Cosmic Model with Qeos and Anisotropic Bianchi Type- I Universe: A Mathematical Study

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Abstract

On enormous scales, the current cosmos is thought to be homogenous and isotropic, but there is evidence of some anisotropy in its early stages. Homogeneous but anisotropic models such as the Bianchi ones are therefore of some interest. The solution to Einstein's field equations for $f(R,T)$gravity has been determined, and it requires the presence of both the cosmological constant and the quadratic equation of state (QEoS),which states that $p = \omega\rho^2 - \rho$ where $\omega \neq 0$. The model first slowed down but is now picking up speed. Also, the model is observed to approach isotropy at late times. We generate expressions for the Hubble parameter and state finder parameter and describe their significance at length. The cosmological model's physical characteristics are also examined. An intriguing aspect of the model is its dynamic cosmological parameter, which is enormous in the early cosmos, shrinks with time, and eventually approaches a constant. This has the potential to be useful in resolving the issue of the cosmological constant. When strings are present, a model of an accelerating universe can be built by presuming a negative value for the constant deceleration parameter and a decreasing vacuum energy density.For sufficiently enormous amounts of time, the threads vanish from the cosmos. Physical and geometrical model characteristics are also covered.

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1. Introduction- The investigation of cosmic strings is a topic that has garnered a lot of interest among cosmologists. It is speculated that these strings are responsible for the density fluctuations that eventually result in the development of galaxies. 1 The point-like particles in the universe are thought to be replaced in string theory by one-dimensional objects called strings. This framework is purely speculative. On distance scales that are greater than the scale of a string, it is assumed that the behavior of a string is identical to that of an ordinary particle. During the time when strings predominated in the universe, fluctuations in the density of particles were caused by the threads themselves. It is conceivable for us to postulate that when strings stop being and particles become significant, the fluctuations will spread in such a way that eventually galaxies would appear as a result of the expansion. This is something that we are able to do.

In 1997, Singh and Desikan did research on FRW models for the cosmological theory that is based on Lyra's geometry. Their work was published in the year 1997. Copeland et al. (2006) have presented for our attention a comprehensive examination of the $f(R)$ hypothesis. The gravitational field equations of the $f(R)$ theory are derived with the assistance of a variational principle of the Einstein-Hilbert type. Dubey and Tripathi considered a cosmological model that included the Friedman equation for the expansion of the universe. This model was taken into consideration by the researchers (2012). They discussed the Newtonian technique after arriving at the Friedman equation using the first rule of thermodynamics as a guide. Myrzakulov (2011) provided evidence to show that the f(R) gravity model is capable of providing an explanation for the acceleration that the cosmos is currently undergoing. Singh and Bishi examined the Bianchi type-I cosmological model in the setting of the gravity quadratic equation of state and the cosmological constant in their research that was published in 2015. The research conducted by Dubey et al.(2018) focused on developing a spatially homogenous Bianchi Type-I massive string cosmological model with bulk viscosity and decreasing vacuum energy density. Recent research conducted by Aditya and Reddy (2018) looked into the dynamics of the Bianchi type-III cosmological model where anisotropic DE and an attracting massive scalar meson field were present. This research was subsequently published in the academic publication known as Scientific Reports. Naidu described the Bianchi type-II modified holographic Ricci DE model, which has an appealingly huge scalar field (2019). Using the $f(R)$ theory of general relativity, Hasmani and Al-Haysah (2019) made an effort to examine spatially homogenous Bianchi type-I cosmological models. This was done in an effort to learn more about these models. In Naidu et al. (2019) .'s study, a Bianchi type-V dark energy cosmological model was investigated in general relativity while taking into account the presence of an attractive massive scalar field. The model was characterized as being spatially homogeneous while yet being anisotropic. Within the framework of the $f(G)$ theory of gravitation, the Bianchi type I cosmological model was investigated by Hatkar et al. (2020) in the presence of quark and weird quark matter that existed in the first second of the early cosmos. This was done in the context of the early universe. Jiten et al. (2021) carried out research on a Bianchi type-III cosmological model. This model featured Lyra geometry, cloud threads, and particles that were interconnected with one another. Singh and Kumar (2021) obtained the proper response for the field circumstances by utilizing a period changing deceleration border, which led to the conclusion that the universe is expanding. In the year 2022, Karim investigated the Bianchi Type-I cosmological model when the Saez-Ballester theory of gravitation was present.

