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Tripled fixed Point Theorem for Mapping in Partially Ordered S-Metric Spaces

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Abstract

Functional analysis is considered as one of the most significant branches of Mathematics. Development of modern physics and functional analysis go simultaneously. The laws of both quantum field theory and mechanics are closely connected to the structure of functional analysis. It can be said at the same time that there observed some significant impact and connection between such physics theoretical frameworks and theauthenticating embodimentof problems and the methodology to find solutions to these problems related to functional analysis. The fixed point theory has been emerged from this branch which focuses on the different utilizing points. It performs a very important part in a number of disciplines like economics, differential equations, functional analysis, artificial intelligence, optimal control, logic programming and topology. Ran and Reurings have considered,“the fixed point theorem for mapping the contraction type mappings in partially ordered metric spaces”. The new concept of triplefixed- point was induced for mapping by Borcut and Berinde very recently. It attained some unique and significant theorems mappings in partially ordered metric spaces. The researcher is making an attempt through currentresearch workto signify“A tripled fixed point theorem for mapping in partially ordered S-metric space”.

Keywords: Tripled Fixed Point, Artificial Intelligence, Functional Analysis, Partially Ordered, Contractive Type Mapping and Metric Space.

1. Introduction

At presentFixed-Point theorem is considered as a very lively area to conduct various research works. With the emergence of fixed point theorem in 1922 by Banach, a number of researchers gave their contribution in the field. This theorem has largely become the area of research in terms of

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generalization now-a-days.In 2004, it is Ran and Reuring has "Investigated the existence of fixed-points in ordered metric spaces." Subsequently, a number offindings have been proved on uniqueness and existence of fixed point in partially ordered metric spaces. Fixed point theory is considered as a process to mix up of topology,geometry, and analysis. The space X is considered to hold the fixed point value for mapping$T_X \to X$ if there exist $x \in X$ sothat$T_x \to x$. In the past few times, this theory is adopted as a highly significanttechnique to study non-linear practices.

Various researches have indicated some familiar findings in partially ordered metric space. Berinde and Borcut have proved many theorems in partially ordered metric space concerned with triple fixed point by its introduction. Variation, enhancement and domain extension to a more common space remains one of the vigorous investigations in fixed point segments. Some researchers have made efforts to generalizethe results of metric space in numerousmeans.

Gahler has introduced the idea of a two metric space in 1963 which is as follow:

Definition1 The triple $(X, d, \leq)$ is called partially ordered metric spaces, if $(X, \leq)$ is a partially ordered set and (X, d) is a metric space.

Definition 2 If (X, d) is a complete metric space, then the triple $(X, d, \leq)$ is called complete partially ordered metric spaces.

Definition 3:A partially ordered metric space $(X, d, \leq)$ is called ordered complete if for each convergent sequence $\{x_n\}_{n=0}^{\infty} \subset X$,either of the following condition holds:

- if x_n is a non-increasing sequence in X such that $x_n \to x$ implies $x \leq x_n$, for all $n \in N$ that is, $x = \inf\{x_n\}$
- if x_n is a non-decreasing sequence in X such that $x_n \to x$ implies $x \leq x_n$, for all $n \in N$ that is, $x = \sup\{x_n\}$.

Ex. 1: Consider $\Re$ be the real line. Then $S(x, y, z) = |x - z| + |y - z|$ for all $x, y, z \in R$ is not S-metric. This $S -$ metric is termed general$S -$ metric on$\mathbb{R}$.

Ex. 2: Consider$X = \Re^2$and d an ordinary metric on X,then

$S(x, y, z) = d(x, y) + d(x, z) + d(y, z)$ is an $S -$ metric on X.

Ex.3 : Consider$X = \mathbb{R}n$ and$||\,.\,||$a norm on X,then

$S(x, y, z) = ||y + z - 2x|| + ||y - z||$ is an $S -$ metric on X.

Definition 4: Sedghi, Shobe and H. Zhou also worked on S-metric space.

Suppose (X, S) be $S -$metric space.

