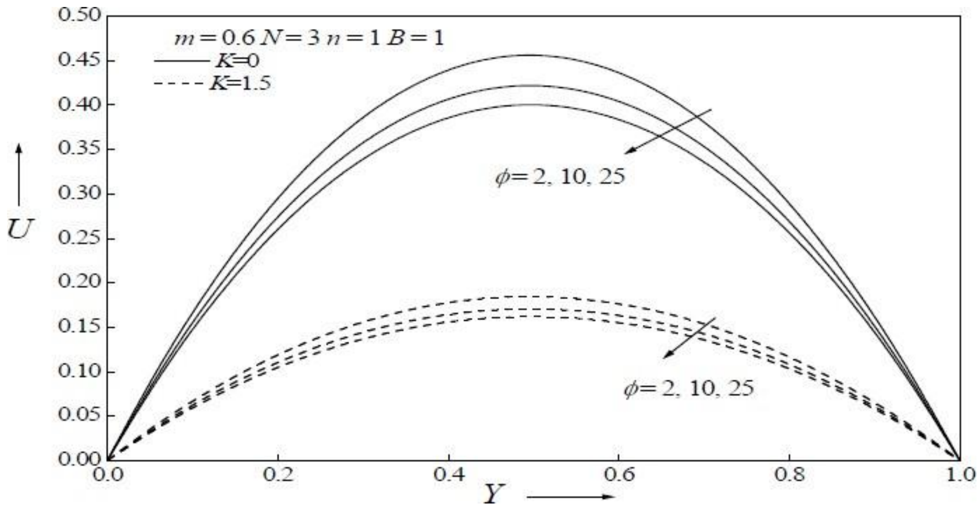

Figure 6: Effects of heat absorption coefficient parameter φ on the velocity.

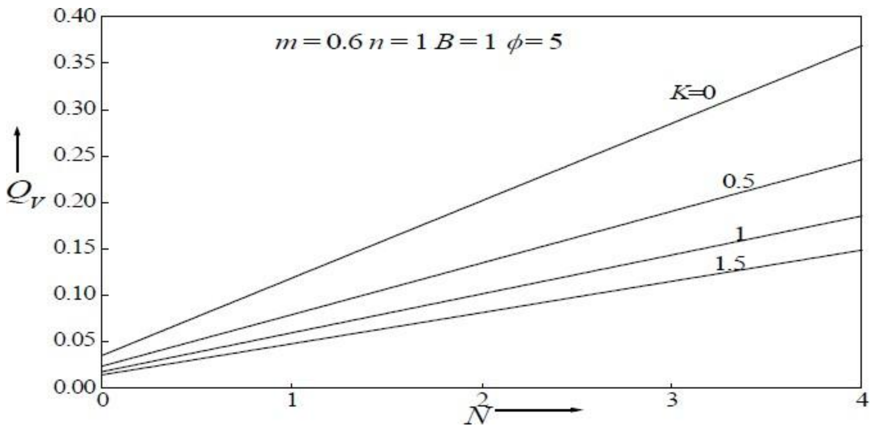

Figure 7: Effects of buoyancy ratio for various vortex viscosity parameter K on thevolume flow rate.

