Environmentally Sustainable Fuzzy Inventory Model for Deteriorating Items for Two Warehouse System Under Inflation and Backorder

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Abstract
In the present scenario of business transaction, the issue of environment is of global concern and researchers have been focussing on this issue at large. Inventory management system involves storages of goods in the form of inventory to be used in fulfilling demand of market. The storage system involves technologies in providing a better environment for enhancing life cycle of products and these highly technological equipment uses energy during operation and release energy which are liable for global warming. Also, during transportation of inventories from one place to other vehicles are used that release carbon emission due to consumption of fuels. Thus, during modelling of an inventory system, it is more concern to minimize the carbon emission and simultaneously energy released so that impact on environment is minimized. Inventory modelling is affected by various factor such as demand pattern and rate of deterioration. In the present paper demand is to be taken as function stock level and inventory model with the concept of two storage system is developed considering time dependent deterioration rate under inflation. The fuzzy model is developed to maximize the profit of the inventory system per unit of time. Numerical example is presented for model validation.

Keywords: Time dependent deterioration rate, stock level dependent demand rate,transportation cost, Carbon emission

1.0 Introduction

There are many researchers developing inventory model incorporating various factors affecting management system of inventory. Modelling have not only issue of managing inventory for demand and supply but also it involves transition of inventory from one place to other while supplying demand. Storage of inventories in business is one of the factors that need to be study deeply along with supply and demand consideration that also involves factors affecting total inventory cost. In the present scenario there is impact of supply chain management on the environment causing human life. In the modern era uses of energy gas been increased with the increase of use of technologies and simultaneously supply chain management producing carbon emission at large due to various activities involved during supply chain. Most of the classical inventory models considers various type of demands pattern such as constant, time dependent, stock dependent and price dependent. Deterioration is one of the most important phenomena of every product during which occurs during storage periods. Some author considered that the utility of goods remains constant during its normal storage periods but in general, certain product, such as food, vegetables, fruits, medicine, blood, fish, alcohol, gasoline and radioactive chemicals deteriorates during normal storage period. The problem of deteriorating inventory has received considerable attention since the establishment of modelling system.

In a highly competitive global market, some time demand become high due in the market either due to uncertainty of market or due to natural calamite and may be due to seasonal requirement. In this case the issue of shortages may arise and to control such situation buffer inventory utilizes to deal with situation. It is not always possible to fulfil complete demand of shortage and therefore partial fulfilment of shortage is of more concern that is known as partial backlogging pattern. In the case of partial backlogging some portion of demand is supplied in the next cycle of inventory system. The shortages of inventory in the market affect the total cost of inventory system and therefore become more concern of modelling.

In addition, with deterioration and shortages of inventory, the present market is also affected due to the inflation in the price of inventory and other activities. The period in which distributor offers discount on items purchased in bulk to get maximum profit and minimize the total inventory cost. During offered discount time period, retailers attracted to order more inventory as compared to normal situation and thus need extra space to store the products purchased in offered period. To store extra inventories purchased one required sufficient space in the market but in metro cities there is lack of space in the main market and highly rented so businessman arrange space for away from the main market place on rental basis apart from his own house of business on temporary basis. This arrangement is termed as two warehouse management system of inventory control. Generally, it is assumed that the rental ware house provides better storage facilities as compared to own ware-house and hence rent is more and deterioration rate is less. To reduce rental cost charged for having utilize long period businessman consume inventory kept in rental house first and then supply is made from own warehouse to fulfil market demand.

Many researchers have developed inventory models using two ware-houses management system. Goswami and Chaudhuri (1992), Bhunia and Maiti (1994); Banerjee and Agrawal (2008) and many more developed inventory models considering system of two-warehouse inventory incorporating various type of demand pattern. In their studies they have considered time varying demand rate either increases or decreases with time.Wu (2001) developed an inventory model incorporating ramp type demand rate, weibull deterioration distribution rate and partially backlogged shortages system. Giri,Jalan and Chaudhary (2003) used an exponential ramp type function to represent demand. Swati Agrawal & Snigdha Banarjee (2011) established a two ware-house inventory model with ramp type demand and partially backlogged shortages. In the recent years researches such as Hui-Ling Yang (2012) & Hui-Ling Yang and Chun-Tao Chang (2013) developed inflationary inventory model with various combinations. R. Kumar and A. K. Vats (2014) developed a deterministic inventory model for single ware house incorporating demand rate as quadratic function of time under inflation. In the most of research papers, researchers have considered that deterioration rate in the both ware-houses are constant. K.V.S. Sarma (1983), developed a deterministic inventory model for deteriorating items with two level of storage and an optimum release rule. T.A. Murdeshwar, Y,S. Sathe(1985) introduced a lot size model with two level of storage. U.Dave (1988) worked on the EOQ models with two level of storage. T.P.M.Pakala and K.K.Archary (1992a) developed a discrete time inventory model for deteriorating items with two warehouses system of inventory management. In the recent years two warehouse inventory model for deteriorating item with exponential demand rate and permissible delay in payment has been developed by Naresh Kumar Kaliraman et. al. (2016). Optimal inventory model for a two-warehouse inventory model under time dependent demand quadratic demand rate is established by M. Srinivasa Reddya and R. Venkateswarlub (2018).

