An Extended VIKOR Method for Interval-Valued Trapezoidal Intuitionistic Fuzzy Decision Making

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Abstract
Real-world problems frequently have several non-commensurable and contradictory criteria, and there may be no solution that meets all of them at the same time. VlseKriterijumskaOptimizacija I KompromisnoResenje (VIKOR) is a technique developed to solve the decision making problems with these criteria. This paper extends themulti-criteria decision making (MCDM) technique VIKOR for interval-valued trapezoidal intuitionistic fuzzy sets (IVTrIFSs). A new ranking method based on Jaccard distance is proposed and is applied in this process. Further, the proposed IVTrIFS VIKOR is tested by applying it to solve the best green supplier problem. The results are compared with the other existing methods. The comparison study revealed that the proposed method is both effective and applicable.

Keywords: **-:** Interval-valued trapezoidal Intuitionistic fuzzy set (IVTrIFS), VIKOR, MCDM, distance based ranking method.

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1. Introduction

Multi-criteria decision making is an important area of study in decision theory. MCDM is used to select the most appropriate alternative from a set of possible alternatives using a predefined set of criteria. However, because of the ambiguity and complexity present in real life, assessing a situation's associated traits with accuracy and certainty is difficult. As uncertainty and ambiguity are to be expected in the decision-making process, especially when experts establish criteria weights and evaluate alternatives using these weights, modelling uncertainty and ambiguity is becoming increasingly important in MCDM problems [1]. Intuitionistic fuzzy sets (IFSs) are one among them. Many generalisations of Atanassov Intuitionistic fuzzy numbers [2] (IFSs) have been proposed since their inception. Among those generalizations interval-valued intuitionistic fuzzy numbers [3] (IVIFNs), intuitionistic triangular fuzzy numbers [4] (ITFNs), intuitionistic trapezoidal fuzzy numbers [5] (ITFNs) and interval-valued trapezoidal intuitionistic fuzzy numbers (IVTrIFNs) are some which acquired significance in decision making.

IVTrIFNs are defined on consecutive set of real numbers unlike IVIFNs, and hence deals the uncertain and vague information in a decision problem more than IVIFN [2, 6, 7] and also better than TrFNs, TrIFN [8]. The IVTrIFN can delicately and effectively describe decision information using various dimensions and units. As a result, IVTrIFNs play an important role in scientific research as well as practical applications. The definition of IVTrIFNs was provided by S.P. Wan in 2011 [9]. The membership and non-membership values of a trapezoidal number are intervals rather than exact numbers in the construction of this fuzzy number. He defined operational laws as well as some IVTrIFNs score and accuracy ranking functions. Wu and Liu [10] proposed a method for ranking IVTrIFNs that uses the experts' risk attitude to develop a new score and expected accuracy functions. Dong and Wan [8] proposed a new ranking method based on IVTrIFN expectation and expectant score. On IVTrIFNs, Wan [9], Wu and Liu [10], and Wei [11] defined some arithmetic and geometric aggregation operators. Dong and Wan [8] also defined some generalised aggregation operators to address IVTrIFS decision-making problems.

The VIKOR method performs well in processing the complex systems with multiple criteria. It finds a compromise solution using the initial (supplied) weights. When there are competing criteria, this technique is intended to rank and select the best from a set of options. VIKOR's ranking index is based on a specific metric of "closeness" to the "ideal" solution. As a result, numerous researchers proposed fuzzy VIKOR in a variety of domains [1,4,5,12,13,14,15]. Alkafaas et al. [1] and Hajiheydari et al. [12] developed IF VIKOR and applied it in decision making problems. Khan et al. [5] developed VIKOR method for Pythagorean fuzzy sets using dissimilarity measure. Narayanamoorthy et al. [4] extended VIKOR method for Interval-valued Hesitant IF sets based on entropy measure. Liu & Qin [13] extended VIKOR method for Interval-valued linguistic IF numbers using entropy measure. The goal of this research is to create a decision model for dealing with uncertainty and ambiguity by incorporating the superiority of IVTrIFSs in VIKOR.

The paper is organised as follows: in section 2, various concepts related to IVTrIFS are presented. Section 3 proposes an IVTrIF extended VIKOR decision method based on Jaccard distance measure. Section 4 investigates a problem of selecting best green supplier problem to demonstrate the model's applicability. The last section contains the conclusion.

2. Preliminaries

In this section, the definition of IVITrFS is given and some operations and distance measures on IVTrIFSs from literature are reviewed.

Definition 1: Interval-valued trapezoidal intuitionistic fuzzy set[4]

Let $\tilde{\alpha} = ([a, b, c, d]; [m_{\tilde{\alpha}}^L, m_{\tilde{\alpha}}^U]; [n_{\tilde{\alpha}}^L, n_{\tilde{\alpha}}^U])$ is an IVITrFS, its membership and non-membership functions are defined as follows:

$$m_{\tilde{\alpha}}^U(x) = \begin{cases} \frac{x-a}{b-a} m_{\tilde{\alpha}}^U & \text{if } a \le x < b \\ m_{\tilde{\alpha}}^U & \text{if } b \le x \le c \\ \frac{d-x}{d-c} m_{\tilde{\alpha}}^U & \text{if } c < x \le d \\ 0 & \text{otherwise.} \end{cases} \qquad m_{\tilde{\alpha}}^L(x) = \begin{cases} \frac{x-a}{b-a} m_{\tilde{\alpha}}^L & \text{if } a \le x \le b \\ m_{\tilde{\alpha}}^L & \text{if } b \le x \le c \\ \frac{d-x}{d-c} m_{\tilde{\alpha}}^L & \text{if } c < x \le d \\ 0 & \text{otherwise.} \end{cases}$$