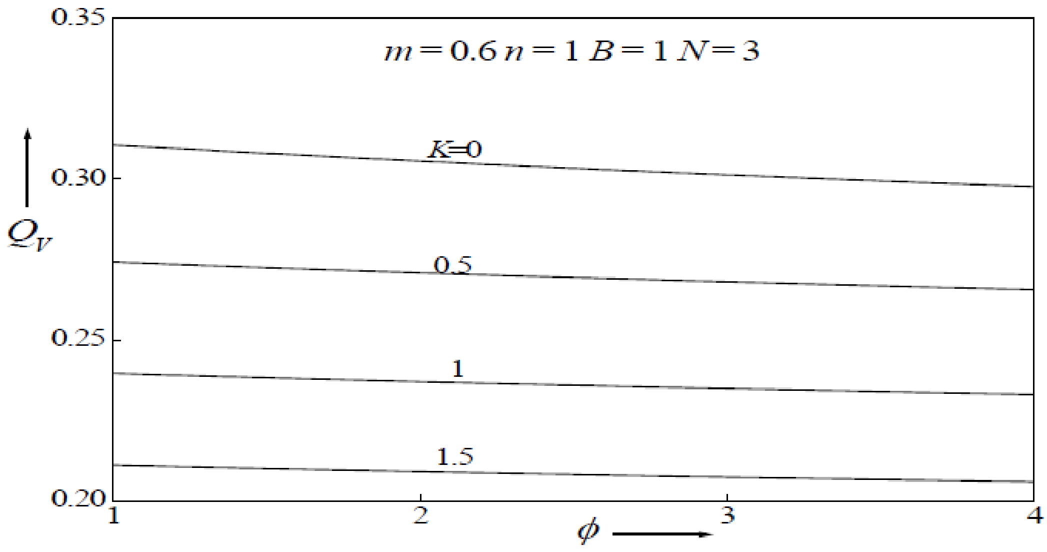

Figure 8: Effects of heat absorption coefficient for various vortex viscosity parameter Kon the volume flow rate.4b

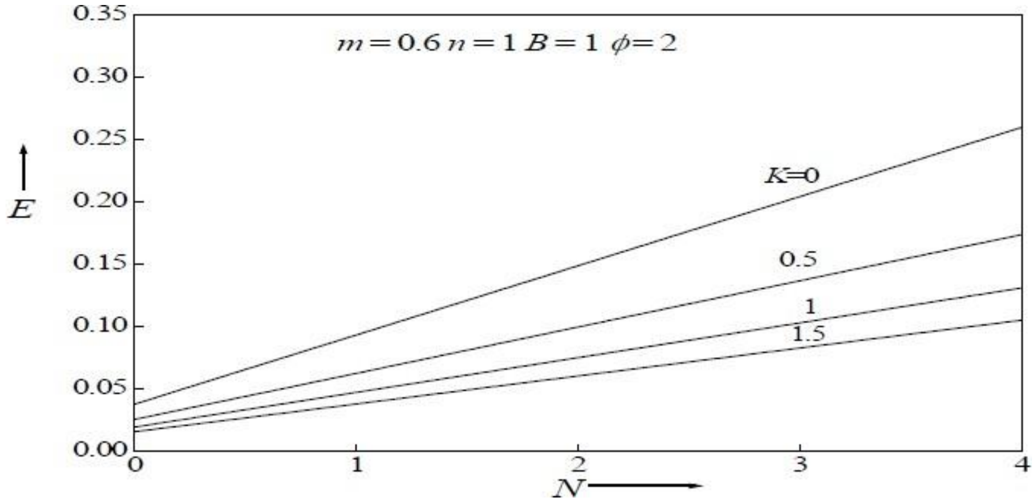

Figure 9: Effects of buoyancy ratio various vortex viscosity parameter K on the totalheat rate.