In the case of uncertainty to deal with fluctuating situation, Zadeh L.A. (1965) first introduced word fuzzy and discussed a set theory to deal such situations. Membership and non-membership of an element in a set is discussed by Bellman and Zadeh L.A. (1970). Decision systems for inventory management and production planning is introduced by Silver and Peterson (1985). Two-storage inventory model for deteriorating items with price dependent demand and shortages under partial backlogged in fuzzy approach is developed by Susanta Kumar Indrajitsingha et. al. (2019). A sustainable production-inventory model with imperfect quality under preservation technology and quality improvement investment is developed by Arash Sepehri et. al.(2021). Advance and delay in payments with the price-discount inventory model for deteriorating items under capacity constraint and partially backlogged shortages is developed by Avijit Duary at.el.(2022). An Inventory Model for Non-Instantaneously Deteriorating Items with Nonlinear Stock-Dependent Demand, Hybrid Payment Scheme and Partially Backlogged Shortages is developed by Md Al-Amin Khan et. al. (2022).

Indrajitsingha S. K. et al. (2019) in a two storage inventory model for deteriorating items have introduced price dependent demand and shortages under partial backlogged in fuzzy approach and Graded Mean Integration Representation (GMIR) method is used to defuzzyfy fuzzy model. Kaur P. and Kumar V. (2021) has developed fuzzy inventory model under two storage system for deteriorating items with selling price dependent demand rate and shortages and included vague nature of holding cost. The environmental issue is of great concern in the modern era and need to be discussed at large. Also, motivated by above papers, and incorporating present issues of energy and carbon emission this paper is enriched under inflation for two storage system with partially backlogging shortages which is decreasing rate of waiting time The model is solved numerically by maximizing profit function for the total cycle length.

2.0 ASSUMPTIONS AND NOTATION

The mathematical model of the two-storage inventory problem is based on the following assumption and notations.

2.1 Assumptions

1. Planning horizon is infinite.
2. Replenishment rate is instantaneous.

3. Rented Warehouse unlimited Storage capacity.
4. Initial inventory level is zero and lead time is negligible.
5. Partially backlogged shortages is allowed in the next cycle.
6. In OW and RW, deterioration rate is time dependent given as $x(t) = a\ t$ and $y(t) = b\ t$ respectively.
7. The holding cost in RW and OW is differ by a constant and RW charges more than expenses occur in OW.
8. The deteriorated units neither repaired nor replaced during cycle period.
9. Instant deterioration occurs during storage system.
10. Inventory system consider a single item and the demand rate is function of on hand inventory level i.e. $g = (S_i(t)$
11. Transportation cost is function of distance travelled from RW to OW warehouse.i.e. $t^c(d) = k\ d$ where k is constant scale factor of distance and d is distance from RW to OW
12. Carbon emission releases during holding products in stock, deterioration and transportation activities throughout cycle period.

2.2 Notations

The following notation is used throughout the paper:

Demand rate（units/unit time）stock dependent and given as

$$D = \begin{cases} \alpha + \beta\, g & \text{if } g > 0 \\ \alpha & \text{if } g < 0 \end{cases}$$

R_1	Inventory level in RW at $t = 0$
W_f	Finite capacity of OW.
$x(t)$	Variable deterioration rate in OW given as $x(t) = a\ t\ where\ 0 < a < 1$.
$y(t)$	Variable deterioration rate in RW given as $y(t) = b\ t\ where\ 0 < b < 1$.
$B_{bac}(t)$	Partial backordering rate which decreases exponentially for waiting time and Defined as $B_{bac}(t)= e^{-\sigma t}$
M_{max}	Maximum inventory backlogged
A	Ordering cost per order
h_{co}	Holding cost per unit per unit time in RW
h_{cr}	Holding cost per unit per unit time in OW
h_{sc}	Inventory shortages cost

h_{lc}	Inventory lost sales cost
c_{ip}	Cost of inventory purchased
h_{dcr}	Inventory deterioration cost in RW
h_{dco}	Inventory deterioration cost in OW
$\tilde{A}$	Fuzzy Ordering cost per order
$\tilde{h}_{hcr}$	Fuzzy Holding cost per unit per unit time in RW
$\tilde{h}_{hco}$	Fuzzy Holding cost per unit per unit time in OW
$\tilde{h}_{sc}$	Fuzzy Inventory shortages cost
$\tilde{h}_{lc}$	Fuzzy Inventory lost sales cost
$\tilde{h}_{dcr}$	Fuzzy Inventory deterioration cost in RW
$\tilde{h}_{dco}$	Fuzzy Inventory deterioration cost in OW
c^e	Cost of emission per unit per unit of time
e^h	Quantity of emission produced per unit per unit of time
c_v	Vehicle capacity
c^{et}	Cost of emission due to transportation
Q_{max}	The maximum order quantity for a cycle length.
$S_i(t)$	Maximum inventory level at any time for i=1, 2, 3 etc. in RW and OW.
t_1	Length of time when inventory vanishes in RW
t_2	Length of time when inventory vanishes in OW
T	Length of the ordering cycle.
r	Inflation rate.
$TP^{fun}(t_1, t_2, , T)$	Total inventory profit function per unit of time per cycle.