2. Mathematical Model:

We take into account the Bianchi type-I line element, defined as

$$ds^2 = -dt^2 + {A_1}^2 dx^2 + {A_2}^2 dy^2 + {A_3}^2 dz^2 \quad (1)$$

In which a, b, and $c$are functions of cosmic time t in the metric system. The following dynamical equations are derived from the field equations of $f(R,t)$gravity for the Bianchi type-I (1) space time :

$$\frac{\ddot{A_2}}{A_2} + \frac{\ddot{A_3}}{A_3} + \frac{\dot{A_2}\dot{A_3}}{A_2 A_3} = \Lambda - (1+2\lambda)p \quad (2)$$

$$\frac{\ddot{A_1}}{A_1} + \frac{\ddot{A_3}}{A_3} + \frac{\dot{A_1}\dot{A_3}}{A_1 A_3} = \Lambda - (1+2\lambda)p \quad (3)$$

$$\frac{\ddot{A_1}}{A_1} + \frac{\ddot{A_2}}{A_2} + \frac{\dot{A_1}\dot{A_2}}{A_1 A_2} = \Lambda - (1+2\lambda)\rho \quad (4)$$

$$\frac{\dot{A_1}\dot{A_2}}{A_1 A_2} + \frac{\dot{A_2}\dot{A_3}}{A_2 A_3} + \frac{\dot{A_3}\dot{A_1}}{A_3 A_1} = \Lambda + (1+2\lambda)\rho \quad (5)$$

where the ordinary derivative with respect to cosmic time t is indicated by an overdot.

For the sake of this investigation, we have taken into account the quadratic equation of state in the form

$$p = \omega\rho^2 - \rho \quad (6)$$

In this case, $\omega \neq 0$ is a constant, and the quadratic nature of the equation of state remains unaffected by this type of study.

There are four equations in this system, and the unknowns are labelled $A_1, A_2, A_3, p, \rho, \Lambda$. Adding two more physically feasible relations yields the full solution. Let's think about Power law relationships, which are grounded on physics. $V = V_0 t^{3n}$

The constant positive number n stands for. We have produced two alternative models of the Bianchi type-I universe depending on the expansion law used.

$$A_1 = c_1 V^{1/3}\, exp\left[\frac{2d_1+d_2}{3}\int\frac{dt}{V}\right] \tag{7}$$

$$A_2 = c_2 V^{1/3} exp\left[\frac{d_2-d_1}{3}\int\frac{dt}{V}\right] \tag{8}$$

$$A_3 = c_3 V^{1/3}\, exp\left[-\frac{d_1+2d_2}{3}\int\frac{dt}{V}\right] \tag{9}$$

where c_1,c_2, and c_3 as well as d_1 and $d_2$are arbitrary integration constants that must be satisfied.

$c_1 c_2 c_3$ = 1

$$\frac{\ddot{A_2}}{A_2}+\frac{\ddot{A_3}}{A_3}+\frac{\dot{A_2}\dot{A_3}}{A_2A_3}=\Lambda-(1+2\lambda)(\omega\rho^2-\rho)$$

$$\frac{\dot{A_2}\dot{A_3}}{A_2A_3}=\Lambda-(1+2\lambda)(\omega\rho^2-\rho)-\frac{\ddot{A_2}}{A_2}-\frac{\ddot{A_3}}{A_3}$$

$$\frac{\dot{A_1}\dot{A_2}}{A_1A_2}+\frac{\dot{A_2}\dot{A_3}}{A_2A_3}+\frac{\dot{A_3}\dot{A_1}}{A_3A_1}=\Lambda+(1+2\lambda)\rho$$

$$\frac{\dot{A_1}\dot{A_2}}{A_1A_2}-\frac{\ddot{A_2}}{A_2}-\frac{\ddot{A_3}}{A_3}+\frac{\dot{A_3}\dot{A_1}}{A_3A_1}=\Lambda+(1+2\lambda)\rho-\Lambda+(1+2\lambda)(\omega\rho^2-\rho)$$

$$=(1+2\lambda)\rho+(1+2\lambda)(\omega\rho^2-\rho)=(1+2\lambda)(\rho+\omega\rho^2-\rho)$$

$$=\omega\rho^2(1+2\lambda)$$

$$\rho^2=\frac{1}{\omega(1+2\lambda)}\left[\frac{\dot{A_1}\dot{A_2}}{A_1A_2}+\frac{\dot{A_3}\dot{A_1}}{A_3A_1}-\frac{\ddot{A_2}}{A_2}-\frac{\ddot{A_3}}{A_3}\right] \tag{10}$$