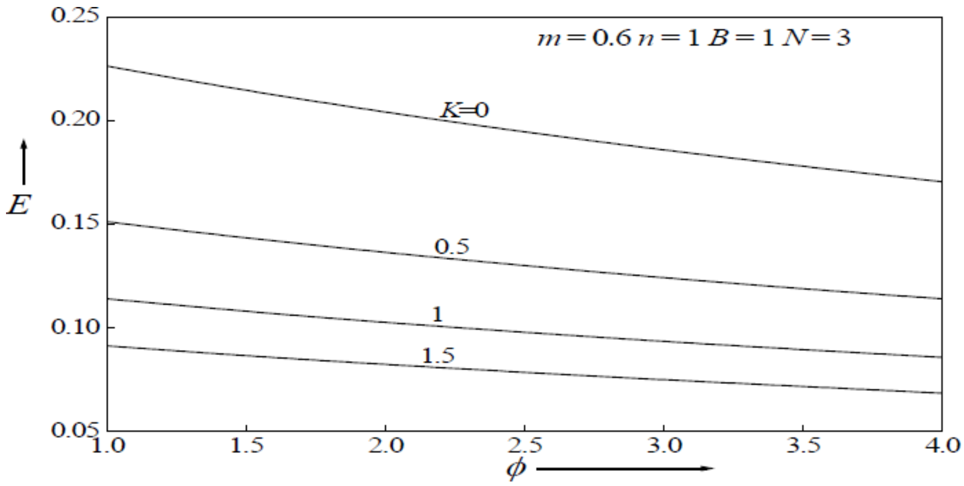

Figure 10: Effects of heat absorption coefficient for various vortex viscosity parameterK on the total heat rate.

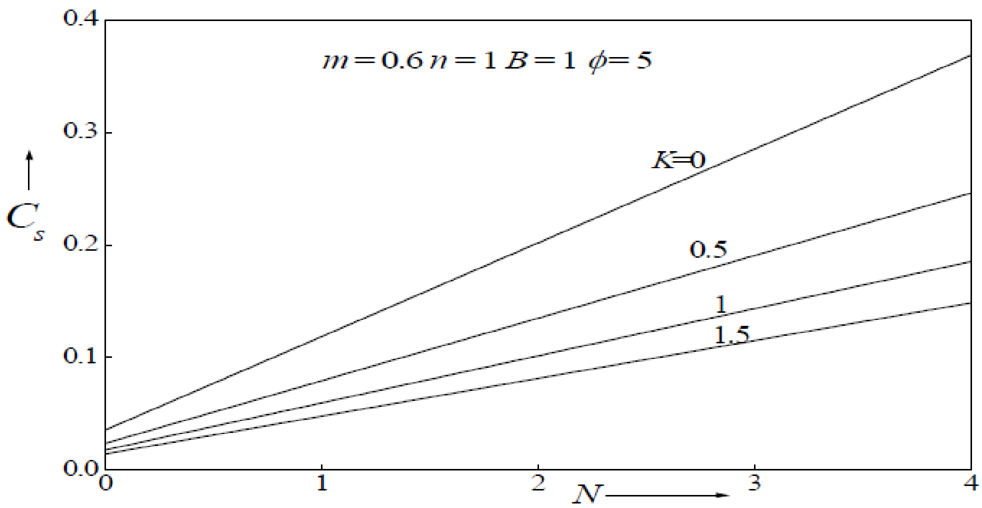

Figure 11: Effects of buoyancy ratio for various vortex viscosity parameter K on thetotal heat species rate.

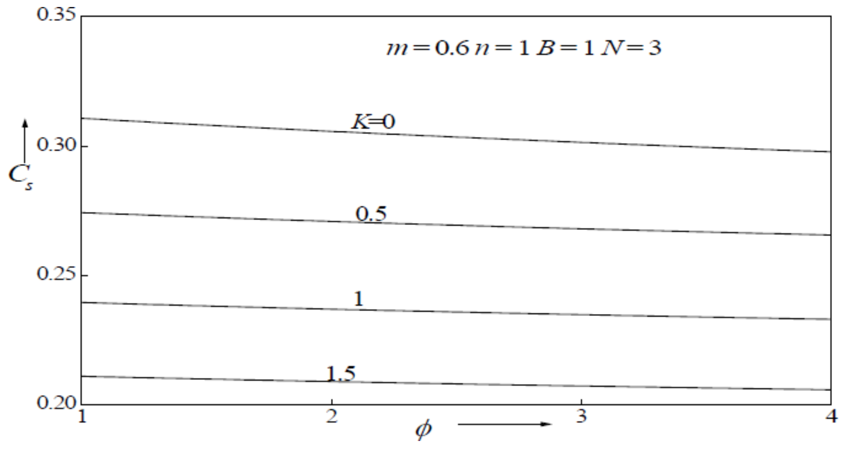

Figure 12: Effects of heat absorption coefficient for various vortex viscosity parameterK on the total heat species rate.