1. Any sequence $\{x_n\}, x \in X$ converges to x iff $S(x_n, x_n, x) \to 0$ as $n \to$. for all $\in > 0$ there exists$n_0 \in N$so that for all$n \geq n_0$, $S(x_n, x_n x) < \in$ and is denoted by $\lim_{n\to\infty} x_n = x$

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2. Any sequencex_n is known as Cauchy sequence if for each $\in > 0$, there exist$n_0 \in N$as for all$n.m \geq n_0$.
3. The $S-$ metric space (X, S) is taken as complete if every Cauchy sequence converges.

Lemma 1.1: Sedghi et. Al. has given a generalized fixed point theorem in $S-$ metric space.

Consider (X, S) be a$S-$ metric space. We have

$$S(x, x, y) = S(y, y, x)$$

Lemma 1.2: According to Sedghi, N. Shobe and A. Aliouche Let (X, S) be an S-metric space. Then we have

$$S(x, x, z) \leq 2S(x, x, y) + S(y, y, z).$$

Lemma 1.3: Sedghi, N. Shobe and A. Aliouche again stated a "generalization of fixed point theorem in S-metric space."

Suppose (X, S) be an $S-$ metric space. If the sequence $\{x_n \in X\}$ converges to x, then x is unique.

Lemma 1.4:"In a generalization of fixed point theorem in S-metric space Sedghi et al.,considered(X, S) be an $S-$metric space. If the sequence $\{x_n\} \in X$ converges to x, then $\{x_n\}$ is a Cauchy sequence."

Lemma 1.5:"Consider(X, S) be a S-metric space. If there exist sequences$\{x_n\}$ and $\{y_n\}$ in X so that$\lim_{n\to\infty} x_n = x$ and$\lim_{n\to\infty} y_n = y$ then$\lim_{n\to\infty} S(x_n, x_n, y_n) = S(x, x, y)$."

Theorem 1: Savitri and Hooda in their research"On tripled fixed point theorem in partially ordered complete metric space." They stated

"Consider $(X \leq)$is partially ordered complete metric space and $F: X^3 \to X$ is a constant mapping that has the mixed monotone property on X and say there exist points x_0, y_0, z_0 with

$$x_0 \leq F(x_o, y_0, z_o), y_0 \geq F(y_o, x_0, y_o), z_o \leq (z_o, y_0, x_o)$$

Proof: Let$p$ and q be non negative real numbers with $p+q<1$sothat

$$d\big(F(x, y, z), F(u, v, w)\big) \leq p\min\{d(F(x, y, z), d(u, v, w), x\}$$

$$+q \min\{d(F(x, y, z), u), d(u, v, w), x\}$$

for all $x, y, z, u, v, w \in X$ with $x \geq u$, $y \leq v$, $z \geq w$.

Then F has a tripled fixed point in X."

The researcher through this paper aims to "generalize the result of Savitri and Hooda into the structure of S-metric space."

Results:

Th^m. 2:

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"Consider $(X \leq)$ is partially ordered complete metric space and $F: X^3 \to X$ is a constant mapping that has the mixed monotone property on X and say there exist points x_0, y_0, z_0 with

$$x_0 \leq F(x_o, y_0, z_o), y_0 \geq F(y_o, x_0, y_o), z_o \leq (z_o, y_0, x_o)$$

Suppose that there exist $\propto$ and β non-negative real numbers with $\propto + \beta < 1$ so that

$S(F((x, y, z), F(x, y, z), F(u, v, w))$

$\leq \propto \min\{S(F(x, y, z), F(x, y, z), x), S(F(u, v, w), F(u, v, w), x\}$

$+\beta \min\{S(F(x, y, z), F(x, y, z), u), S(F(u, v, w), F(u, v, w), u\}$ ….(1)

for all $x, y, z, u, v, w \in X$ with $x \geq u$, $y \leq v$, $z \geq w$. Then F has a tripled fixed point in X."

Proof :"Consider $x_o, y_0, z_o \in X$ with

$x_o \leq F(x_o, y_0, z_o), y_0 \geq F(y_o, x_0, y_o), z_o \leq F(z_o, y_0, x_o)$ ….(2)

Define sequence $\{x_n\}, \{y_n\}, \{z_n\}$ in X so that for all n = 0, 1, 2,……

$x_{n+1} = F(x_n, y_n, z_n), y_{n+1} = F(y_n, x_n, x_n),\ z_{n+1} = F(z_n, y_n, x_n)$ ….(3)

It is claimed that $\{x_n\}\{z_n\}$ are non decreasing and $\{y_n\}$ is non increasing i.e. n = 0, 1, 2, … ..