Its non-membership function is given by

$$n_{\tilde{\alpha}}^{U}(\mathrm{x}) = \begin{cases} \frac{b-x\ +n_{\tilde{\alpha}}^{U}(\mathrm{x-a})}{b-a} & \text{if a} \leq \text{x} < b \\ n_{\tilde{\alpha}}^{U} & \text{if b} \leq \text{x} \leq \text{c} \\ \frac{x-c\ +n_{\tilde{\alpha}}^{U}(\mathrm{d-x})}{d-c} & \text{if c} < x \leq d \\ 0 & \text{otherwise.} \end{cases}$$

$$n_{\tilde{\alpha}}^{L}(\mathrm{x}) = \begin{cases} \frac{b-x\ +n_{\tilde{\alpha}}^{L}(\mathrm{x-a})}{b-a} & \text{if a} \leq \text{x} < b \\ n_{\tilde{\alpha}}^{L} & \text{if b} \leq \text{x} \leq \text{c} \\ \frac{x-c\ +n_{\tilde{\alpha}}^{L}(\mathrm{d-x})}{d-c} & \text{if c} < x \leq d \\ 0 & \text{otherwise.} \end{cases}$$

Where $0 \leq m_{\tilde{\alpha}}^{L} \leq m_{\tilde{\alpha}}^{U} \leq 1$, $0 \leq n_{\tilde{\alpha}}^{L} \leq n_{\tilde{\alpha}}^{U} \leq 1$, $0 \leq m_{\tilde{\alpha}}^{U} + n_{\tilde{\alpha}}^{U} \leq 1$ and $0 \leq m_{\tilde{\alpha}}^{L} + n_{\tilde{\alpha}}^{L} \leq 1$
$a, b, c, d \in R$

$\tilde{\alpha} = ([\mathrm{a, b, c, d}]; [m_{\tilde{\alpha}}^{L}, m_{\tilde{\alpha}}^{U}]; [n_{\tilde{\alpha}}^{L}, n_{\tilde{\alpha}}^{U}])$ is sometimes called as IVITrFN.

Definition 2: Arithmetic operation laws of IVTrIFSs [9]

Let $\widetilde{\alpha_1} = ([a_1, b_1, c_1, d_1]; [m_{\widetilde{\alpha_1}}^{L}, m_{\widetilde{\alpha_1}}^{U}]; [n_{\widetilde{\alpha_1}}^{L}, n_{\widetilde{\alpha_1}}^{U}])$

and $\widetilde{\alpha_2} = ([a_2, b_2, c_2, d_2]; [m_{\widetilde{\alpha_2}}^{L}, m_{\widetilde{\alpha_2}}^{U}]; [n_{\widetilde{\alpha_2}}^{L}, n_{\widetilde{\alpha_2}}^{U}])$ be two IVTrIFSs, then the arithmetical operations of $\widetilde{\alpha_1}$ and $\widetilde{\alpha_2}$ are defined as follows:

(i) $\widetilde{\alpha_1} \oplus \widetilde{\alpha_2} = \big([a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2]; \big[m_{\widetilde{\alpha_1}}^{L} + m_{\widetilde{\alpha_2}}^{L} - m_{\widetilde{\alpha_1}}^{L} m_{\widetilde{\alpha_2}}^{L}, m_{\widetilde{\alpha_1}}^{U} + m_{\widetilde{\alpha_2}}^{U} -$ *mα1Umα2U,nα1Lnα2L,nα1Unα2U*;

(ii) $\widetilde{\alpha_1} \otimes \widetilde{\alpha_2} = \big([a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2]; \big[m_{\widetilde{\alpha_1}}^{L} m_{\widetilde{\alpha_2}}^{L}, m_{\widetilde{\alpha_1}}^{U} m_{\widetilde{\alpha_2}}^{U}\big], \big[n_{\widetilde{\alpha_1}}^{L} + n_{\widetilde{\alpha_2}}^{L} - n_{\widetilde{\alpha_1}}^{L} n_{\widetilde{\alpha_2}}^{L}, n_{\widetilde{\alpha_1}}^{U} +$ *nα2U−nα1Unα2U*;

(iii) $r\widetilde{\alpha_1} = \big([ra_1, rb_1, rc_1, rd_1];\ \big[1 - \big(1 - m_{\widetilde{\alpha_1}}^{L}\big)^{r},\ 1 - \big(1 - m_{\widetilde{\alpha_1}}^{U}\big)^{r}\big], \big[{n_{\widetilde{\alpha_1}}^{L}}^{\mathrm{r}}, {n_{\widetilde{\alpha_1}}^{U}}^{\mathrm{r}}\big]\big)$, $r > 0$;

(iv) $\widetilde{\alpha_1}^{r} = \big([a_1^r, b_1^r, c_1^r, d_1^r];\ \big[{m_{\widetilde{\alpha_1}}^{L}}^{\mathrm{r}}, {m_{\widetilde{\alpha_1}}^{U}}^{\mathrm{r}}\big], \big[1 - (1 - n_{\widetilde{\alpha_1}}^{L})^{r},\ 1 - (1 - n_{\widetilde{\alpha_1}}^{U})^{r}\big]\big), r > 0.$

Definition 3: Distance measures on IVTrIFSs [9,11]

For any two IVTrIFSs, the Hamming distance and Euclidean distance are defined as

Let $\widetilde{\alpha_1} = ([a_1, b_1, c_1, d_1]; [m^L_{\widetilde{\alpha_1}}, m^U_{\widetilde{\alpha_1}}]; [n^L_{\widetilde{\alpha_1}}, n^U_{\widetilde{\alpha_1}}])$

and $\widetilde{\alpha_2} = ([a_2, b_2, c_2, d_2]; [m^L_{\widetilde{\alpha_2}}, m^U_{\widetilde{\alpha_2}}]; [n^L_{\widetilde{\alpha_2}}, n^U_{\widetilde{\alpha_2}}])$ are two IVTrIFSs.