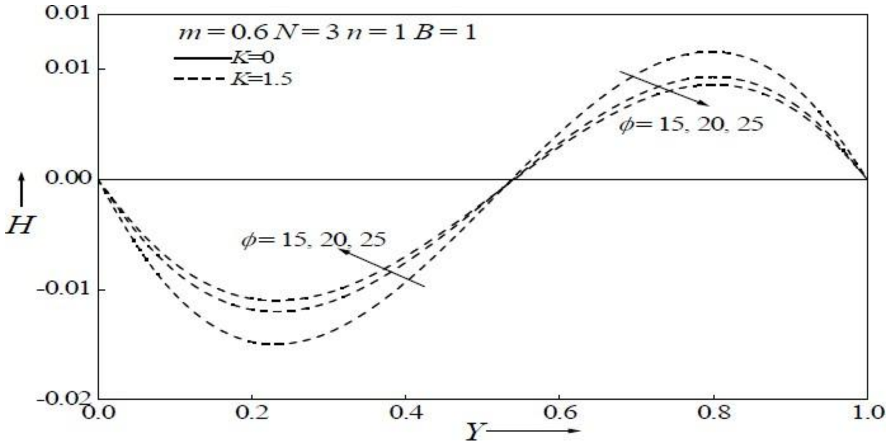

Figure 13: Effects of vortex viscosity parameter on the microrotation velocity.

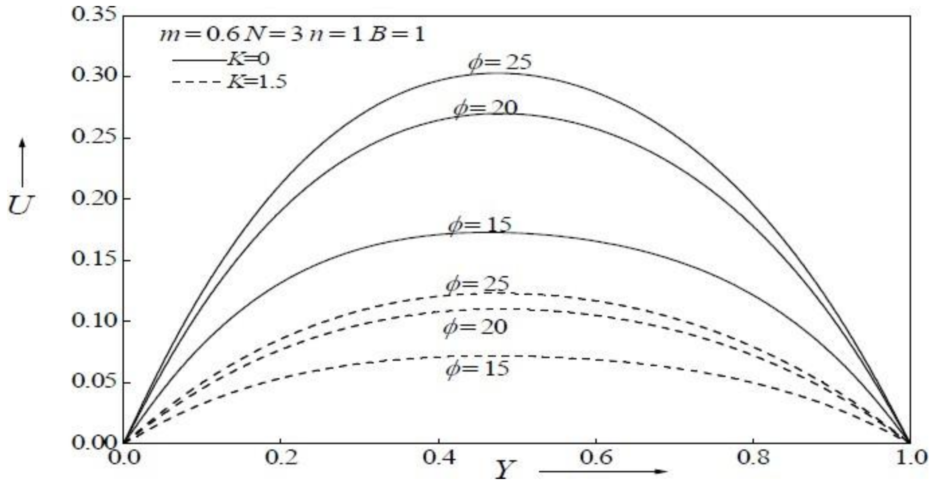

Figure 14: Effects of vortex viscosity parameter on the velocity.

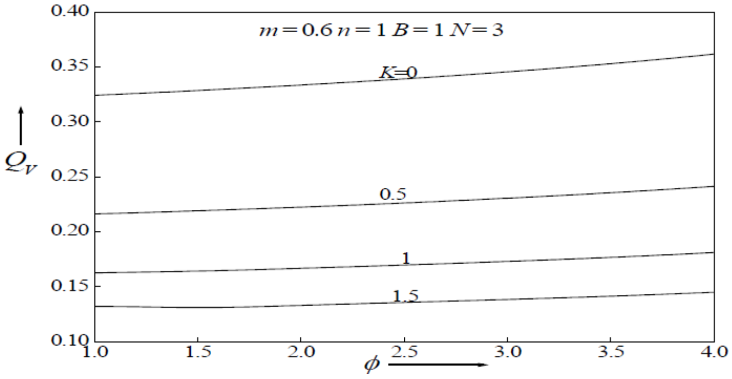

Figure 15: Effects of heat generation and vortex viscosity parameter on the totalvolume flow rate.