Preliminaries:

3.0 Fuzzy Set and Fuzzy Numbers

To deal with uncertainties there is need to define fuzzy set and fuzzy numbers together the method used to defuzyfy fuzzy numbers. The following definitions are given as under

Definition-3.1: Let $B^{\sim}$ is a fuzzy set in a given universal set χ then it is characterised by a membership function $\lambda_{B^{\sim}}(x)$ which associated with each element x a real number in the interval [0 1].The function value $\lambda_{B^{\sim}}(x)$ termed as the grade of membership of x in $B^{\sim}$.

Definition-3.3: Any convex normalized fuzzy subset B˜ on ℜ (Set of real numbers) with membership function $\lambda_{B^{\sim}}\ \Re \rightarrow [0\ \ 1]$ is called fuzzy number. The fuzzy set number $a^{\sim}$ is called Triangular fuzzy number if it is determined by the crisp numbers $(a_{11}\quad a_{12}\quad a_{13})$ where $a_{11} < a_{12} < a_{13}$ with the membership function of the form

$$\lambda_{a^{\sim}}(x) = \begin{cases} \dfrac{x - a_{11}}{a_{12} - a_{11}}, & a_{11} \le x \le a_{12} \\ \dfrac{a_{13} - x}{a_{13} - a_{12}}, & a_{12} \le x \le a_{13} \\ 0, & \text{otherwise} \end{cases}$$

3.4 Signed Distance: Let B˜ be the fuzzy set defined on the ℜ (Set of real numbers), then the signed distance of B˜. Let a˜ = $(a_{11}\ \ a_{12}\quad a_{13})$ be a Triangular fuzzy number, then fuzzy expected value of b˜ using signed distance method is defined by

$$E\,(a^{\sim}) = \frac{(a_{11} + 2\ a_{12} + a_{13})}{4}$$

4.0 Development of Mathematical Model

On the arrival of inventory in the beginning, a fixed amount of W units are kept at OW according to its capacity and the rest amount of inventory R^2 are kept into ranted warehouse. The evolution of stock level in the system is depicted in **Figure-1.** First the amount of inventory kept in RW is consumed to minimize the rent of RW and hence inventory cost, thereafter demand of customers is fulfilled by supplying inventory from OW. In the RW, the level of inventory depleted due to combined effect of demand and deterioration and differential equation governing this situation is given as

$$\frac{\mathrm{d}\,S_1(t)}{\mathrm{dt}} = -g - x(t)\,\{S_1(t\} \quad ; \qquad 0 \le t \le t_1 \qquad (1)$$

With b.c. $S_1(t = 0$ at $t = t_1$. The solution of equation (1) is

$$S_1(t) = \left(\alpha\left\{\left(\mathrm{t}_1 - \frac{\beta}{2}t_1^2 + \frac{a}{6}t_1^3\right) - \left(\mathrm{t} - \frac{\beta}{6}t^2 + \frac{a}{6}t^3\right)\right\}\right)e^{(\beta t - \frac{at^2}{2})} \qquad (2)$$

When inventory vanishes at RW, the demand of customers is fulfilled by supplying goods from OW and inventory of OW in the interval $[0\quad \mathrm{t}_1]$ reduces due to varying deterioration rate only and during the interval $[\mathrm{t}_1\ \ \mathrm{t}_2]$, the inventory reduces due to combined effect of both variable deterioration rate and demand. The situations are governed by the following differential equations

$\frac{dS_2(t)}{dt} = -y(t)\{S_2(t)\}$ $\quad 0 \le t \le t_1$ (3)

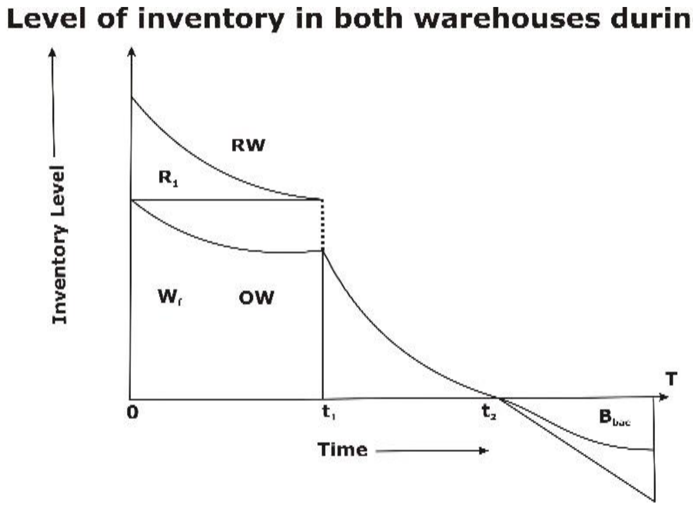

Figure-1: Representing level of inventory with respect to cycle length

$\frac{dS_3(t)}{dt} = -g - y(t)\{S_3(t)\}$; $\quad t_1 \le t \le t_2$
(4)

With b. c. $S_2(0) = W_f$ at $t = 0$ and $S_3(t_2) = 0$ at $t = t_2$. The solution of (3) & (4) are resp.