Measurable potential exists for us as

$$A_1 = c_1 a\, exp\left[\frac{2d_1+d_2}{3}\int\frac{dt}{V_0t^{3n}}\right]=c_1V^{1/3}\, exp\left[-\left(\frac{2d_1+d_2}{3V_0}\right)\frac{t^{-3n+1}}{(3n-1)}\right] \tag{11}$$

$$A_2 = c_2 V^{1/3}\, exp\left[-\left(\frac{d_2-d_1}{3V_0}\right)\frac{t^{-3n+1}}{(3n-1)}\right] \tag{12}$$

$$A_3 = c_3 V^{1/3}\, exp\left[\left(\frac{d_1+2d_2}{3V_0}\right)\frac{t^{-3n+1}}{(3n-1)}\right] \tag{13}$$

3. Cosmological Parameter of the model:

(i) Hubble Parameter

$$H = \frac{1}{3}(H_1 + H_2 + H_3)$$

$$H_1 = \frac{n}{t} + \frac{\dot{A_1}}{A_1}, H_2 = \frac{n}{t} + \frac{\dot{A_2}}{A_2}, H_3 = \frac{n}{t} + \frac{\dot{A_3}}{A_3}$$

One can derive the Hubble parameters in the direction of the galaxy as

$$H_1 = \frac{n}{t} + \frac{\dot{A_1}}{A_1} = \frac{n}{t} + \frac{1}{c_1 V^{\frac{1}{3}} \, exp\left[-\left(\frac{2d_1+d_2}{3V_0}\right)\frac{t^{-3n+1}}{(3n-1)}\right]} c_1 V^{\frac{1}{3}} \, exp\left[-\left(\frac{2d_1 + d_2}{3V_0}\right)\frac{t^{-3n+1}}{(3n-1)}\right].$$

$$\left[\left(\frac{2d_1 + d_2}{3V_0}\right)(3n-1)\frac{t^{-3n}}{(3n-1)}\right] = \frac{\boldsymbol{n}}{\boldsymbol{t}} + \left(\frac{2d_1 + d_2}{3V_0}\right)t^{-3n}$$

$$H_2 = \frac{\boldsymbol{n}}{\boldsymbol{t}} + \left(\frac{d_2 - d_1}{3V_0}\right)t^{-3n}$$

$$H_3 = \frac{\boldsymbol{n}}{\boldsymbol{t}} - \left(\frac{d_1 + 2d_2}{3V_0}\right)t^{-3n}$$

Hubble parameters H_1, H_2 and H_3 are labeled for the x, y and $z$axes.

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$$H = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\boldsymbol{n}}{\boldsymbol{t}} + \frac{(2d_1+d_2+d_2-d_1-d_1-2d_2)}{V_0 t^{3n}} = \frac{n}{t} \tag{14}$$

(ii) Deceleration parameter :

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) = -1 + \frac{1}{n} \tag{15}$$

(iii) Dynamical scalar expansion:

$$\theta = 3H = \frac{3n}{t} \tag{16}$$

(iv) The anisotropy parameter ($\boldsymbol{A_m}$):

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{1}{3}\left[\left(\frac{H_1 - H}{H}\right)^2 + \left(\frac{H_2 - H}{H}\right)^2 + \left(\frac{H_3 - H}{H}\right)^2\right] =$$

$$\frac{1}{3}\left[\left(\frac{\frac{n}{t} + \left(\frac{2d_1+d_2}{3V_0}\right)t^{-3n} - \frac{n}{t}}{H}\right)^2 + \left(\frac{\frac{n}{t} + \left(\frac{d_2-d_1}{3V_0}\right)t^{-3n} - \frac{n}{t}}{H}\right)^2 + \left(\frac{\frac{n}{t} - \left(\frac{d_1+2d_2}{3V_0}\right)t^{-3n} - \frac{n}{t}}{H}\right)^2\right] =$$

