Influence of Thermal Diffusion in a Non-Linear MHD Flow Close to a Stagnation Point

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Abstract

The influence of thermal diffusion with a special reference to stretching parameter in a non-linear MHD heat and mass transfer flow are studied parametrically. The MHD boundary layer equations are solved by adopting Homotopy Perturbation method (HPM) which is controlled by a system of non-linear partial differential equations. The impact of many relevant physical parameters is graphically illustrated and discussed.

Keywords: Soret number, Mass transfer, Schmdit number, Sherwood number

INTRODUCTION

Extensive research and experiments have been conducted in the domain of study by numerous scientists and engineers due to its technological importance and submissions in several engineering appliances that run on MHD principles and also its use in explaining sunspots and solar flares and the formation of stars from interstellar clouds. It also aids in providing a simplified mathematical explanation and justification of the complicated branch of plasma physics related to hot ionized gases (plasmas). The pioneer worker in the field of MHD was Maxwell (1864) and Lorentz (1952). Also, the involvement of Alfven (1942) in the field of MHD is remarkable. Numerous authors have reported analytical and numerical solutions of MHD flow problems which are very interesting due to promising effects of magnetic field on boundary layer are specified by Hayet et al. (2007) came up with a systematic solution of MHD flow of II-Grade fluid over a shrinking plate. Fang and Zhang (2009), Chien-Hsin (2009), Pal and Chatterjee (2011), and

Bhattacharyya (2011) observed the effect of different parameters of Magneto-hydrodynamic flow. Jhankal (2014) discussed MHD flow on boundary layer with a low gradient of pressure on a flat plate by applying the method of Homotopy Perturbation.

In fluid mechanics, the discussion of flow related problems to stagnation point is of great scientific importance due to its numerous applications in technology and engineering. Attia (2006) studied numerically the flow of Stagnation Point on a stretching sheet with heat generation in a porous media, Kazem et al. (2011) has given an improved analytical solution of this problem.

In this research, we develop the Homotopy Perturbation Method (HPM), a semi-exact method that may be used in MHD boundary layer flow with minimal pressure gradient across a flat plate. The major goal of this work is to use the Homotopy Perturbation Method to explore the effects of thermal diffusion, stretching parameter on MHD flow related difficulties to the stagnation point. The basic idea of the present work is developed by considering the influence of thermal diffusion as the generalization of the work of Borgohain K. et.al. (2022).

MATHEMATICAL PLANNING

In the presented Figure.1, the model that represents the significant circumstances with initial velocity u_w, T_w being the temperature and u_∞, $T_\infty(x)$ being the velocity and temperature of the flow external to the boundary layer.

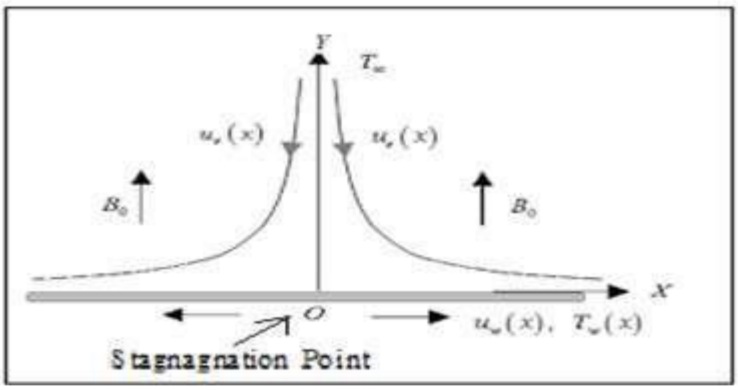

Figure.1 Physical model of the problem

Choudhary et al. [2015] proposed some standard assumptions, based on which the following governing equations defining physical circumstances are evaluated.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}+\frac{\nu}{K}\left[U(x)-u\right]-\frac{\sigma B_0{}^2 u}{\rho} \tag{2}$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right) \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_M\frac{\partial^2 C}{\partial y^2}+D_T\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Whereas the symbols used above have their typical meanings.