$S_2(t) = W_f \; e^{-\frac{b\,t^2}{2}}$; (5)

$S_3(t) = \left(\alpha\left\{\left(t_2 - \frac{\beta}{2}t_2^2 + \frac{\beta}{6}t_3^3\right) - \left(t - \frac{b}{6}t^2 + \frac{\beta}{6}t^3\right)\right\}\right) e^{(bt - \frac{\beta t^2}{2})}$;
(6)

Since at time epoch $t = t_2$ the level of inventory vanishes and due to continuous demand shortages occurs till the next replenishment and shortages backordered partially this is governed by the differential equation

$\frac{d\,S_{sc}(t)}{dt} = -\,B_{bac}(t)(T - t)$

$= -e^{-\sigma(T-t)}$; $\quad t_2 \le t \le T$ (7)

Solution of above equation with B.C. $S_{sc}(t_2) = 0$ at $t = t_2$ is given as

$S_{sc}(t) = \frac{a}{2}(\, e^{-\sigma(T-t_2)} - \, e^{-\sigma(T-t)})$
(8)

Maximum inventory backlogged at $t = T$ is

$$B_{bac} = \frac{\alpha}{2}\left(e^{-\sigma(T-t_2)} - 1\right) \tag{9}$$

Since, initially inventory level in RW is $R_1 = S_1(0)$, therefore we obtain from eq. (2)

$$R_1 = \left(\alpha\left\{\left(t_1 - \frac{\beta}{2}t_1^2 + \frac{\alpha}{6}t_1^3\right)\right\}\right) \tag{10}$$

Continuity in OW at $t = t_1$ gives $I_2(t_1) = I_3(t_1)$ therefore

$$W_f = \left(\alpha\left\{\left(t_2 - \frac{b\beta}{2}t_2^2 + \frac{b}{6}t_3^3\right) - \left(t_1 - \frac{\alpha}{2}t_1^2 + \frac{b}{6}t_1^3\right)e^{\beta t_1}\right\}\right) \tag{11}$$

Now the maximum ordered quantity per cycle is obtained as

$$Q_{max} = R_1 + W_f + B_{bac} \tag{12}$$

Thus, the total present worth inventory cost during the cycle length, consist of the following costs elements

- Ordering cost (CO)
- Inventory Purchase Cost (IPC)
- Inventory holding cost in Rented Warehouse (IHR)
- Inventory holding cost in Own Warehouse (IHO)
- Deterioration cost in Rented Warehouse (DCR)
- Deterioration cost in Own Warehouse (DCO)
- Shortages cost (SHC)
- Lost sales cost (LSC)
- Transportation Cost (TRC)
- Emission cost due to holding inventory (ECI)
- Emission cost due to deteriorating inventory (ECD)
- Emission cost due to transportation activities (ETA)
- Sales revenue (SR)

The above cost are given as under

Ordering cost is

$$CO = Ae^{-rt}$$

Inventory holding cost in RW is

$$IHR = h_{hcr}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right)$$

Inventory holding cost in OW is

$$\mathrm{IHO} = \mathrm{h}_{hco}\left(\int_0^{t_1} e^{-rt}S_2(t)d\mathrm{t} + \int_{t_2}^{T} e^{-rt}S_3(t)d\mathrm{t}\right)$$

Total cost of inventory deteriorated in RW during storage period

$$\mathrm{DCR} = \mathrm{h}_{dcr}\left\{e^{-rt}\,\mathrm{R}_1 - \int_0^{t_1} e^{-rt}D\;d\mathrm{t}\right\}$$

Total cost of inventory deteriorated in OW during storage period

$$\mathrm{DCO} = \mathrm{h}_{dco}\left\{e^{-rt}\mathrm{W}_\mathrm{f} - \int_{t_1}^{t_2} e^{-rt}D\;d\mathrm{t}\right\}$$

Total cost of inventory shortages during $[\mathrm{t}_2\ \mathrm{T}]$ is

$$\mathrm{SHC} = \mathrm{c}_{\mathrm{sc}}\int_{t_2}^{T} e^{-rt}\,\{S_{sc}(t)\}d\mathrm{t}$$

Total cost of inventory lost sales during $[\mathrm{t}_2\ \mathrm{T}]$ is

$$\mathrm{LSC} = \mathrm{c}_{\mathrm{lc}}\int_{t_2}^{T} e^{-rt}(1 - \mathrm{B}_{\mathrm{bac}}\,(\mathrm{T}-\mathrm{t}))d\mathrm{t}$$

Inventory purchase Cost

$\mathrm{IPC} = c_{ip}\ \mathrm{Q}_{\max}$

Cost incurred against Green House Gas emission produced in warehouse due to hold stock during complete cycle in RW

$$COE_{RW} = \frac{c^e e^h}{T}\left(\int_0^{t_1} e^{-rt}\,S_1(t)d\mathrm{t}\right)$$

Cost incurred against Green House Gas emission produced in warehouse due to hold stock during complete cycle in OW

$$COE_{OW} = \frac{c^e\ e^h}{T}\left(\int_0^{t_1} e^{-rt}S_2(t)d\mathrm{t} + \int_{t_2}^{T} e^{-rt}S_3(t)d\mathrm{t}\right)$$