The Hamming distance of $\widetilde{\alpha_1}$ and $\widetilde{\alpha_2}$ is defined as:

$$d_H(\widetilde{\alpha_1}, \widetilde{\alpha_2}) = \frac{1}{8}\begin{pmatrix} \left|\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)a_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)a_2\right| + \left|\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)a_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)a_2\right| + \\ \left|\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)b_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)b_2\right| + \left|\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)b_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)b_2\right| \\ +\left|\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)c_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)c_2\right| + \left|\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)c_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)c_2\right| \\ + + \left|\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)d_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)d_2\right| + \left|\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)d_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)d_2\right| \end{pmatrix}$$

The Euclidean distance of $\widetilde{\alpha_1}$ and $\widetilde{\alpha_2}$ is defined as:

$$d_E(\widetilde{\alpha_1}, \widetilde{\alpha_2}) = \frac{1}{2\sqrt{2}}\left(\sqrt{\begin{matrix} \left(\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)a_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)a_2\right)^2 + \left(\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)a_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)a_2\right)^2 + \\ \left(\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)b_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)b_2\right)^2 + \left(\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)b_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)b_2\right)^2 \\ + \left(\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)c_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)c_2\right)^2 + \left(\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)c_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)c_2\right)^2 \\ + \left(\left(m^L_{\widetilde{\alpha_1}} - n^U_{\widetilde{\alpha_1}}\right)d_1 - \left(m^L_{\widetilde{\alpha_2}} - n^U_{\widetilde{\alpha_2}}\right)d_2\right)^2 + \left(\left(m^U_{\widetilde{\alpha_1}} - n^L_{\widetilde{\alpha_1}}\right)d_1 - \left(m^U_{\widetilde{\alpha_2}} - n^L_{\widetilde{\alpha_2}}\right)d_2\right)^2 \end{matrix}}\right)$$

When ranking, the distance from the origin is calculated, and the set with the greatest distance receives the better ranking.

Definition 4: Score and Accuracy functions of IVTrIFS [9]

The score and accuracy functions of $\tilde{\alpha}$ is respectively defined by:

$$S_x(\tilde{\alpha}) = \frac{m^L_{\tilde{\alpha}} + m^U_{\tilde{\alpha}} - n^L_{\tilde{\alpha}} - n^U_{\tilde{\alpha}}}{2}$$

$$H_x(\tilde{\alpha}) = \frac{m^L_{\tilde{\alpha}} + m^U_{\tilde{\alpha}} + n^L_{\tilde{\alpha}} + n^U_{\tilde{\alpha}}}{2}$$

Definition 5: Expected functions of IVTrIFSs [10]

The score expected function of $\tilde{\alpha}$:

$$I\big(S_x(\tilde{\alpha})\big) = \frac{S_x(\tilde{\alpha})}{2}[(1-\delta)(a+b) + \delta(c+d)]$$

The accurate expected function of $\tilde{\alpha}$:

$$I\big(H_x(\tilde{\alpha})\big) = \frac{H_x(\tilde{\alpha})}{2}[(1-\delta)(a+b) + \delta(c+d)]$$

Where $S_x(\tilde{\alpha})$ and $H_x(\tilde{\alpha})$ are as definition 4.

The set with the highest score is considered the best. If the scores are equal, compare the sets with accuracy values; the set with lower accuracy will be ranked higher. When both the score and the accuracy are equal, the sets are considered equal.

Definition 6: Value and Ambiguity Index of IVTrIFSs [16]

The value index and ambiguity index of IVTrIFS were defined based on membership values and non-membership values.

Value index of IVTrIFS

The value index for an IVITrFN$\tilde{\alpha} = ([\mathrm{a,b,c,d}]; [m_{\tilde{\alpha}}^{L}, m_{\tilde{\alpha}}^{U}]; [n_{\tilde{\alpha}}^{L}, n_{\tilde{\alpha}}^{U}])$ is given as

$$V(\tilde{\alpha}) = kS_x(\tilde{\alpha}) + (1-k)(1 - H_x(\tilde{\alpha}))$$

Where $k \in [0,1]$, $S_x(\tilde{\alpha})$, $H_x(\tilde{\alpha})$are score and accuracy functions respectively.

Ambiguity index of IVTrFS

The ambiguity index for an IVITrFN$\tilde{\alpha} = ([\mathrm{a,b,c,d}]; [m_{\tilde{\alpha}}^{L}, m_{\tilde{\alpha}}^{U}]; [n_{\tilde{\alpha}}^{L}, n_{\tilde{\alpha}}^{U}])$ is given as

$$V(\tilde{\alpha}) = \frac{(d-a) - 2(b-c)}{6}(1 + S_x(\tilde{\alpha}) - H_x(\tilde{\alpha}))$$

Where $k \in [0,1]$, $S_x(\tilde{\alpha})$, $H_x(\tilde{\alpha})$are score and accuracy functions respectively

Definition 7: Jaccard distance [17]

The Jaccard distance between two sets is given by

$$d_{JD}(A_1, A_2) = 1 - J_{SI}(A_1, A_2)$$

where $J_{SI}(A_1, A_2) = \frac{|A_1 \cap A_2|}{|A_1 \cup A_2|}$ is the Jaccard similarity index.