$$\frac{1}{3}\left[\left(\frac{\left(\frac{2d_1+d_2}{3V_0}\right)t^{-3n}}{H}\right)^2 + \left(\frac{\left(\frac{d_2-d_1}{3V_0}\right)t^{-3n}}{H}\right)^2 + \left(\frac{-\left(\frac{d_1+2d_2}{3V_0}\right)t^{-3n}}{H}\right)^2\right] =$$

$$\frac{1}{27{V_0}^2H^2}\left(4{d_1}^2+{d_2}^2+4d_1d_2+{d_2}^2+{d_1}^2-2d_1d_2+{d_1}^2+4{d_2}^2+4d_1d_2\right)t^{-6n}$$

$$A_m=\frac{1}{27{V_0}^2H^2}\left(6{d_1}^2+6{d_2}^2+6d_1d_2\right)t^{-6n}$$

$$=\frac{2t^2}{9{V_0}^2n^2}\left({d_1}^2+{d_2}^2+d_1d_2\right)t^{-6n}=\frac{2}{9{V_0}^2n^2}\left({d_1}^2+{d_2}^2+d_1d_2\right)t^{-6n+2} \quad (17)$$

(v) Shear Scalar (σ):

$$\sigma=\frac{1}{2}\left(\frac{\dot{A_1}^2}{{A_1}^2}+\frac{\dot{A_2}^2}{{A_1}^2}+\frac{\dot{A_3}^2}{{A_1}^2}\right)-\frac{\theta^2}{6}$$

$$\sigma=\frac{1}{2}\frac{t^{-6n}}{9{V_0}^2}\left[4{d_1}^2+{d_2}^2+4d_1d_2+{d_2}^2+{d_1}^2-2d_1d_2+{d_1}^2+4{d_2}^2+4d_1d_2\right]-\frac{1}{6}9.\frac{t^2}{n^2}$$

$$\sigma=\frac{1}{18}\frac{t^{-6n}}{{V_0}^2}\left[6{d_1}^2+6{d_2}^2+6d_1d_2\right]-\frac{3}{2}\frac{t^2}{n^2}=\frac{1}{3}\frac{t^{-6n}}{{V_0}^2}\left({d_1}^2+{d_2}^2+d_1d_2\right)-\frac{3}{2}\frac{t^2}{n^2} \quad (18)$$

(vi) The state-finder pair $\{r,s\}$:

$$V=a^3=V_0t^{3n}\Longrightarrow a={V_0}^{1/3}t^n$$

$$\dddot{a}=V_0^{\frac{1}{3}}n(n-1)(n-2)t^{n-3}$$

$$r=\frac{\dddot{a}}{a}\times\frac{1}{H^3}=\frac{V_0^{\frac{1}{3}}n(n-1)(n-2)t^{n-3}}{{V_0}^{1/3}t^n}\times\frac{t^3}{n^3}=\frac{(n-1)(n-2)}{n^2}=\frac{n^2-3n+2}{n^2} \quad (19)$$

$$s=\frac{r-1}{3\left(\frac{q-1}{2}\right)}=\frac{\frac{n^2-3n+2}{n^2}-1}{3\left(\frac{-1+\frac{1}{n}-1}{2}\right)}=\frac{\frac{-3n+2}{n^2}}{3\left(\frac{1-2n}{2n}\right)}=\frac{2(2-3n)}{3n(1-2n)} \quad (20)$$

(vii) Energy density:

From equation (5)

$$\rho^2=\frac{1}{\omega(1+2\lambda)}\left[\frac{\dot{A_1}\dot{A_2}}{A_1A_2}+\frac{\dot{A_3}\dot{A_1}}{A_3A_1}-\frac{\ddot{A_2}}{A_2}-\frac{\ddot{A_3}}{A_3}\right]$$