Cost of transportation incurred during complete cycle length

$$CO_{tr} = \frac{1}{T}\left\{\frac{(Q_{max} - W_f)}{v_c}d\right\}$$

Cost incurred against Green House Gas emission produced due to transportation activities during complete cycle in OW

$$EM_{Cost} = \frac{c^{te}}{T}\left\{\frac{(Q_{max} - W_f)}{v_c}d\right\};$$

Average inventory cost of the system per cycle length

$TP^{fun}(t_1,t_2,,T) = [Revenue\ earned - (Ordering\ Cost + Purchasing\ cost + Holding\ Cost\ ijn\ RW + Holding\ cost\ in\ OW + detreioration\ Cost\ in\ RW + Deterioration\ Cost\ in\ OW + Emission\ Cost\ in\ RW\ due\ to\ holding\ stock + Emission\ Cost\ in\ OW\ due\ to\ holding\ stock + Emission\ cost\ due\ to\ deterioration\ in\ RW + Emission\ cost\ due\ to\ deterioration\ in\ OW + Transportaion\ Cost + Emission\ cost\ due\ to\ transportation)]$

$TP^{fun}(t_1,t_2,,T) = [(\mathrm{CO} + IPC + \mathrm{IHR} + IHO + DCR + DCO + SHC + LHC + COE_{RW} + COE_{OW} + CO_{tr} + EM_{Cost})]$

Hence the total relevant inventory cost per unit of time during cycle length is given by

$$TP^{fun}(t_1,t_2,,T) = \frac{1}{T}\Big[\mathrm{A}e^{-rt} + c_{ip}\,\mathrm{Q}_{\max} + \mathrm{h}_{hcr}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \mathrm{h}_{hco}\left(\int_0^{t_1} e^{-rt} S_2(t)dt + \int_{t_2}^{T} e^{-rt} S_3(t)dt\right) + \mathrm{h}_{dcr}\left\{e^{-rt}\,\mathrm{R}_1 - \int_0^{t_1} e^{-rt} D\,dt\right\} + \mathrm{h}_{hco}\left\{e^{-rt}\mathrm{W_f} - \int_{t_1}^{t_2} e^{-rt} D\,dt\right\} + \mathrm{c_{sc}}\int_{t_2}^{T} e^{-rt}\{S_{sc}(t)\}dt + \mathrm{c_{lc}}\int_{t_2}^{T} e^{-rt}\big(1 - \mathrm{B_{bac}}(\mathrm{T}-t)\big)dt + \frac{c^e e^h}{T}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \frac{c^e e^h}{T}\left(\int_0^{t_1} e^{-rt} S_2(t)dt + \int_{t_2}^{T} e^{-rt} S_3(t)dt\right) + \frac{1}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\} + \frac{c^{te}}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\}\Big] \quad (13)$$

5.0 Optimality condition

The optimal problem can be formulated as

Minimize $TP^{fun}(t_1,t_2,,T)$

Subject to: $(t_1 > 0, t_2 > 0, T > 0)$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial\, TP^{fun}(t_1,t_2,,T)}{\partial t_1}=0; \quad \frac{\partial\, TP^{fun}(t_1,t_2,,T)}{\partial t_2}=0; \quad \frac{\partial TP^{fun}(t_1,t_2,,T)}{\partial \mathrm{T}} = 0; \quad (14)$$

Solving equation (14) for $\mathrm{t}_1, \mathrm{t}_2$ and T, the optimal value of decision variables can be obtained and denoted as $\mathrm{t}_1^*, \mathrm{t}_2^*$ and T^*. Using values of decision variables obtained as above, average inventory cost from equation (13) may calculated.

6.0 Numerical examples:

To analyse the model, we consider following examples in case of crisp and fuzzy model in appropriate unit given in example-1 and example-2 respectively.

Example-1: Consider the following set of values of parameters: $\alpha = 50, \beta = 0.25,\ A = 1000, r = 0.06, h_{hcr} = 6.0, h_{hco} = 3.0,\ h_{hco} = 15,\ h_{dcr} = 18,\ c_{sc} = 15,\ c_{lc} = 25\ ;\ a = 0.003,\ b = 0.005, \sigma = 0.6, W_f = 100,\ c^e = 0.30\ \ e^h = 0.15;\ c^{et} = 0.50; c_v = 200\ ;\ c_{ip} = 90$ in an appropriate unit of parameters. The optimal results obtained from the model itself and for the other cases discussed in the model as well, have shown in Table-1.Sensitivity analysis can be performed by changing the value of one parameter at a time and keeping values of other parameters unchanged. Result is shown in Table-1.

Table-1

Cost function	t_1^*	t_2^*	T^*	Total relevant inventory cost
$\Pi(t_1, t_2, T)$	1.5334	2.8987	3.7866	1091.47

Since the model represented developed and represented by equation (1) is highly non-linear and not easy to prove convexity of the model algebraically. Therefore, using values of decision variables and taking cycle length as constant at a time convexity of the model is represented in Figure-2 with the help of 3D graph drawn by Mathematica-9.0 Software.