3. Proposed VIKOR method for IVTrIFSs information

In this section, we define the union and intersection of IVTrIFSs and Jaccard distance between two IVTrIFSs. Further, an extended VIKOR approach for IVTrIFSs is presented.

Definition 8: Set operations on IVTrIFNs

Let $\widetilde{\alpha_1} = ([a_1, b_1, c_1, d_1]; [m_{\widetilde{\alpha_1}}^{L}, m_{\widetilde{\alpha_1}}^{U}]; [n_{\widetilde{\alpha_1}}^{L}, n_{\widetilde{\alpha_1}}^{U}])$

and $\widetilde{\alpha_2} = ([a_2, b_2, c_2, d_2]; [m_{\widetilde{\alpha_2}}^{L}, m_{\widetilde{\alpha_2}}^{U}]; [n_{\widetilde{\alpha_2}}^{L}, n_{\widetilde{\alpha_2}}^{U}])$ be two IVTrIFNs, then

$$\widetilde{\alpha_1} \cup \widetilde{\alpha_2} = \begin{pmatrix} [\max(a_1, a_2), \max(b_1, b_2), \max(c_1, c_2), \max(d_1, d_2)]; \\ [\max\left(m^L_{\widetilde{\alpha_1}}, m^L_{\widetilde{\alpha_2}}\right), \max\left(m^U_{\widetilde{\alpha_1}}, m^U_{\widetilde{\alpha_2}}\right), \min\left(n^L_{\widetilde{\alpha_1}}, n^L_{\widetilde{\alpha_2}}\right), \min\left(n^U_{\widetilde{\alpha_1}}, n^U_{\widetilde{\alpha_2}}\right) \end{pmatrix}$$

$$\widetilde{\alpha_1} \cap \widetilde{\alpha_2} = \begin{pmatrix} [\min(a_1, a_2), \min(b_1, b_2), \min(c_1, c_2), \min(d_1, d_2)]; \\ [\min\left(m^L_{\widetilde{\alpha_1}}, m^L_{\widetilde{\alpha_2}}\right), \min\left(m^U_{\widetilde{\alpha_1}}, m^U_{\widetilde{\alpha_2}}\right), \max\left(n^L_{\widetilde{\alpha_1}}, n^L_{\widetilde{\alpha_2}}\right), \max\left(n^U_{\widetilde{\alpha_1}}, n^U_{\widetilde{\alpha_2}}\right) \end{pmatrix}$$

Definition 9: Jaccard Distance on IVTrIFNs

Let $\widetilde{\alpha_1}$ and $\widetilde{\alpha_2}$ be any two IVTrIFNs then the Jaccard distance between $\widetilde{\alpha_1}$ and $\widetilde{\alpha_2}$ is denoted as $d_{JSM}(\widetilde{\alpha_1}, \widetilde{\alpha_2})$ is defined as follows:

$$d_{JD}(\widetilde{\alpha_1}, \widetilde{\alpha_2}) = 1 - \frac{|\widetilde{\alpha_1} \cap \widetilde{\alpha_2}|}{|\widetilde{\alpha_1} \cup \widetilde{\alpha_2}|} \qquad (1)$$

The modulus represents the Euclidean distance d_e from origin{[0,0,0,0],[0,0],[0,0]}.

3.1. IVTrIFS VIKOR method

In this section, an extended VIKOR approach based on the proposed new distance measure on IVTrIFSs is presented. The proposed model involves the following steps to solve IVTrIFSs MCDM problems.

A finite set of 'm' criteria $\{C_1, C_2, \ldots, C_m\}$ for a set of 'n' alternatives $\{A_1, A_2, \ldots, A_n\}$;a set of decision makers $\{dm_1, dm_2, \ldots, dm_r\}$ are assigned for evaluation. The weight vectorsof decision makers and criterion are represented by $(w_{dm_1}, w_{dm_2}, \ldots, w_{dm_r})$ where $w_{dm_l} \geq 0$, $l = 1,2, \ldots$ r such that $\sum_{l=1}^{r} w_{dm_l}$=1, and t = $(w_{C_1}, w_{C_2}, \ldots, w_{C_m})$ where $w_{C_j} \geq 0$, $j = 1,2, \ldots$ m and $\sum_{j=1}^{m} w_{C_j}$=1.

Assume that the decision maker provides the rating values corresponding to each alternative in terms of an IVITrFS. Let $\tilde{p}_{ij}^l$ be the estimation of the alternative A_i, $\mathrm{i} = 1,2, \ldots \mathrm{n}$ for criteria C_j, $\mathrm{j} = 1,2, \ldots \mathrm{m}$ by the decision maker dm_l, $l = 1,2, \ldots r$.

For example, the decision maker l evaluates the alternative A_i under C_j and gives estimate using the IVTrIFS as $([a_{\tilde{p}_{ij}^l}, b_{\tilde{p}_{ij}^l}, c_{\tilde{p}_{ij}^l}, d_{\tilde{p}_{ij}^l}];\ [m^L_{\tilde{p}_{ij}^l}, m^U_{\tilde{p}_{ij}^l}];\ [n^L_{\tilde{p}_{ij}^l}, n^U_{\tilde{p}_{ij}^l}])$. The most possible value is from b to c. The membership degree for the most possible value $[b, c]$ is $[m^L_{\tilde{p}_{ij}^l}, m^U_{\tilde{p}_{ij}^l}]$, non-membership degree is $[n^L_{\tilde{p}_{ij}^l}, n^U_{\tilde{p}_{ij}^l}]$.