$$\rho^2=\frac{1}{\omega(1+2\lambda)}\left[\frac{t^{-6n}\left({d_2}^2-2{d_1}^2+d_1d_2\right)}{9{V_0}^2}+\frac{t^{-6n}\left(2{d_1}^2+5d_1d_2+2{d_2}^2\right)}{9{V_0}^2}-\frac{t^{-6n}\{6nt^{3n-1}V_0(2d_1+d_2-6nt^{3n-1}V_0)+18nt^{6n-2}{V_0}^2(3n-1)+2{d_1}^2+5{d_2}^2+2d_1d_2-6nt^{3n-1}V_0d_1-3nt^{3n-1}V_0d_2\}}{9{V_0}^2}\right]$$

$$\rho^2=\frac{t^{-6n}}{9{V_0}^2\omega(1+2\lambda)}\left[\{6nt^{3n-1}V_0(2d_1+d_2-6nt^{3n-1}V_0)+18nt^{6n-2}{V_0}^2(3n-1)+2{d_1}^2+8{d_2}^2+\right.$$

8d1d2−6nt3n−1V0d1−3nt3n−1V0d2 (21)

(vii) Pressure:

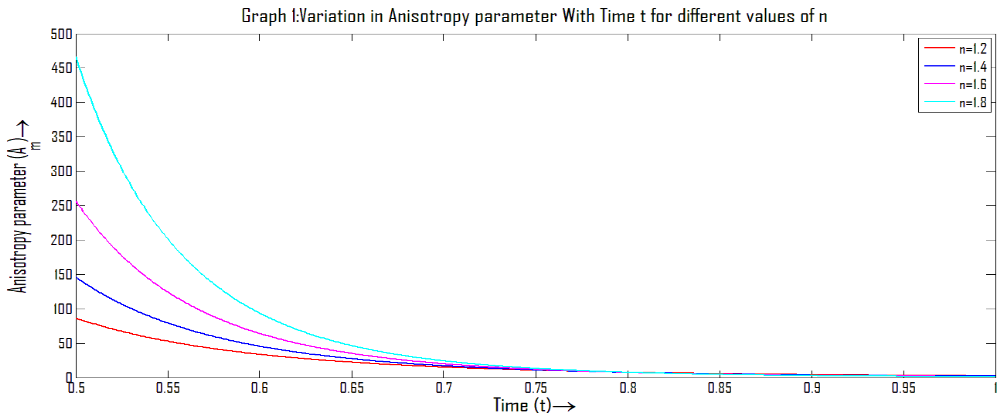
$$p = \omega\rho^2 - \rho$$

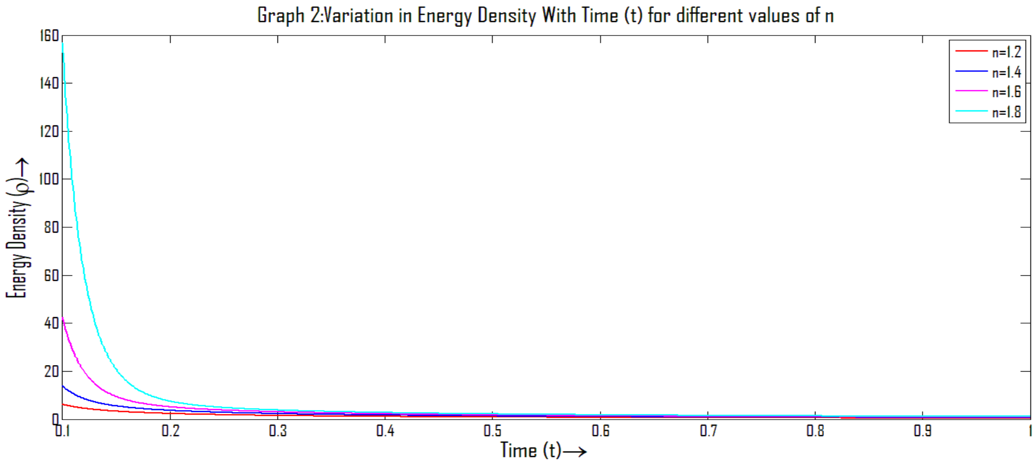
$$p = \frac{t^{-6n}}{9V_0{}^2(1+2\lambda)}[\{6nt^{3n-1}V_0(2d_1 + d_2 - 6nt^{3n-1}V_0) + 18nt^{6n-2}V_0{}^2(3n-1) + 2d_1{}^2 + 8d_2{}^2 + 8d_1d_2 - 6nt^{3n-1}V_0d_1 - 3nt^{3n-1}V_0d_2\}] - \sqrt{\frac{t^{-6n}}{9V_0{}^2\omega(1+2\lambda)}[\{6nt^{3n-1}V_0(2d_1 + d_2 - 6nt^{3n-1}V_0) + 18nt^{6n-2}V_0{}^2(3n-1) + 2d_1{}^2 + 8d_2{}^2 + 8d_1d_2 - 6nt^{3n-1}V_0d_1 - 3nt^{3n-1}V_0d_2\}]} \quad (22)$$