Boundary Conditions are

$$u=u_w\left(x\right)=cx; v=0; T=T_w; C=C_w \text{ for } y=0$$

$$u=u_e\left(x\right)=ax; T\to T_\infty, C\to C_\infty \text{ for } y\to\infty \tag{5}$$

The stream function $\psi(x,y)$ is given by

$$u=\frac{\partial\psi}{\partial y}; v=-\frac{\partial\psi}{\partial x} \tag{6}$$

To standardize the flow model, the following non dimensional terms are being incorporated:

$$\left.\begin{aligned}&\psi(x,y)=\sqrt{c\nu}\ \text{x}\ f(\eta);\ \eta=\sqrt{\frac{c}{\nu}}y; T=T_\infty+\left(T_w-T_\infty\right)\theta\left(\eta\right)\\&C=C_\infty+\left(C_w-C_\infty\right)\phi\left(\eta\right)\end{aligned}\right\} \tag{7}$$

The following non linear coupled differential equations are produced by applying (6), (7) in equation (2), (3), and (4)

$$f'''\left(\eta\right)+f\left(\eta\right)f''\left(\eta\right)-f'^2\left(\eta\right)-\left(\lambda+M\right)f'\left(\eta\right)+C\left(C+\lambda\right)=0 \tag{8}$$

$$\theta''\left(\eta\right)+Pr\,f\left(\eta\right)\theta'\left(\eta\right)+Pr\,B\theta\left(\eta\right)=0 \tag{9}$$

$$\phi''\left(\eta\right)+Sc\,f\left(\eta\right)\phi'\left(\eta\right)=-Sr\,\theta''\left(\eta\right) \tag{10}$$

Subject to boundary conditions

$$\left.\begin{array}{l} f(\eta)=0; f^{/}(\eta)=1: \theta(\eta)=1; \phi(\eta)=1 \text{ as } \eta=0 \\ \\ f^{/}(\eta)\to \square, \theta(\eta)\to 0; \phi(\eta)\to 0 \text{ as } \eta\to\infty \end{array}\right\} \tag{11}$$

Equations (8), (9) and (10) can be rewritten as

$$\left.\begin{array}{l} f^{///}+f\,f^{/}-f^{/2}-M_1 f^{/}+M_2=0 \\ \theta^{//}+Pr\,f\,\theta^{/}-M_3\theta=0 \\ \phi^{//}+Sc\,f\,\phi^{/}=-Sr\,\theta'' \end{array}\right\} \tag{12}$$

Where $\lambda=\dfrac{\nu}{cK}$ is the porosity parameter, $M=\dfrac{B_0^2\sigma}{\rho c}$ is the magnetic parameter, $Pr=\dfrac{\mu C_p}{k}$ is the Prandtl number, $C=\dfrac{a}{c}$ is the stretching parameter , $B=\dfrac{Q}{c\rho C_p}$ is the heat absorpation parameter, $Sc=\dfrac{\nu}{D_M}$ is the Schmidt number and $Sr=\dfrac{D_T(T_w-T_\infty)}{D_M(C_w-C_\infty)}$ is the Soret number.

By virtue of HPM, the equations (10),(11),(12) can be written as follows:

$$(1-p)\left(f^{///}-M_1 f^{/}\right)+p\left(f^{///}+f.f^{/}-f^{/2}-M_1.f^{/}\right)=-M_2 \tag{13}$$

$$(1-p)\left(\theta^{//}-M_3\theta\right)+p\left(\theta^{//}+Pr.f.\theta^{/}-M_3\theta\right)=0 \tag{14}$$

$$(1-p)\phi^{//}+p\left(\phi^{//}+Sc\,f\,\phi^{/}+Sr\,\theta''\right)=0 \tag{15}$$

Let us consider f, θ and ϕ as

$$\left.\begin{array}{l} f=f_0+p.f_1+p^2 f_2+\ldots.. \\ \theta=\theta_0+p.\theta_1+p^2\theta_2+\ldots.. \\ \phi=\phi_0+p.\phi_1+p^2\phi_2+\ldots.. \end{array}\right\} \tag{16}$$