$x_n \leq x_{n+1,} y_n \geq y_{n+1}, z_n \leq z_{n+1}$ ….(4)

From (2) and (3), we have for n = 0,

$x_o \leq F(x_o, y_0, z_o), y_0 \geq F(y_o, x_0, y_o), z_o \leq F(z_o, y_0, x_o)$

$$x_{n+1} = F(x_n, y_n, z_n), y_{n+1} = F(y_n, x_n, x_n),\ z_{n+1} = F(z_n, y_n, x_n)$$

This implies $x_0 \leq x_{1,} y_0 \geq y_1, z_0 \leq z_1$

So eqn. 4hold for n = 0."

"Consider that eqn. 4 holds for some $n \in \mathbf{N}$. It is shown that eqn. 4 is true for n + 1.

By mixed monotone property of F, we have:

$$x_{n+2} = F(x_{n+1}, y_{n+1}, z_{n+1}) \geq F(x_n, y_{n+1}, z_{n+1}) \geq F(x_n, y_n, z_{n+1}) \geq F(x_n, y_n, z_n)$$
$$y_{n+2} = F(y_{n+1}, x_{n+1}, y_{n+1}) \leq F(y_n, x_{n+1}, y_{n+1}) \leq F(y_n, x_n, y_{n+1}) \leq F(y_n, x_n, y_n)$$
$$z_{n+2} = F(z_{n+1}, y_{n+1}, x_{n+1}) \geq F(z_n, y_{n+1}, x_{n+1}) \geq F(z_n, y_n, x_{n+1}) \geq F(z_n, y_n, x_n)$$

Hencethrough mathematical induction, equation (4) holds for $n \in N$.

So,

$$x_0 \leq x_1 \leq x_2 \leq \ldots . \leq x_n \leq x_{n+1} \ldots .,$$

$$y_0 \geq y_1 \geq y_2 \geq \ldots . . \geq y_n \geq y_{n+1} \ldots ..$$

$$z_0 \leq z_1 \leq z_2 \leq \ldots . . \leq z_n \leq z_{n+1} \ldots ..$$

As $x_n \geq x_{n-1}, y_n \leq y_{n-1}, z_n \geq z_{n-1}$ from (1)

$S\big(F(x_n, y_n, z_n), F(x_n, y_n, z_n), F(x_{n-1}, y_{n-1}, z_{n-1})\big)$

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$\leq$

$\propto \min\{S(F(x_n,y_n,z_n),F(x_n,y_n,z_n),S(F(x_{n-1},y_{n-1},zx_{n-1}),F(x_{n-1},y_{n-1},z_{n-1}),x_n)\}$

$+\beta\min\{S(F(x_n,y_n,z_n),F(x_n,y_n,z_n),x_{n-1}),S(F(x_{n-1},y_{n-1},z_{n-1}),F(x_{n-1},y_{n-1},z_{n-1}),x_{n-1})\}$

$=\propto \min\{S(x_{n+1},x_{n+1},x_n),S(x_n,x_n,x_n)\}+\beta\min\{S(x_{n+1},x_{n+1},x_{n-1}),S(x_n,x_n,x_{n-1})\}$

$=\beta S(x_{n,}x_{n,}x_{n-1})$

Hence, $S(x_{n+1},x_{n+1},x_n)\leq\beta S(x_n,x_n,x_{n-1})$ ….(5)

Again as $y_n\leq y_{n-1},x_n\geq x_{n-1}$ from (1)

$S\big(F(y_{n-1},x_{n-1},y_{n-1}),F(y_{n-1},x_{n-1},y_{n-1}),F(y_n,x_n,y_n)\big)$

$\leq$

$\propto \min\{S(F(y_{n-1},x_{n-1},y_{n-1}),F(y_{n-1},x_{n-1},y_{n-1}),y_{n-1},S(F(y_n,x_n,y_n),F(y_n,x_n,y_n),y_{n-1})\}$

$+\beta\min\{S(F(y_{n-1},x_{n-1},y_{n-1}),F(y_{n-1},x_{n-1},y_{n-1}),y_n),S(F(y_n,x_n,y_n),F(y_n,x_n,y_n),y_n)\}$