(viii) Cosmological constantΛ:

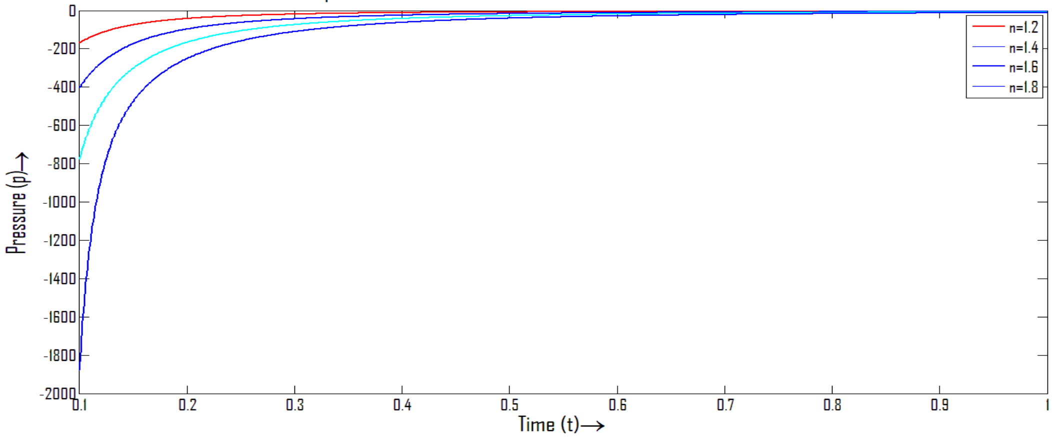
$$\Lambda = -\frac{t^{-6n}}{3V_0{}^2}(d_1{}^2 + d_2{}^2 + d_1d_2) - (1 + 2\lambda)\sqrt{\frac{t^{-6n}}{9V_0{}^2\omega(1+2\lambda)}((6nt^{3n-1}V_0(2d_1 + d_2 - 6nt^{3n-1}V_0) + 18nt^{6n-2}V_0{}^2(3n-1) + 2d_1{}^2 + 8d_2{}^2 + 8d_1d_2 - 6nt^{3n-1}V_0d_1 - 3nt^{3n-1}V_0d_2))} \quad (23)$$