Then the following steps are being followed to find the best alternative.

Step 1: Construction of decision matrix.

The performance of each alternative under each criteria is calculated as

$$\tilde{P}_{ij} = \frac{1}{l}(\tilde{p}_{ij}^{1} \oplus \tilde{p}_{ij}^{2} \oplus \ldots \oplus \tilde{p}_{ij}^{l})$$

And the decision matrix is defined as $D = [\tilde{p}_{ij}]_{n\times m}$.

Step 2: Calculate the normalized decision matrix as follows:

$$\tilde{p}_{ij} = \frac{\tilde{P}_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{P}_{ij}^{2}}}$$

Step 2: Identify the positive ideal solution (PIS) $\tilde{p}_j^+$ and the negative ideal solution (NIS) $\tilde{p}_j^-$, $j = 1,2, \ldots m$ for all the criteria.

If the criteria *j* represents a benefit, then

$$\tilde{p}_j^+ = \begin{pmatrix} \left[\max_i a_{\tilde{p}_{ij}}, \max_i b_{\tilde{p}_{ij}}, \max_i c_{\tilde{p}_{ij}}, \max_i d_{\tilde{p}_{ij}}\right]; \\ \left[\max_i m_{\tilde{p}_{ij}}^L, \max_i m_{\tilde{p}_{ij}}^U\right]; \left[\min_i n_{\tilde{p}_{ij}}^L, \min_i n_{\tilde{p}_{ij}}^U\right] \end{pmatrix}$$

$$\tilde{p}_j^- = \begin{pmatrix} \left[\min_i a_{\tilde{p}_{ij}}, \min_i b_{\tilde{p}_{ij}}, \min_i c_{\tilde{p}_{ij}}, \min_i d_{\tilde{p}_{ij}}\right]; \\ \left[\min_i m_{\tilde{p}_{ij}}^L, \min_i m_{\tilde{p}_{ij}}^U\right]; \left[\max_i n_{\tilde{p}_{ij}}^L, \max_i n_{\tilde{p}_{ij}}^U\right] \end{pmatrix}$$

If the criteria *j* represents a cost, then

$$\tilde{p}_j^+ = \begin{pmatrix} \left[\min_i a_{\tilde{p}_{ij}}, \min_i b_{\tilde{p}_{ij}}, \min_i c_{\tilde{p}_{ij}}, \min_i d_{\tilde{p}_{ij}}\right]; \\ \left[\min_i m_{\tilde{p}_{ij}}^L, \min_i m_{\tilde{p}_{ij}}^U\right]; \left[\max_i n_{\tilde{p}_{ij}}^L, \max_i n_{\tilde{p}_{ij}}^U\right] \end{pmatrix}$$

$$\tilde{p}_j^- = \begin{pmatrix} \left[\max_i a_{\tilde{p}_{ij}}, \max_i b_{\tilde{p}_{ij}}, \max_i c_{\tilde{p}_{ij}}, \max_i d_{\tilde{p}_{ij}}\right]; \\ \left[\max_i m_{\tilde{p}_{ij}}^L, \max_i m_{\tilde{p}_{ij}}^U\right]; \left[\min_i n_{\tilde{p}_{ij}}^L, \min_i n_{\tilde{p}_{ij}}^U\right] \end{pmatrix}$$

Step 3: Calculate the values of group utility S_i and indivisible regret R_i values for

$i = 1,2,..n.$

$$S_i = \sum_{j=1}^{m} \frac{w_{C_j} d_{JD}(\tilde{p}_j^+ - \tilde{p}_{ij})}{d_{JD}(\tilde{p}_j^+ - \tilde{p}_j^-)}$$

$$R_i = \max_{\mathrm{j}} \frac{w_{C_j} d_{JD}(\tilde{p}_j^+ - \tilde{p}_{ij})}{d_{JD}(\tilde{p}_j^+ - \tilde{p}_j^-)}$$

The smaller values of S_i and R_i represents to the better average and the worse group scores respectively.

Step 4: Determine the VIKOR index value Q_i values for $each i = 1,2, \ldots n$

$$Q_i = \frac{v(S_i - S^*)}{(S^- - S^*)} + \frac{(1-v)(R_i - R^*)}{(R^- - R^*)}$$

Where,$S^* = \min_i S_i$,$S^- = \max_i S_i$, $R^* = \min_i R_i$, $R^- = \max_i R_i$ and v is the weight of the decision maker strategy. Here suppose that $v = 0.5$.

Step 5: RankA_i , by ordering the values S_i, R_i and Q_i in decreasing order.

The alternative that has minimum Q_i is the best alternative.

1. Numerical example

In this section, the proposed extended VIKOR method is applied to solve the problem a real-world problem from the Wu and Liu [10]. The problem is to choose the best green supplier. The company selected three suppliers (alternatives) for evaluation under four criteria:

Product quality ($C1$)

Technology capability ($C2$)

Pollution control ($C3$)

Environmental management ($C4$).

Weights of each criterion are given as:

$$W_{C_1} = ([0.3\,, 0.4\,, 0.5\,, 0.6]; [0.3\,, 0.5], [0.1\,, 0.2])$$

$$W_{C_2} = ([0.3\,, 0.4\,, 0.5\,, 0.6]; [0.4\,, 0.5], [0.3\,, 0.4])$$

$$W_{C_3} = ([0.2\,, 0.4\,, 0.5\,, 0.6]; [0.4\,, 0.6], [0.2\,, 0.4])$$

$$W_{C_4} = ([0.4\,, 0.5\,, 0.7\,, 0.8]; [0.3\,, 0.4], [0.2\,, 0.4])$$

The decision makers dm_1,dm_2, and dm_3, are chosen from three different departments namely: production, purchasing, and quality inspection. Assessments of the three suppliers by decision makers based on each criterion are given in the form of IVTrIFSs.