From the location to the qualifying restrictions that follow

$f_0(0)=0,\ f_0'(0)=1,\ f_0'(6)=C;\ \theta_0(0)=1,\ \theta_0(6)=0,\ \phi_0(0)=1,\ \phi_0(6)=0$ at $\eta=0$ $f_1(0)=0$, $f_1'(0)=0$, $f_1'(6)=0$; $\theta_1(0)=0$, $\theta_1(6)=0$; $\phi_1(0)=0$, $\phi_1(6)=0$ at $\eta\to\infty$

(17)

(In the boundary layer theory, these are $\eta\to\infty$ replaced by $\eta=6$ those at in concurrent practice)

By using (16) and (17) and by equating the terms containing and not containing p, the equations (13), (14) and (15) becomes

$$f_0(\eta)=C_1+C_2e^{\sqrt{M_1}\eta}+C_3e^{-\sqrt{M_1}\eta}+\frac{M_2}{M_1}\eta \tag{20}$$

$$\theta_0(\eta)=C_4e^{\sqrt{M_3}\eta}+C_5e^{-\sqrt{M_3}\eta} \tag{21}$$

$$\phi_0(\eta)=\left(1-\frac{1}{6}\eta\right) \tag{22}$$

$$f_1(\eta)=C_6+C_7e^{\sqrt{M_1}\eta}+C_8e^{-\sqrt{M_1}\eta}+A_{10}\eta^2+A_{11}\eta^3+A_{12}\eta-\left[A_{15}e^{\sqrt{M_1}\eta}-A_{16}e^{-\sqrt{M_1}\eta}\right]$$

$$-\eta\left[E_1e^{\sqrt{M_1}\eta}+E_2e^{-\sqrt{M_1}\eta}\right]+\eta^2\left[A_{17}e^{\sqrt{M_1}\eta}+A_{18}e^{-\sqrt{M_1}\eta}\right] \tag{23}$$

$$\theta_1(\eta)=C_9e^{\sqrt{M_3}\eta}+C_{10}e^{-\sqrt{M_3}\eta}+\eta\left[A_{39}e^{\sqrt{M_3}\eta}+A_{40}e^{-\sqrt{M_3}\eta}\right]-\eta^2\left[A_{35}e^{\sqrt{M_3}\eta}-A_{36}e^{-\sqrt{M_3}\eta}\right]$$

$$-\left[A_{31}e^{M_4\eta}-A_{32}e^{-M_4\eta}\right]+\left[A_{33}e^{M_5\eta}-A_{34}e^{-M_5\eta}\right] \tag{24}$$

$$\phi_1(\eta)=A_{13}\eta+A_7+\frac{Sc}{6}\left(\frac{C_1\eta^2}{2}+\frac{C_2}{M_1}e^{\sqrt{M_1}\eta}+\frac{C_3}{M_1}e^{-\sqrt{M_1}\eta}+\frac{M_2}{6M_1}\eta^3\right)$$
$$-Sr\left(C_4e^{\sqrt{M_3}\eta}+C_5e^{-\sqrt{M_3}\eta}\right) \tag{25}$$

Hence, the solutions are obtained by ignoring higher order perturbed terms to get:

$$f(\eta)=f_0+pf_1$$
$$\theta(\eta)=\theta_0+p\theta_1$$
$$\phi(\eta)=\phi_0+p\phi_1$$

The terminologies for viscous drage in terms of skin friction (τ) , the coefficient of rate of heat transfer (Nu) and the coefficient of rate of mass transfer (Sh) are given by:

$$\tau=\left(\frac{\partial f}{\partial \eta}\right)_{\eta=0}=\sqrt{M_1}\left(C_2-C_3\right)+\frac{M_2}{M_1}+p\left[\sqrt{M_1}\left(C_7-C_8\right)\right]+A_{12}-\left(A_{13}+A_{14}\right)$$
$$-2\sqrt{M_1}\left(A_{15}+A_{16}\right)+\left(A_{19}-A_{20}\right)$$

$$Nu=-\left(\frac{\partial \theta}{\partial \eta}\right)_{\eta=0}=-\sqrt{M_3}\left(C_4-C_5\right)+p\left[\sqrt{M_1}\left(C_9-C_{10}\right)\right]$$
$$+A_{39}+A_{40}-M_3\left(A_{31}-A_{32}\right)$$

$$Sc=-\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=0}=\frac{1}{6}-p\,A_{18}$$