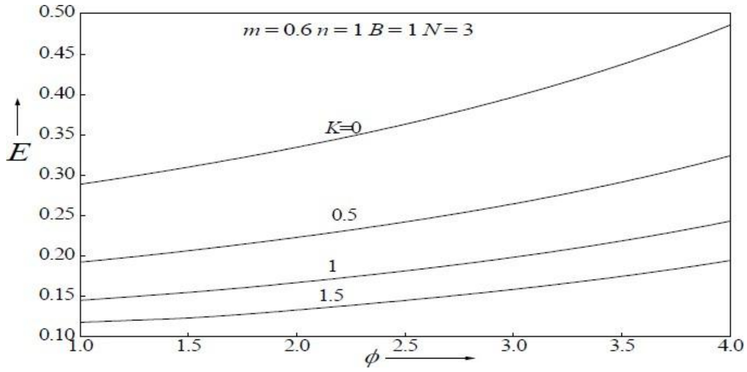

Figure 16: Effects of heat generation and vortex viscosity parameter on the total heatrate.

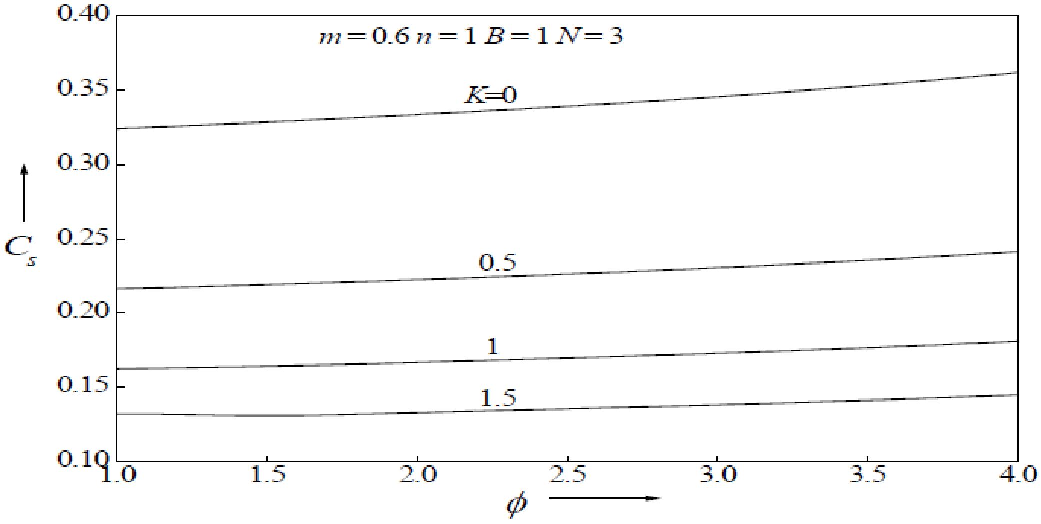

Figure 17: Effects of heat absorption and vortex viscosity parameter on the total speciesrate.

Conclusion

Mixed convective flow and heat transfer of micropolar fluid in a vertical channel with asymmetric wall temperatures and concentrations are solved analytically. The magnitude of microrotational velocity tends to increase as the vortex viscosity parameter is increased. Increasing the vortex viscosity parameter tends to decrease the fluid velocity in the vertical channel. The velocity increases as the heat generation coefficient increases, and as the vortex viscosity parameter increases, the velocity decreases. The effect of the vortex viscosity parameter and buoyancy ratio on the microrotation velocity and velocity is similar to that of heat absorption and is thus not graphically depicted.

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