Defuzzification of criteria weights

The defuzzified values of W_{C_j} are calculated using Score expected function (Definition 6) and are given as:

$$W_{C_1} = 0.113$$

$$W_{C_2} = 0.045$$

$$W_{C_3} = 0.085$$

$$W_{C_4} = 0.03$$

Then the normalized weights are calculated using $w_{C_j} = \frac{W_{C_j}}{\sum_{j=1}^{4} W_{C_j}}$

The normalized weights for criteria are computed and presented as:

$$w_{C_1} = 0.413$$

$$w_{C_2} = 0.165$$

$$w_{C_3} = 0.312$$

$$w_{C_4} = 0.11$$

Step 1: The performance of each alternative is calculated and the decision matrix is constructed, given in table 1[10].

Table 1: Decision matrix

Criteria	A_1	A_2	A_3
C_1	([0.4 , 0.5 , 0.6 , 0.7]; [0.4,0.5],[0.3,0.4])	([0.36 , 0.5 , 0.6 ,0.73]; [0.3 , 0.6],[0.3 , 0.4])	([0.43 , 0.56 , 0.66 , 0.76]; [0.3 , 0.4],[0.5 , 0.6])
C_2	([0.33 , 0.46 , 0.6 , 0.73]; [0.2 , 0.4],[0.2 , 0.4])	([0.33 , 0.46 , 0.6 , 0.8]; [0.4 , 0.5],[0.3 , 0.4])	([0.2 , 0.33,0.43 , 0.53]; [0.4 , 0.5],[0.2 , 0.5])
C_3	([0.33 , 0.46 , 0.56 , 0.7]; [0.3 , 0.4],[0.3 , 0.4])	([0.23 , 0.36 , 0.5 , 0.66]; [0.4 , 0.6],[0.2 , 0.4])	([0.2 , 0.33 , 0.43 , 0.53]; [0.4 , 0.5],[0.3 , 0.4])
C_4	([0.23 , 0.36 , 0.5 , 0.63]; [0.5 , 0.6],[0.2 , 0.4])	([0.4 , 0.56 , 0.66 , 0.8]; [0.3 , 0.4],[0.1 , 0.4])	([0.33 , 0.43 , 0.53 , 0.66]; [0.2 , 0.5],[0.3 , 0.5])

Step 2: The normalized decision matrix is calculated and presented in Table 2:

Table 2: Normalized decision matrix

Criteria	A_1	A_2	A_3
C_1	([0.35 , 0.44 , 0.53 , 0.62]; [0.4 , 0.5],[0.3 , 0.4])	([0.31 , 0.44 , 0.53 , 0.64]; [0.3 , 0.6],[0.3 , 0.4])	([0.34 , 0.45 , 0.53 , 0.61]; [0.3 , 0.4],[0.5 , 0.6])
C_2	([0.29 , 0.41 , 0.54 , 0.66]; [0.2 , 0.4],[0.2 , 0.4])	([0.28 , 0.41 , 0.52 , 0.69]; [0.4 , 0.5],[0.3 , 0.4])	([0.25 , 0.42 , 0.54 , 0.67]; [0.4 , 0.5],[0.2 , 0.5])
C_3	([0.31 , 0.43 , 0.52 , 0.66]; [0.3 , 0.4],[0.3 , 0.4])	([0.24 , 0.38 , 0.53 , 0.70]; [0.4 , 0.6],[0.2 , 0.4])	([0.25 , 0.42 , 0.54 , 0.67]; [0.4 , 0.5],[0.3 , 0.4])
C_4	([0.25 , 0.39 , 0.54 , 0.69]; [0.5 , 0.6],[0.2 , 0.4])	([0.32 , 0.44 , 0.53 , 0.64]; [0.3 , 0.4],[0.1 , 0.4])	([0.32 , 0.42 , 0.52 , 0.65]; [0.2 , 0.5],[0.3 , 0.5])

Step 2: The positive ideal solution (PIS) $\tilde{p}_j^+ j = 1,2,3,4$ with respect to each criteria is identified as below:

$$\tilde{p}_1^+ = ([0.35 ,0.45, 0.53, 0.64]; [\ 0.4 ,0.6]; [0.3,0.4])$$

$$\tilde{p}_2^+ = ([0.29 ,0.42, 0.54, 0.69]; [\ 0.4 ,0.5]; [0.2,0.4])$$

$$\tilde{p}_3^+ = ([0.31 ,0.43, 0.54, 0.7]; [\ 0.4 ,0.6]; [0.2,0.4])$$

$$\tilde{p}_4^+ = ([0.32 ,0.44, 0.54, 0.7]; [\ 0.5 ,0.6]; [0.1,0.4])$$

and the negative ideal solution (NIS) $\tilde{p}_j^-$ with respect to each criteria is

$$\tilde{p}_1^- = ([0.31 ,0.44, 0.53, 0.61]; [\ 0.3 ,0.4]; [0.5,0.6])$$

$$\tilde{p}_2^- = ([0.25 ,0.41, 0.52, 0.66]; [\ 0.2 ,0.4]; [0.3,0.5])$$

$$\tilde{p}_3^- = ([0.24 ,0.38, 0.52, 0.66]; [\ 0.3 ,0.4]; [0.3,0.4])$$

$$\tilde{p}_4^- = ([0.25 ,0.39, 0.52, 0.65]; [\ 0.2 ,0.5]; [0.3,0.5])$$

Step 3: The values of group utility S_i and indivisible regret R_i values for $i = 1,2,3$ are calculated using eq. (1) and given in table 3.