RESULTS AND FINDINGS

In this study, the problem of boundary layer for MHD flow placed vertically in presence of heat and mass transfer is considered by HPM. The obtained outcomes are revealed graphically and are compared with the accurate solutions. The result shows that the estimated solution obtained in this paper has an exceptional concurrence with the work done by Borgohain K. et.al. (2022).The mathematical results are obtained for various values of physical parameters with the fixed value of Homotopy Perturbation Parameter (p= 0.1) implanted in the flow system.

The character of species concentration due to magnetic field and stretching parameter is revealed in figure 2-3. Fig. 2 demonstrates that the behaviour of the species is decelerated under the action of strength of the applied magnetic field. The figure predicts that the species concentration decreases to zero in the infinite direction. Figure 3 depicts the consequence of stretching parameter on profile of concentration. It is noted that the concentration profile is continuously reduced for varying the stretching parameter (C = 0.5, 1, 1.5) i.e. the concentration distribution is continuously moves down on account of stretching parameter.

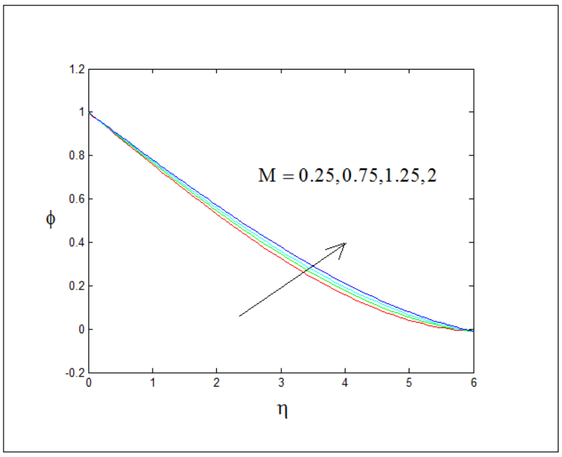

Figure 2: Concentration versus η for λ=3, C=1.5, B=0.1, Pr=0.1, Sc=.60, Sr=0.1, p=0.1

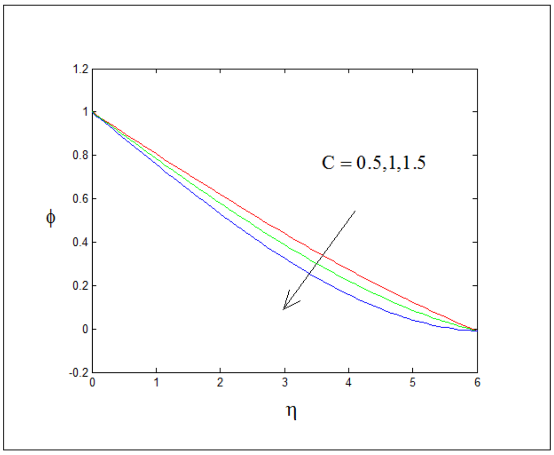

Figure 3: Concentration versus η for λ=3, M=0.25, B=0.1, Pr=0.1, Sc=.60, Sr=0.1, p=0.1

The influence of Soret number and Schmdit number is demonstrated in fig. 3-4. Figure 4 shows that the concentration level of the fluid drops down due to thermal diffusion. Figure 5 presents the variation of the species concentration under the influence of Schmdit number. The figure predicts that the species concentration decreases to zero in the infinite direction. This phenomenon physically states that for enlarging the mass diffusivity of the flow, the species concentration gets mounted up.