$=\propto \min\{S(y_n,y_n,y_{n-1}),S(y_{n+1},y_{n+1},y_{n-1})\}+\beta\min\{S(y_n,y_n,y_n),S(y_{n+1},y_{n+1},y_n)\}$

$=\propto S(y_n,y_n,y_{n+1})$

Therefore $S(y_{n+1},y_{n+1},y_{n-1})\leq\propto S(y_n,y_n,y_{n-1})$ (6)

Finally as $z_n\geq z_{n-1},y_n\leq y_{n-1},x_n\geq x_{n-1}$ from (1)

$S\big(F(z_n,y_n,x_n),F(z_n,y_n,x_n),F(z_{n-1},y_{n-1},x_{n-1})\big)$

$\leq$

$\propto \min\big\{S\big(F(z_n,y_n,x_n),F(z_n,y_n,x_n),S(F(z_{n-1},y_{n-1},x_{n-1}),F(z_{n-1},y_{n-1},x_{n-1})\big)\big\}$

$+\beta\min\big\{S\big(F(z_n,y_n,x_n),F(z_n,y_n,x_n),z_{n-1}),S(F(z_{n-1},y_{n-1},x_{n-1}),F(z_{n-1},y_{n-1},x_{n-1})\big)\big\}$

$=\propto \min\{S(z_{n+1},z_{n+1},z_n),S(z_n,z_n,z_n)\}+\beta\min\{S(z_{n+1},z_{n+1},z_{n-1}),S(z_n,z_n,z_{n-1})\}$

$=\beta S(z_{n,}z_{n,}z_{n-1})$

Therefore, $S\big(z_{n+1,}z_{n+1,}z_n\big)\leq\ \beta S(z_{n,}z_{n,}z_{n-1})$ ….(7)

By Adding (5), (6) and (7) we get

$S(x_{n+1},x_{n+1},x_n)+S(y_{n+1},y_{n+1},y_{n-1})+S(z_{n+1},z_{n+1},z_n)$

$\leq\beta S\big((x_n,x_n,x_{n-1})+\alpha S(y_n,y_n,y_{n-1})+\beta S(z_n,z_n,z_{n-1})\big)$

$=\beta[S(x_n,x_n,x_{n-1})+S(z_n,z_n,z_{n-1})]+\alpha S(y_n,y_n,y_{n-1})$

$\leq(\propto+\beta)[S(x_n,x_n,x_{n-1})+S(z_n,z_n,z_{n-1})]+(\propto+\beta)S(y_n,y_n,y_{n-1})$

$=\ (\propto+\beta)[S(x_n,x_n,x_{n-1})+S(y_n,y_n,y_{n-1})+S(z_n,z_n,z_{n-1})]$

Let $A=\alpha+\beta<1$. Then

$S(x_{n+1},x_{n+1},x_n)+S(y_{n+1},y_{n+1},y_{n-1})+S(z_{n+1},z_{n+1},z_n)$

$\leq A[S(x_n,x_n,x_{n-1})+S(y_n,y_n,y_{n-1})+S(z_n,z_n,z_{n-1})]$

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$\leq \mathrm{A}^2[\mathrm{S}(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}-2}) + \mathrm{S}(\mathrm{y}_{\mathrm{n}-1}, \mathrm{y}_{\mathrm{n}-1}, \mathrm{y}_{\mathrm{n}-2}) + \mathrm{S}(\mathrm{z}_{\mathrm{n}-1}, \mathrm{z}_{\mathrm{n}-1}, \mathrm{z}_{\mathrm{n}-2})]$

$\leq \mathrm{A}^2[\mathrm{S}(\mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}-1}, \mathrm{x}_{\mathrm{n}-2}) + \mathrm{S}(\mathrm{y}_{\mathrm{n}-1}, \mathrm{y}_{\mathrm{n}-1}, \mathrm{y}_{\mathrm{n}-2}) + \mathrm{S}(\mathrm{z}_{\mathrm{n}-1}, \mathrm{z}_{\mathrm{n}-1}, \mathrm{z}_{\mathrm{n}-2})]$