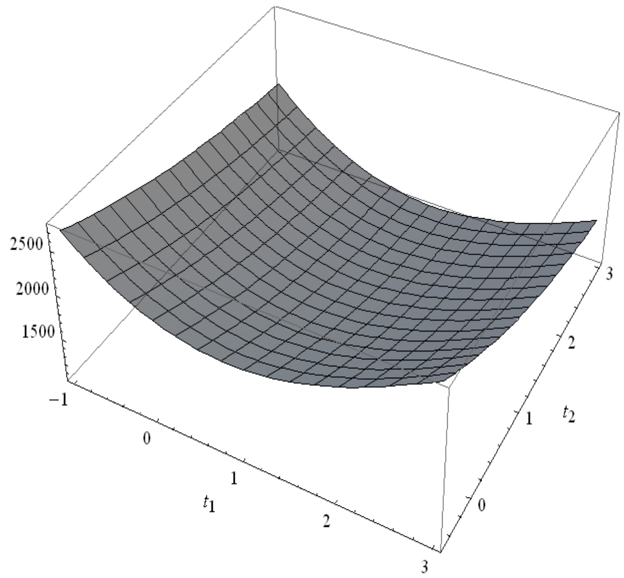

Figure-2: Representing convexity of the two ware-house crisp model

With the help of 2D graph drawn using Mathematica-9.0 minimum average inventory cost with respect to cycle length is shown in Figure-3.

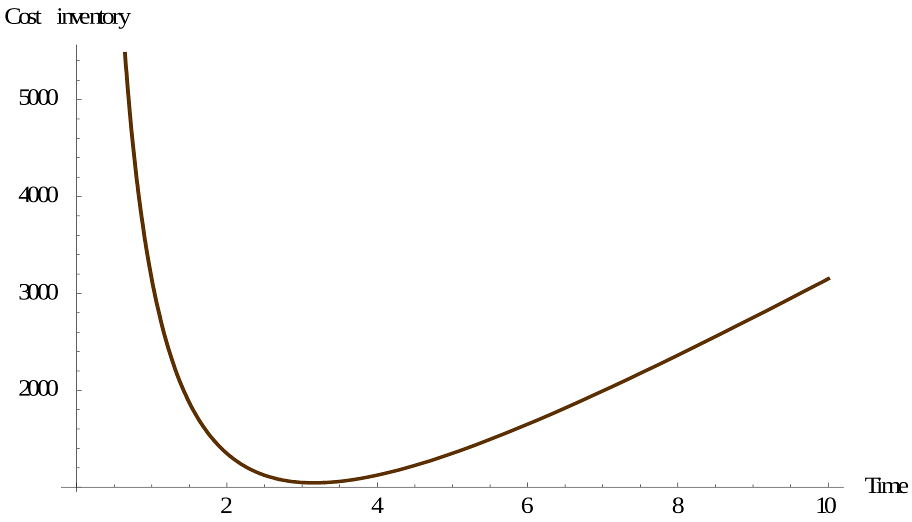

Figure-3: Graph representing cycle length (In Time) verses Average Inventory cost

7.0 Fuzzy Model

After development of crisp model a corresponding fuzzy model has been developed and example is given to validate the model and results are compared in the observation section.

$$\tilde{T}P^{fun}(t_1, t_2, , T) = [\tilde{C}O + IPC + \tilde{I}HR + \tilde{I}HO + \tilde{D}CR + \tilde{D}CO + \tilde{S}HC + \tilde{L}HC + COE_{RW} + COE_{OW} + CO_{tr} + EM_{Cost})]$$

Hence the total relevant fuzzy inventory cost per unit of time during cycle length is given by

$$\tilde{T}P^{fun}(t_1, t_2, , T) = \frac{1}{T}\Big[\tilde{A}e^{-rt} + c_{ip}\, Q_{\max} + \tilde{h}_{hcr}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \tilde{h}_{hco}\left(\int_0^{t_1} e^{-rt} S_2(t)dt + \int_{t_2}^{T} e^{-rt} S_3(t)dt\right) + \tilde{h}_{hdr}\left\{e^{-rt}\, R_1 - \int_0^{t_1} e^{-rt} D\, dt\right\} + \tilde{h}_{hdo}\left\{e^{-rt} W_f - \int_{t_1}^{t_2} e^{-rt} D\, dt\right\} + \tilde{c}_{sc}\int_{t_2}^{T} e^{-rt}\, \{S_{sc}(t)\}dt + \tilde{c}_{lc}\int_{t_2}^{T} e^{-rt}\big(1 - B_{bac}\,(T - t)\big)dt + \frac{c^e e^h}{T}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \frac{c^e\, e^h}{T}\left(\int_0^{t_1} e^{-rt} S_2(t)dt + \int_{t_2}^{T} e^{-rt} S_3(t)dt\right) + \frac{1}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\} + \frac{c^{te}}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\}\Big] \quad (12)$$

Where $\tilde{A} = (A_1, A_2, A_3)$; $\tilde{h}_{hcr} = (h_{hcr1}, h_{hcr2}, h_{hcr3})$; $\tilde{h}_{hco} = (h_{hco1}, h_{hco2}, h_{hco3})$

$\tilde{h}_{hdr} = (h_{hdr1}, h_{hdr2}, h_{hdr3})$; $\tilde{h}_{hdo} = (h_{hdo1}, h_{hdo2}, h_{hdo3})$; $\tilde{c}_{sc} = (c_{sc1}, c_{sc2}, c_{sc3})$; $\tilde{c}_{lc} = (c_{lc1}, c_{lc2}, c_{lc3})$

To defuzzyfy equation signed distance method is used

$$defTP^{fun}(t_1, t_2, , T) = \frac{1}{4\,T}[TP^{fun1}(t_1, t_2, , T) + 2\, TP^{fun2}(t_1, t_2, , T) + TP^{fun3}(t_1, t_2, , T)\ ]$$