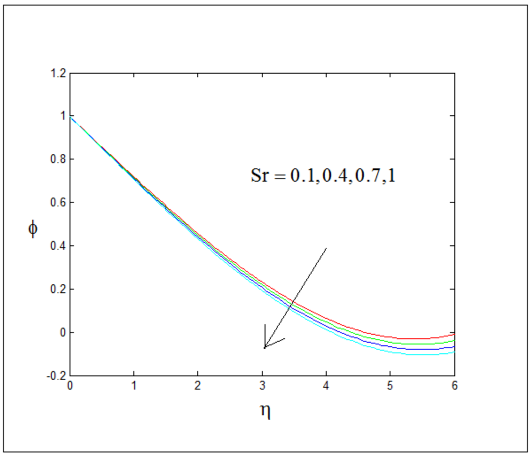

Figure 4: Concentration versus η for λ=3, C=1.5, B=0.1, Pr=0.1, Sc=.60, M=0.25, p=0.1

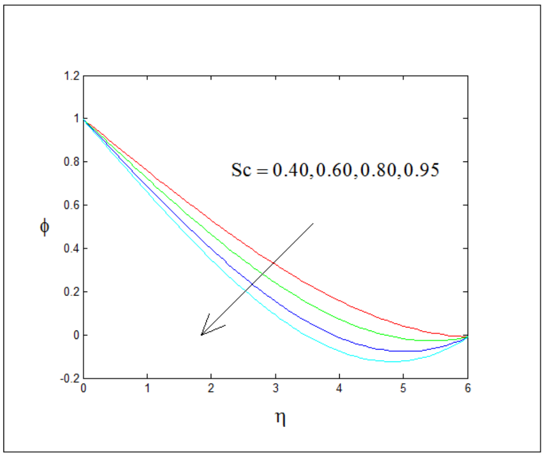

Figure 5: Concentration versus η for λ=3, C=1.5, B=0.1, Pr=0.1, M=.25, Sr=0.1, p=0.1

Variations of mass flux against magnetic field and stretching parameter are verified in figures 6 and 7. It is continent from both the figures that the strength of the applied magnetic field rises the mass flux and the stretching parameter made the co-efficient of rate of mass transfer to minimize.

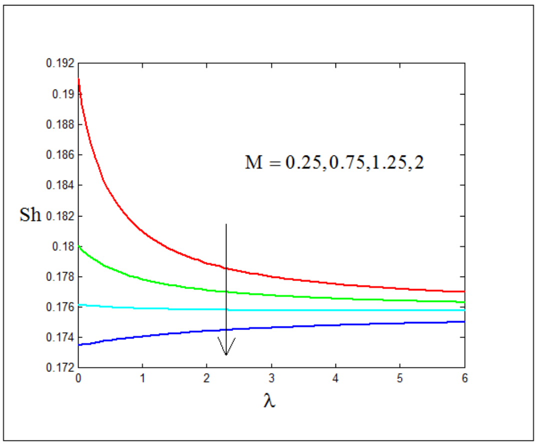

Figure 6: Sherwood number versus λ for C=1.5, B=0.1, Pr=0.1, Sc=.60, Sr=0.1, p=0.1

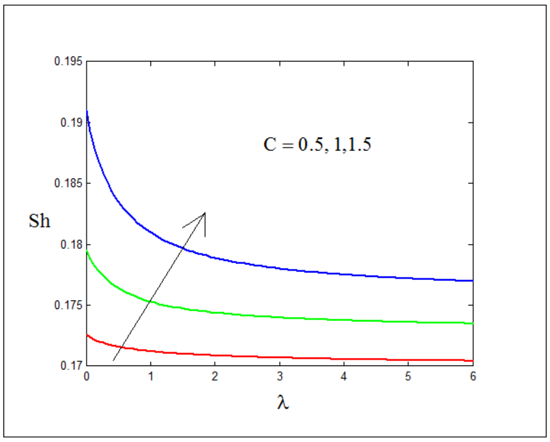

Figure 7: Sherwood number versus λ for M=0.25, B=0.1, Pr=0.1, Sc=.60, Sr=0.1, p=0.1

Consequences of thermal diffusion and mass diffusion on Sherwood number are demonstrated in figures 8-9. From these two figures, it is evident that thermal diffusion made the mass flux to accelerate while mass diffusion controls the rate of mass transfer.