$= \beta[\mathrm{S}(\mathrm{x}_\mathrm{n}, \mathrm{x}_\mathrm{n}, \mathrm{x}_{\mathrm{n}-1}) + \mathrm{S}(\mathrm{z}_\mathrm{n}, \mathrm{z}_\mathrm{n}, \mathrm{zx}_{\mathrm{n}-1}) + \propto \mathrm{S}(\mathrm{y}_\mathrm{n}, \mathrm{y}_\mathrm{n}, \mathrm{y}_{\mathrm{n}-1})]$

$\leq (\propto + \beta)[\mathrm{S}(\mathrm{x}_\mathrm{n}, \mathrm{x}_\mathrm{n}, \mathrm{x}_{\mathrm{n}-1}) + \mathrm{S}(\mathrm{z}_\mathrm{n}, \mathrm{z}_\mathrm{n}, \mathrm{z}_{\mathrm{n}-1})] + (\propto + \beta)\mathrm{S}(\mathrm{y}_\mathrm{n}, \mathrm{y}_\mathrm{n}, \mathrm{y}_{\mathrm{n}-1})]$

$= (\propto + \beta)[S(x_n, x_n, x_{n-1}) + S(y_n, y_n, y_{n-1}) + S(z_n, z_n, z_{n-1})]$

In addition by lemma 1.2, we have for all $n \leq m$

$S(x_n, x_n, x_m) + S(y_n, y_n, y_m) + S(z_n, z_n, z_m)\backslash$

$\leq (2S(x_n, x_n, x_{n+1}) + 2S(y_n, y_n, y_{n+1}) + 2S)$

$+ S(x_{n+1}, x_{n+1}, x_n) + S(y_{n+1}, y_{n+1}, y_n) + S(z_{n+1}, z_{n+1}, z_n)$

$\leq (2S(x_n, x_n, x_{n+1}) + 2S(y_n, y_n, y_{n+1}) + 2S(z_n, z_n, z_{n+1}))$

$+ (2S(x_{n+1}, x_{n+1}, x_n) + 2S(y_{n+1}, y_{n+1}, y_{n-1}) + 2S(z_{n+1}, z_{n+1}, z_n))$

$+ \ldots + [2S(x_{m-2}, x_{m-2}, x_{m-1}) + 2S(y_{m-2}, y_{m-2}, y_{m-1}) + 2S(z_{m-2}, z_{m-2}, z_{m-1})]$

$\ldots + [2S(x_{m-2}, x_{m-2}, x_{m-1}) + 2S(y_{m-2}, y_{m-2}, y_{m-1}) + 2S(z_{m-2}, z_{m-2}, z_{m-1})]$

$+(S\big((x_{m-1}, x_{m-1}, x_m) + S(y_{m-1}, y_{m-1}, y_m) + S(z_{m-1}, z_{m-1}, z_m)\big)$

$\leq (2S(x_n, x_n, x_{n+1}) + 2S(y_n, y_n, y_{n+1}) + 2S(z_n, z_n, z_{n+1}))$

$+ \ldots + [2S(x_{m-1}, x_{m-1}, x_m) + 2S(y_{m-1}, y_{m-1}, y_m) + 2S(z_{m-1}, z_{m-1}, z_m)]$

$\leq 2[(A^n + A^{n+} + \ldots . A^{m-1})(S(x_0, x_0, x_1) + S(y_0, y_0, y_1) + S(z_0, z_0, z_1))]$

$\leq \frac{2A^n}{1-A}[S(x_0, x_0, x_1) + S(y_0, y_0, y_1) + S(z_0, z_0, z_1)]$

Since $A < 1$, taking limit as $n, m \to \infty$, we get

$\lim_{n,m \to \infty} S(x_n, x_n, x_m) + S(y_n, y_n, x_m) + S(z_n, z_n, z_m) = 0$

This means that

$\lim_{n,m \to \infty} S(x_n, x_n, x_m) = S(y_n, y_n, x_m) = S(z_n, z_n, z_m) = 0$

Therefore x_n, y_n and $z_n$are Cauchy's sequence in X.

X is complete so there exists $x, y, z \in X$ so $that$ $n \to \infty, x_n \to x, y_n \to y, z_n \to z$.