For example: $S_1 = \sum_{j=1}^{4} \frac{w_{C_j} d_{JD}\left(\tilde{p}_j^+, \tilde{p}_{ij}\right)}{d_{JD}\left(\tilde{p}_j^+, \tilde{p}_j^-\right)}$

$$= \frac{w_{C_1} d_{JD}(\tilde{p}_1^+, \tilde{p}_{11})}{d_{JD}(\tilde{p}_1^+, \tilde{p}_1^-)} + \frac{w_{C_2} d_{JD}(\tilde{p}_2^+, \tilde{p}_{12})}{d_{JD}(\tilde{p}_2^+, \tilde{p}_2^-)} + \frac{w_{C_3} d_{JD}(\tilde{p}_1^+, \tilde{p}_{13})}{d_{JD}(\tilde{p}_3^+, \tilde{p}_3^-)} + \frac{w_{C_4} d_{JD}(\tilde{p}_4^+, \tilde{p}_{14})}{d_{JD}(\tilde{p}_4^+, \tilde{p}_4^-)}$$

Finding $d_{JD}(\tilde{p}_1^+, \tilde{p}_{11}) = 1 - \frac{|\tilde{p}_1^+ \cap \tilde{p}_{11}|}{|\tilde{p}_1^+ \cup \tilde{p}_{11}|}$

$\tilde{p}_1^+ \cap \tilde{p}_{11} = ([0.35,0.44,0.53,0.62]; [0.4,0.5], [0.3,0.4])$and

$$\tilde{p}_1^+ \cup \tilde{p}_{11} = [0.35,0.45,0.53,0.64]; [0.4,0.6], [0.3,0.4])$$

Then $|\tilde{p}_1^+ \cap \tilde{p}_{11}| = d_e(\tilde{p}_1^+ \cap \tilde{p}_{11}, N) = 0.098$ where $N = ([0,0,0,0]; [0,0], [0,0])$

And $|\tilde{p}_1^+ \cup \tilde{p}_{11}| = d_e(\tilde{p}_1^+ \cup \tilde{p}_{11}, N) = 0.114$

Then $d_{JD}(\tilde{p}_1^+, \tilde{p}_{11}) = 0.1439$

Similarly, $d_{JD}(\tilde{p}_1^+, \tilde{p}_1^-) = 0.3412$

$$\frac{w_{C_1} d_{JD}(\tilde{p}_1^+, \tilde{p}_{11})}{d_{JD}(\tilde{p}_1^+, \tilde{p}_1^-)} = \frac{0.413 * 0.1439}{0.3412} = 0.174$$

Similarly we have,

$\frac{w_{C_2} d_{JD}(\tilde{p}_2^+, \tilde{p}_{12})}{d_{JD}(\tilde{p}_2^+, \tilde{p}_2^-)} = 0.074; \frac{w_{C_3} d_{JD}(\tilde{p}_3^+, \tilde{p}_{13})}{d_{JD}(\tilde{p}_3^+, \tilde{p}_3^-)} = 0.26$ and $\frac{w_{C_4} d_{JD}(\tilde{p}_4^+, \tilde{p}_{14})}{d_{JD}(\tilde{p}_4^+, \tilde{p}_4^-)} = 0.063$

$$S_1 = 0.174 + 0.074 + 0.26 + 0.063 = 0.571$$

Table 3: Utility measure and Regret measure

Alternatives $\boldsymbol{A_i}$	**Utility measure** $\boldsymbol{S_i}$	**Regret measure** $\boldsymbol{R_i}$
A_1	0.571	0.26
A_2	0.172	0.067
A	0.852	0.509

Step 4: The VIKOR index Q_i values are calculated for $i = 1,2, \dots n$

$S^* = 0.172, S^- = 0.852$, $R^* = 0.067$, $R^- = 0.509$ and $v = 0.5$

Table 4: VIKOR index

Alternatives $\boldsymbol{A_i}$	**VIKOR index** $\boldsymbol{Q_i}$
A_1	0.5109
A_2	0
A_3	1

Step 5: Rank the alternatives by sorting the values S_i, R_i and Q_i in decreasing order.

Among the given alternatives, $A_2$is the option with the lowest Q_i. Hence, A_2 is the best alternative and is selected as a compromise solution. The order of precedence is $A_2 > A_1 > A_3$.

The results show that the developed method is strictly ranking the alternatives when compared to Wu and Liu [10] and is coinciding with results of Sireesha and Himabindu[16].

Conclusion

The information involving in decision making process is incomplete and uncertain often. Therefore, it is difficult to assert the accuracy about the performance of the alternatives, and hence it is suitable to assess them as IVTrIFSs. Hence, in this paper we extended the VIKOR with IVTrIFSs using a new distance Jaccard on IVTrIFSs. The group utility S_i and indivisible regret R_i are calculated using Jaccard distance. A problem of selecting best supplier problem is solved using the proposed method to demonstrate the effectiveness and feasibility of the proposed IVTrIF-VIKOR method. Further the result analysis is done comparing with existing methods. It is observed that the proposed method is effectively ranking the alternatives compared to some of the existing methods. This proves that the proposed method can effectively solve problems with inadequate and ambiguous information, making it applicable to all decision-making issues.