Where

$TP^{fun1}(t_1, t_2,, T) = \frac{1}{T}\Big[A_1 e^{-rt} + c_{ip}\, \mathrm{Q_{max}} + \mathrm{h}_{hcr1}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \mathrm{h}_{hco1}\left(\int_0^{t_1} e^{-rt} S_2(t)d\mathrm{t} + \int_{t_2}^{T} e^{-rt} S_3(t)d\mathrm{t}\right) + \mathrm{h}_{hdr1}\left\{e^{-rt}\, \mathrm{R_1} - \int_0^{t_1} e^{-rt} D\, d\mathrm{t}\right\} + \mathrm{h}_{hdo1}\left\{e^{-rt}\mathrm{W_f} - \int_{t_1}^{t_2} e^{-rt} D\, dt\right\} + \mathrm{c_{sc1}} \int_{t_2}^{T} e^{-rt}\, \{S_{sc}(t)\}dt + \mathrm{c_{lc1}} \int_{t_2}^{T} e^{-rt}\big(1 - \mathrm{B_{bac}}\,(\mathrm{T} - \mathrm{t})\big)dt + \frac{c^e e^h}{T}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \frac{c^e\, e^h}{T}\left(\int_0^{t_1} e^{-rt} S_2(t)dt + \int_{t_2}^{T} e^{-rt} S_3(t)dt\right) + \frac{1}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\} + \frac{c^{te}}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\}\Big]$

$TP^{fun2}(t_1, t_2,, T) = \frac{1}{T}\Big[A_2 e^{-rt} + c_{ip}\, \mathrm{Q_{max}} + \mathrm{h}_{hcr2}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \mathrm{h}_{hco2}\left(\int_0^{t_1} e^{-rt} S_2(t)d\mathrm{t} + \int_{t_2}^{T} e^{-rt} S_3(t)d\mathrm{t}\right) + \mathrm{h}_{hdr2}\left\{e^{-rt}\, \mathrm{R_1} - \int_0^{t_1} e^{-rt} D\, d\mathrm{t}\right\} + \mathrm{h}_{hdo2}\left\{e^{-rt}\mathrm{W_f} - \int_{t_1}^{t_2} e^{-rt} D\, dt\right\} + \mathrm{c_{sc2}} \int_{t_2}^{T} e^{-rt}\, \{S_{sc}(t)\}dt + \mathrm{c_{lc1}} \int_{t_2}^{T} e^{-rt}\big(1 - \mathrm{B_{bac}}\,(\mathrm{T} - \mathrm{t})\big)dt + \frac{c^e e^h}{T}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \frac{c^e\, e^h}{T}\left(\int_0^{t_1} e^{-rt} S_2(t)dt + \int_{t_2}^{T} e^{-rt} S_3(t)dt\right) + \frac{1}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\} + \frac{c^{te}}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\}\Big]$

$TP^{fun3}(t_1, t_2,, T) = \frac{1}{T}\Big[A_3 e^{-rt} + c_{ip}\, \mathrm{Q_{max}} + \mathrm{h}_{hcr3}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \mathrm{h}_{hco3}\left(\int_0^{t_1} e^{-rt} S_2(t)d\mathrm{t} + \int_{t_2}^{T} e^{-rt} S_3(t)d\mathrm{t}\right) + \mathrm{h}_{hdr3}\left\{e^{-rt}\, \mathrm{R_1} - \int_0^{t_1} e^{-rt} D\, d\mathrm{t}\right\} + \mathrm{h}_{hdo3}\left\{e^{-rt}\mathrm{W_f} - \int_{t_1}^{t_2} e^{-rt} D\, dt\right\} + \mathrm{c_{sc3}} \int_{t_2}^{T} e^{-rt}\, \{S_{sc}(t)\}dt + \mathrm{c_{lc3}} \int_{t_2}^{T} e^{-rt}\big(1 - \mathrm{B_{bac}}\,(\mathrm{T} - \mathrm{t})\big)dt + \frac{c^e e^h}{T}\left(\int_0^{t_1} e^{-rt}\, S_1(t)dt\right) + \frac{c^e\, e^h}{T}\left(\int_0^{t_1} e^{-rt} S_2(t)dt + \int_{t_2}^{T} e^{-rt} S_3(t)dt\right) + \frac{1}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\} + \frac{c^{te}}{T}\left\{\frac{(Q_{max} - W_f)}{v_c} d\right\}\Big]$

8.0 Optimality condition

The optimal problem can be formulated as

$$\text{Minimize } \tilde{T}P^{fun}(t_1, t_2,, T)$$

$$\text{Subject to: } (t_1 > 0, t_2 > 0, T > 0)$$

To find the optimal solution of the equation the following condition must be satisfied

$$\frac{\partial \tilde{T}P^{fun}(t_1,t_2,,T)}{\partial t_1} = 0; \quad \frac{\partial \tilde{T}P^{fun}(t_1,t_2,,T)}{\partial t_2} = 0; \quad \frac{\partial \tilde{T}P^{fun}(t_1,t_2,,T)}{\partial \mathrm{T}} = 0; \qquad (16)$$

Example-2: Consider the following set of values of parameters: $\alpha = 50, \beta = 0.25, \tilde{A} = (900, 1000, 1100)$ $r = 0.06, \tilde{h}_{hcr} = (5.0,6.0,7.0); \tilde{h}_{hco} = (2.0,3.0,4.0); \tilde{h}_{hdr} = (14,15,16); \tilde{h}_{hdo} = (16,18,20); \tilde{c}_{sc} = (14,15,16); \tilde{c}_{lc} = (24,25,26); a = 0.003, b = 0.005, \sigma = 0.6, W_f = 100, c^e = 0.30 \quad e^h = 0.15; c^{et} = 0.50; c_v = 200; c_{ip} = 90$ in an appropriate unit of parameters. The optimal results obtained from the model itself and for the other cases discussed in the model as well, have shown in Table-1.Sensitivity analysis can be performed by changing the value of one parameter at a time and keeping values of other parameters unchanged.