Therefore by using limit$n \to \infty$ in eqn. 3, we have

$x = \lim_{n\to\infty} F(x_{n-1}, y_{n-1}, z_{n-1}) = F\left(\lim_{n\to\infty} x_{n-1}. \lim_{n\to\infty} y_{n-1}. \lim_{n\to\infty} z_{n-1}\right) = F(x, y, z)$

$y = \lim_{n\to\infty} F(y_{n-1}, x_{n-1}, y_{n-1}) = F\left(\lim_{n\to\infty} y_{n-1}. \lim_{n\to\infty} x_{n-1}. \lim_{n\to\infty} y_{n-1}\right) = F(y, x, y)$

$z = \lim_{n\to\infty} F(z_{n-1}, y_{n-1}, x_{n-1}) = F\left(\lim_{n\to\infty} z_{n-1}. \lim_{n\to\infty} y_{n-1}. \lim_{n\to\infty} x_{n-1}\right) = F(z, y, x)$

Hence we get$F(x, y, z) = x, F(y, x, y), F(z, y, x) = z$ and F has a triple fixed pt."

Let us now support our resultwith the help of an example.

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Example 4:"Suppose $X = [0,1]$ $x \leq y \leftrightarrow x, y \in [0,1]$ with usual oreder $\leq$.

Suppose $(X, \leq, S)$ be partially ordered S-metric (complete)with usual S metric as expressed in Ex. 1 above as:

$S(x, y, z) = |x - z| + |y - z|$,

So $S(x, y, z) = |x - y| + |x - y|$

$=2|x - y|$

$= 2d(x, y)$

Define $F: X \times X \times X \to X$ by

$$F(x,y,z) = \begin{cases} \frac{x-y}{8}, & if\ x \geq y \\ \frac{x-y}{8}, & if\ y \geq z \\ \frac{x-y}{8}, & if\ z \geq x \\ \frac{1}{8}, & other\ cases \end{cases}$$

Then F shows mixed monotone property and is continuous."

Suppose$x_0 = y_0 = z_0 = 0$ such that

$x_0 = 0 \leq F(0,0,0) = F(x_0, y_0, z_0), y_0 = 0 \geq F(0,0,0) = F(y_0, x_0, z_0)$ and $z_0 = 0 \leq F(0,0,0)$
$= F(z_0, y_0, x_0)$

Next it is shown that the mapping$F$ satisfies eqn. (1) for $\alpha = \beta = \frac{1}{8}$

For $x, y, z, u, v, w \in X, x \geq u, y \leq v, z \geq w$ so that eight cases as discussed under hold:

1. $x = u, y < v, z = w$
2. $x = u, y < v, z > w$
3. $x = u, y = v, z = w$
4. $x = u, y < v, z > w$
5. $x > u, y < v, z = w$
6. $x > u, y < v, z > w$
7. $x > u, y = v, z > w$
8. $x > u, y = v, z = w$

If (x, y, z)= (0, 0, 1) and (u, v, w) = (0, 1, 1), It is considered for the possibility of case for the max. and min. values of$x, y, z, u, v, w \in X$ $using$ eq 1.

Clearly L.H.S. of eqn. (1) is 0:

$$\mathrm{S(F(x.y,z), F(x.y,z), F(u,v,w)) = 2d\big(F(x.y,z), F(u,v,w)\big) = 0}$$

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R.H.S. of eqn. (1) is given by

$$= \frac{1}{8}\min\{S(F(x.y,z),\ F(x.y,z),x),\ S(F(u,v,w),x)\} + \frac{1}{8}\min\{S(F(x.y,z),F(x.y,z),u),$$

$$S(F(u,v,w),F(u,v,w),u)\}$$

$$\frac{1}{8}\min\{2d(F(x.y,z),F(x.y,z),x),2d(F(u,v,w),x)\} + \frac{1}{8}\min\{2d(F(x.y,z),F(x.y,z),u),2d(F(u,v,w),F(u,v,w),u)\}$$

$$= \frac{1}{8}\min\left\{2\left(\frac{1}{8},0\right),2\left(\frac{1}{8},0\right)\right\}+\frac{1}{8}\min\left\{2\left(\frac{1}{8},0\right),2\left(\frac{1}{8},0\right)\right\}=\frac{2}{32}=\frac{1}{16}.$$

Hence (1) is satisfied and proved asF has a tripled fixed point.

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