4. Results and Discussion-

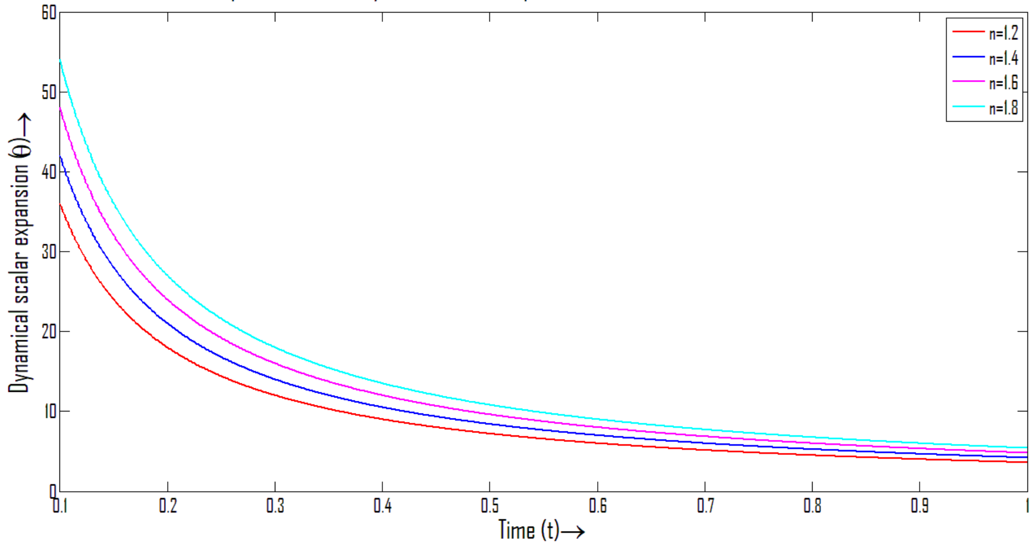




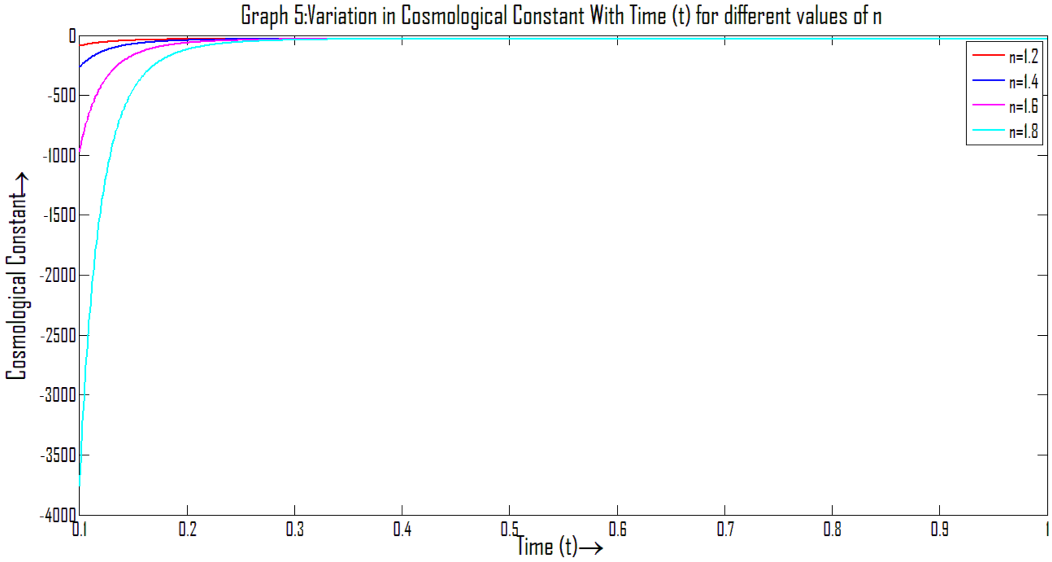
Graph 3:Variation in Pressure With Time (t) for different values of n



Graph 4:Variation in Dynamical scalar expansion With Time (t) for different values of n



Graph 5:Variation in Cosmological Constant With Time (t) for different values of n



The variation of the anisotropy parameter and the energy density against time t is depicted in graphs (1) and (2) and is shown for different values of n as shown in the figures. When we looked into this further, we found that A_m and tend to become ρ as t gets closer and closer to infinity. Both the energy density and the anisotropy parameter go up as the number of particles in the system increases. Based on graphs (3) and (4), it can be shown that when n increases, the pressure and the dynamical scalar expansion both go in opposite directions. The fluctuation of the cosmological constant (Λ) against time $(t)$is depicted by graph (5) for a variety of different values as shown in the graph. It has been noticed that the cosmological constant approaches zero as the passage of time progresses and decreases as the number of cosmic particles in existence increases.

5. Concluding Remarks-

We offer the Bianchi type-I cosmological model in $f(R,T)$ modified gravity. This model takes into account the presence of a cosmic constant as well as a quadratic equation of state (QEoS).In spite of the model's development, its anisotropy remains unchanged. A decrease in energy density (ρ) is seen as a function of time (t), with tending to zero as time progresses, and a similar trend is seen in the pressure p, which is noted to be negative and to tend toward zero as time progresses. It is also well known that the cosmological constant, Λ has a negative value; nevertheless, in this particular instance, we can observe that, for power laws, tends to zero as time goes on.The decelerating phase of expansion is defined by the invariant value of the deceleration parameter q for $t \to 0$. Some parameters used in cosmology, including distance and state finder parameters, were also discussed. We conclude that the state-finder parameters $\{r, s\}$converge on the $(r \to 1, s \to 0)$ value of the ΛCDM model at later times.

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