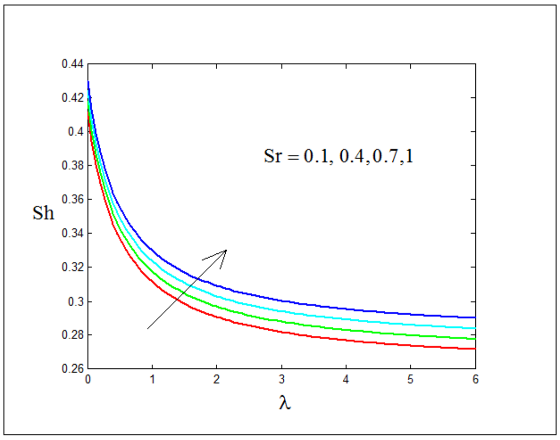

Figure 8: Sherwood number versus λ for C=1.5, B=0.1, Pr=0.1, Sc=.60, M=0.25, p=0.1

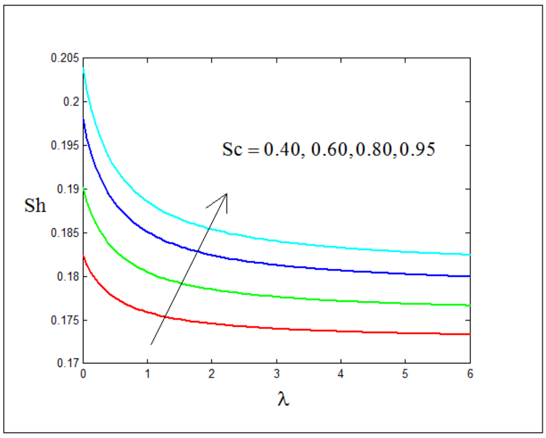

Figure 9: Sherwood number versus λ for C=1.5, B=0.1, Pr=0.1, Sr=0.1, M=0.25, p=0.1

Comparison of results

For comparing the results of the present work, the results of Borgohain K. et.al. (2022) are used. Comparing figures 10 and 11 with figure 3 and 6 of the work done by Borgohain K. et.al. (2022), it is observed that the same kind of behaviour is occured due to the implementation of Schmdit number in concentration profile. i.e. there is a significant effect of mass diffusion on this profile. The concentration distribution is almost similar making an admirable fact with the findings investigated by Borgohain K. et.al. (2022) and the present authors.

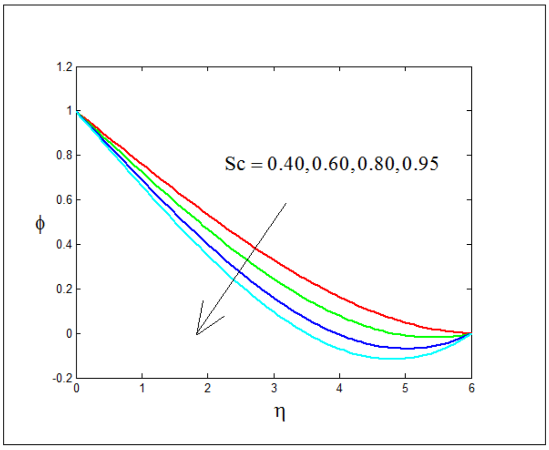

Figure 10: Concentration versus η for λ =3, C=1.5, B=0.1, Pr=0.1, M=.25, Sr=0, p=0.1

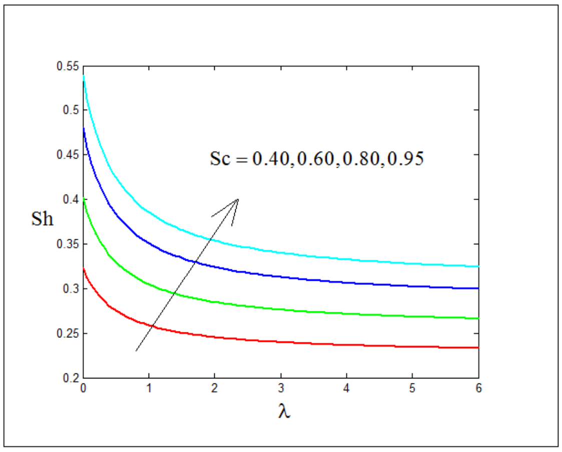

Figure 11: Sherwood number versus λ for C=1.5, B=0.1, Pr=0.1, Sr=0, M=0.25, p=0.1