Table-1

Cost function	t_1^*	t_2^*	T^*	Total relevant inventory cost
$\Pi(t_1, t_2, T)$	0.5125	1.8256	2.7248	1074.19

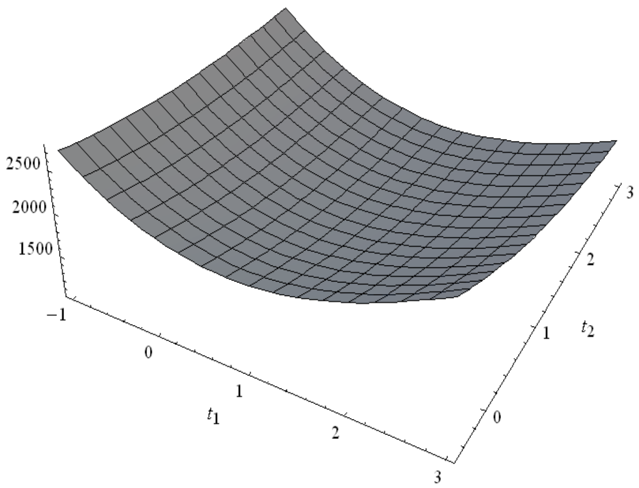

Figure-5: Representing convexity of the two ware-house crisp model

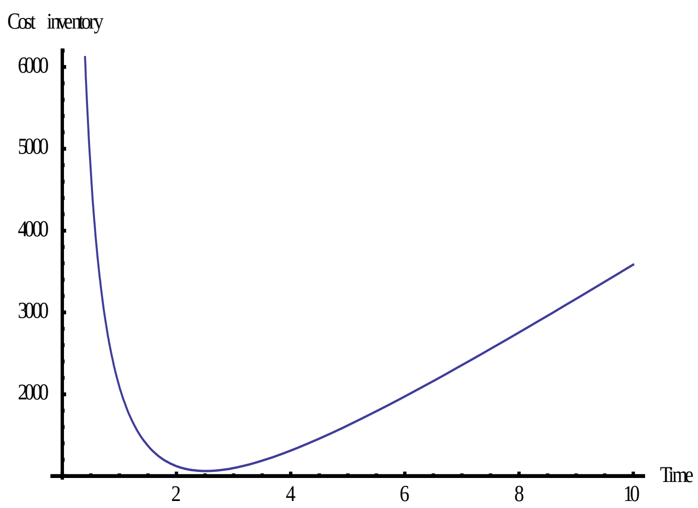

Figure-6: Graph representing cycle length (In Time) verses Average Inventory cost

From Table-1 and Table-2, the following observation is being listed:

1) The model with developed in fuzzy environment has low cost as compared to crisp model also fuzzy model is more supportive in vide selection range of parameters considered as of fuzzy nature.
2) The total length of business is lower in fuzzy case as compared to crisp one.
3) The convexity of proposed crisp inventory model with fixed value of $T^* = 3.7866$ is shown by graph depicted in Figure-2.
4) The convexity of proposed crisp inventory model with fixed value of $T^* = 2.7248$ is shown by graph depicted in Figure-5.
5) The convexity of inventory-time graph depicted in Figure-3 in case of crisp model shows that there exists a unique point where, average inventory cost is minimum.
6) The convexity of inventory-time graph depicted in Figure-6 in case of fuzzy model shows that there exists a unique point where, average inventory cost is minimum.
7) Model has incorporated emission produced from inventories kept during warehouses and also during transportation of vehicles and a emission cost has been charged to reduce the level of inventory by investing this amount in technology.

Conclusions: In this paper, deterministic inventory model incorporating inflation and partial backordering policy with time dependent deterioration rate and stock dependent demand has been developed. Two warehouse storage policy has been used in storing excess quantity of goods purchased under consideration that own warehouse has limited capacity of space. The optimization technique is used to derive the optimum replenishment policy i.e., to minimize the total relevant cost of the inventory system. Numerical example is presented taken in appropriate unit for validation on model efficacy. The crisp model developed has been extended to the fuzzy model where more wide range is available to consider the values of parameter which helps to deal uncertain situation of present or future demand. Fuzzy model has been defuzyfied with the help of signed distance method and using this method average inventory cost has been calculated in the case of fuzzy model. Further this paper may be extended by incorporating other types of demand pattern such as exponentially increasing or decreasing demand with aim of reduction of carbon emission which is more concern of present era of research and another extension of this model may be done for a bulk release pattern and consideration of displaying the stock and incorporation of advertising activities also enriched the model applicability when incorporated during extension of present model.

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