References

1. Alkafaas S.S, Fattouh M, Masoud R, Nada O (2020) “Intuitionistic Fuzzy VIKOR Method for Facility Location Selection Problem” International Journal of Engineering Research & Technology (IJERT), ISSN: 2278-0181, Vol. 9 Issue 08.
2. Atanassov K, Gargov G (1989), “Interval valued intuitionistic fuzzy sets”, Fuzzy Sets and Systems, Volume 31, Issue 3, pp.343-349, 10.1016/0165-0114(89)90205-4.
3. Atanassov KT (1986), “Intuitonistic fuzzy sets,” Fuzzy Sets and Systems, vol. 20, pp. 87–96.
4. Narayanamoorthy S ,Geetha S , Rakkiyappana R , Young Hoon J (2019),” Interval-valued intuitionistic hesitant fuzzy entropy based VIKOR method for industrial robots selection” Expert Systems with Applicationsvol. 121, pp.28-37.
5. Khan M J, Ali M I, Kumam, P, Kumam, W, Aslam, M, Alcantud, JCR(2022), “Improved generalized dissimilarity measure-based VIKOR method for Pythagorean fuzzy sets”, Int J Intell Syst, vol. 37, pp.1807- 1845. doi:10.1002/int.22757.

6.Maoying T and Jing L (2013), "Some aggregation operators with interval valued intuitionistic trapezoidal fuzzy numbers and their application in multiple attribute decision making", AMO—Advanced Model Optimization, Vol. 15, No. 2.

7.Wan S.P(2013), "Power average operators of trapezoidal intuitionistic fuzzy numbers and application to multi-attribute group decision making", Applied Mathematical Modelling, Volume 37, Issue 6, Pages 4112-4126, 10.1016/j.apm.2012.09.017.

8.Dong J and Wan S (2015), "Interval-valued trapezoidal intuitionistic fuzzy generalized aggregation operators and application to multi attribute group decision making", Scientia Iranica E, vol. 22(6), pp. 2702–2715.

9.Wan S P (2011), "Multi-attribute decision making method based on interval-valued intuitionistic trapezoidal fuzzy number," Control and Decision, vol. 26, no. 6, pp. 857–860, 2011.

10.Wu J and Liu Y (2013), "An approach for multiple attribute group decision making problems with interval-valued intuitionistic trapezoidal fuzzy numbers," Computers & Industrial Engineering, vol. 66, no. 2, pp. 311–324.

11.Wei G (2015), "Approaches to interval intuitionistic trapezoidal fuzzy multiple attribute decision making with incomplete weight information", International Journal of Fuzzy Systems, vol. 17(3), pp. 484–489.

12.Hajiheydari N, Mohammad S Delgosha(2020), "Extended Intuitionistic Fuzzy VIKOR Method in Group Decision Making: The Case of Vendor Selection Decision" World Academy of Science, Engineering and Technology International Journal of Social and Business Sciences Vol.14, No.5.

13.Peide Liu, Xiyou Qin (2017), "An Extended VIKOR Method for Decision Making Problem with Interval-Valued Linguistic Intuitionistic Fuzzy Numbers Based on Entropy" Informatica, Vol. 28, No. 4, pp.665–685.

14.Rasim M, Alguliyev, Ramiz M, Aliguliyev, Rasmiyya S, Mahmudova (2015), "Multicriteria Personnel Selection by the Modified Fuzzy VIKOR Method", The Scientific World Journal, vol. 2015, Article ID 612767, 16 pages, https://doi.org/10.1155/2015/612767.

15.Salimian S, Seyed M M , and Jurgita A(2022) "An Interval-Valued Intuitionistic Fuzzy Model Based on Extended VIKOR and MARCOS for Sustainable Supplier Selection in Organ Transplantation Networks for Healthcare Devices" Sustainability 2022, 14, 3795. https://doi.org/10.3390/su14073795.

16.Sireesha V and Himabindu K (2016) "An ELECTRE Approach for Multi-criteria Interval-Valued Intuitionistic Trapezoidal Fuzzy Group Decision Making Problems", Advances in fuzzy systems, Article ID 1956303, 17 pages.

17.Levandowsky M and Winter D (1971), "Distance between sets", Nature, Vol. 234, pp: 34-35.

18.De, P. K., & Das, D. (2014), "A study on ranking of trapezoidal intuitionistic fuzzy numbers", International Journal of Computer information systems and industrial management applications, 6, pp.437-444.

19.Seker S (2020), "A novel interval-valued intuitionistic trapezoidal fuzzy combinative distance-based assessment (CODAS) method", Soft Computing, vol. 24 (3), pp. 2287–2300. DOI: https://doi.org/10.1007/s00500-019-04059-3.

20.Shu M.H, Cheng C.H , and Chang J.R (2006), "Using intuitionisticfuzzy sets for fault-tree analysis on printed circuit board assembly," Microelectronics Reliability, vol. 46,no.12,pp. 2139-2148.

21.Stephen D and FanyE(2020), "Similarity measures on generalized interval-valued trapezoidal intuitionistic fuzzy number", Malaya Journal of Matematik, Vol. S, No. 1, 313-318, https://doi.org/10.26637/MJM0S20/0059.

22. Talib A.M (2020), "Fuzzy VIKOR Approach to Evaluate the Information Security Policies and Analyze the Content of Press Agencies in Gulf Countries", Journal of Information Security, Vol.11 No.4, 10.4236/jis.2020.114013.

23.Wang J.Q (2008), "Overview on fuzzy multi-criteria decision-making approach," Control and Decision, vol.23, pp.601-606.