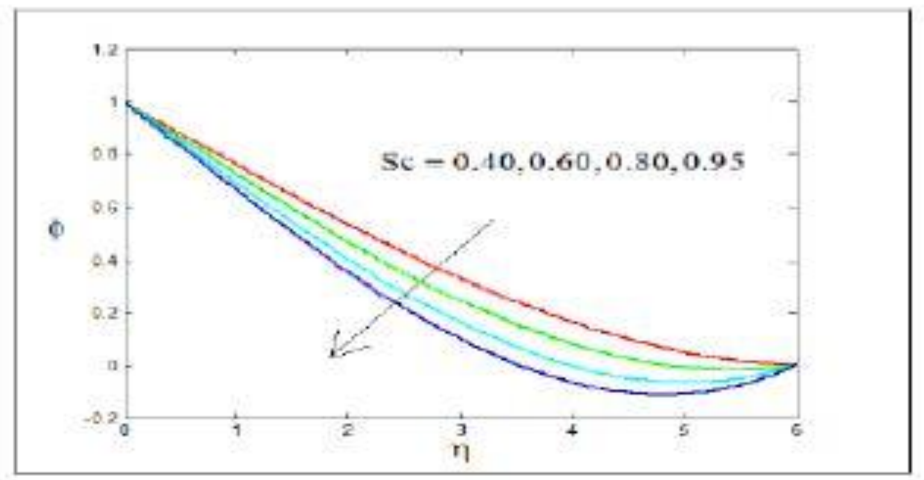

Figure 3: Concentration against η with M=0.25, λ=3, C=1.5, P=0.1

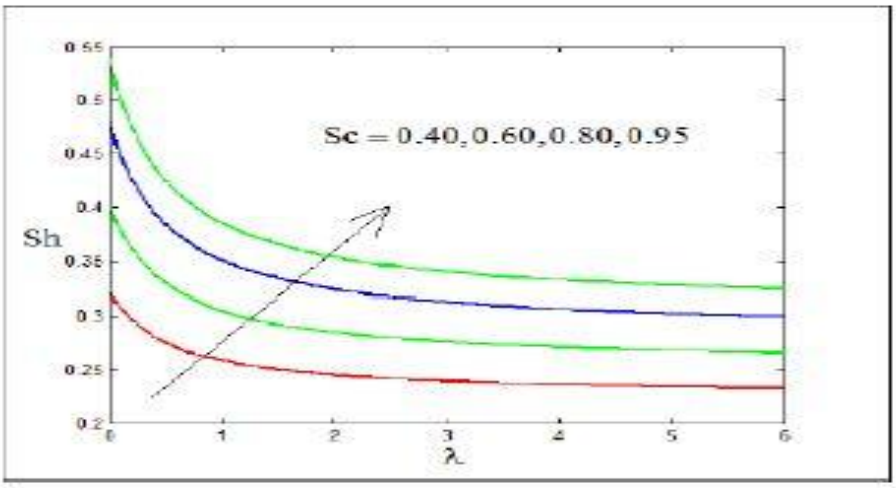

Figure 6: Sherwood number against λ with M=0.25, C=1.5, P=0.1

Concluding Remarks

(a) The behaviour of the species is decelerated under the action of strength of the applied magnetic field and stretching parameter..

(b) The concentration level of the fluid falls due to thermal diffusion and schmdit parameter.

(c) The strength of the applied magnetic field raises the mass flux and the stretching parameter made the co-efficient of rate of mass transfer to minimize.

(d) Thermal diffusion made the mass flux to accelerate while mass diffusion controls the rate of mass transfer.

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